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Calculating B_K using HYP staggered fermions

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We give an update on our calculation of B_K using HYP-smeared valence staggered quarks. We have results for B_K at tree-level on several coarse MILC lattices ($a \approx 0.12$ fm) and one of the fine lattices ($a \approx 0.09$ fm), using 10 light valence quarks ranging down to $m_s^{\text{phys}}/10$. We have generalized staggered chiral perturbation theory to our mixed action setup, and outline the results. We explain our present fitting strategy, and give some preliminary results.

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1. Introduction

An accurate result for the kaon B-parameter is important both for its phenomenological impact [1] and as a bellwether of the success in incorporating chiral symmetry and controlling systematics in lattice calculations. Calculations using several different fermion methods are underway, with the present best result using domain-wall fermions [2]. We are pursuing a calculation using improved staggered fermions. This has the advantage of being computationally cheap, but the challenge of dealing with the effects of taste-breaking in a context where there is operator mixing [3].

We use the standard staggered action with HYP-smeared links for our valence fermions, which reduces taste-breaking by a factor of 3 compared to asqtad quarks [4, 5]. We use the MILC lattices generated with $2+1$ flavors of asqtad sea quarks. For the coarse MILC lattices, on which we focus here, the resultant taste-breaking in the pion masses, while reduced compared to asqtad quarks, remains large enough that we must use the standard power-counting of staggered chiral perturbation theory, in which $a^2 \approx p^2$. The complications that this introduces have been explained in Ref. [3].

Our method for calculating B_K using wall sources is explained in Ref. [6]. For each lattice we use 10 valence quark masses running from $\approx m_s^{\rm phys}$ down to $\approx m_s^{\rm phys}/10$ in equal steps, and we calculate B_K and m_K for the Goldstone taste for all 55 quark-mass combinations. Results for one-loop matching factors using the mixed action are not yet complete so we use tree-level matching. For this, and other reasons to be explained, all results obtained here should be regarded as very preliminary. They are essentially our first pass at fitting the data, which we are using to inform subsequent fitting and to determine where improvements in statistics are needed.

2. Staggered chiral perturbation theory for a mixed action

Staggered chiral perturbation theory (SχPT) [7, 8] incorporates discretization errors into the chiral expansion, and in particular includes the effects of taste-breaking and rooting. For our lightest kaons, such effects are comparable to those coming from the explicit chiral symmetry breaking due to quark masses, and thus enter SχPT at LO. A major effect in SχPT is that chiral loops (which begin at next-to-leading-order [NLO]) must be evaluated with the masses of the pions of the appropriate tastes, rather than a common mass. For B_K, which involves an insertion of the weak Hamiltonian, one must also deal with mixing between operators having different tastes. This leads to a significantly larger number of unknown coefficients multiplying NLO terms than are present in the continuum, as explained in Ref. [3].

The SχPT analysis of Ref. [3] does not apply directly to our set-up, however, because we use a mixed action. Following the methods developed in other mixed-action contexts [9], we have generalized the results of Ref. [3] to our setup. This turns out to be straightforward. Here we only give a summary—further details will be presented in Ref. [10].

There are three classes of effects resulting from using a mixed action. The first, which corresponds to the short-distance parts of sea-quark loops, is simply that the coefficients multiplying various terms in the SχPT expression for B_K will change. Since these coefficients were previously unknown, however, this has no practical impact—one trades one set of unknown coefficients for another.

The second class comes from loop diagrams involving mixed pions—those composed, say, of a valence (HYP) quark and a sea (asqtad) antiquark. These, however, are absent for B_K at NLO.

The third class involves loop diagrams with pions composed of sea-quarks alone. For B_K at NLO these loops all include "hairpin" vertices, for this is the only way in which the long-distance part of sea-quark loops can enter. The effect of using a mixed action then boils down to the need to distinguish between three types of hairpin vertices—valence-valence, valence-sea and sea-sea—all of which would be the same if the same valence and sea quarks were being used. It turns out that this has an impact only for tastes V and A (the taste singlet hairpin coupling to a particle which is being integrated out anyway), so that one ends up with 6 hairpin parameters (3 for each of two tastes) instead of 2. We name the hairpin parameters $\delta_B'^{vv}$, $\delta_B'^{vs}$ and $\delta_B'^{ss}$, with $B = A$ or V, and the superscript indicating the types of quark involved. They appear at NLO in the combinations

$$\delta_B^{MA1} = (\delta_B'^{vs})^2/\delta_B'^{ss}, \quad \delta_B^{MA2} = \left[\delta_B'^{vv}\delta_B'^{ss} - (\delta_B'^{vs})^2\right]/\delta_B'^{ss}. \tag{2.1}$$

These only enter in the expressions for non-degenerate quarks—those for degenerate quarks in the kaon do not involve hairpin vertices. Specifically, in the contribution denoted $\mathscr{M}^{PQ}_{\rm disc}$ in Ref. [3], and given in eq. (50) of that paper, one must, for both $B = V$ and A, make the substitution

$$\delta_B' \to \delta_B^{MA1}, \tag{2.2}$$

and add the following new term

$$a^2\delta_B^{MA2}\frac{2C_\chi^{2B} + C_\chi^{3B}}{\pi^2 f^4}\left(2\frac{\ell(Y_B) - \ell(X_B)}{Y_B - X_B} + \widetilde{\ell}(X_B) + \widetilde{\ell}(Y_B)\right). \tag{2.3}$$

Here we use the notation X_B (Y_B) for the mass-squared of the flavor non-singlet valence pion with taste B and composition $\bar{x}x$ ($\bar{y}y$), where the kaon itself has the composition $\bar{x}y$ (and taste P). In the notation of Ref. [3] $X_B = m^2_{X_B}$ and $Y_B = m^2_{Y_B}$. The functions ℓ and $\widetilde{\ell}$ are chiral logarithms, and are defined in Ref. [3].

The original hairpins, $\delta_B'^{ss}$, enter through their (unchanged) contributions to the masses of the flavor-singlet mesons, η_B and η_B'.

The hairpin parameters are a measure of taste-symmetry breaking, and we expect them to satisfy a similar hierarchy to that we observe in the pion spectrum, namely $\delta_B'^{vv}/\delta_B'^{ss} \approx 1/3$. In words, hairpins correspond to quark-antiquark pairs communicating through intermediate gluons, and the taste-breaking component is, by construction, reduced for our HYP-smeared valence quarks. Based on this argument we also expect that $(\delta_B'^{vs}/\delta_B'^{ss})^2 \approx 1/3$. Combining these expectations we find

$$\delta_B^{MA1} \approx \delta_B'^{ss}/3 \quad \text{and} \quad \delta_B^{MA2} \approx 0. \tag{2.4}$$

If these expectations are accurate, then using a mixed action has essentially no impact on chiral fitting, because the new terms proportional to δ_B^{MA2} [eq. (2.3)] can be dropped, and the terms proportional to δ_B^{MA1} were present anyway with an unknown coefficient.

3. Fitting strategy

The NLO expression for B_K in partially-quenched mixed-action SχPT takes the form

$$B_K = \sum_{i=1}^{16} c_i f_i, \tag{3.1}$$

in which f_i are known functions and c_i are coefficients to be determined. Of the 16 unknown coefficients, 1 (c_0) appears at LO (and is the value of B_K when $a \to 0$ and then the chiral limit is taken), 4 are the NLO low-energy constants (LECs) present in the continuum, and the remaining 11 are LECs due to lattice artifacts and truncated perturbative matching. For degenerate quarks these numbers reduce to 9 coefficients $= 1$ LO $+ 3$ continuum LECs $+ 5$ "lattice LECs". This counting is for a single lattice spacing—the dependence of the artifacts on a involves a mix of a^2, α^2 and $\alpha^2 a^2$ [3].

The functions f_i depend on the masses of the flavor-non-singlet valence pions of all tastes and all compositions ($\bar{x}x$, $\bar{y}y$ and $\bar{x}y$), which we determine as part of our calculation [5]. In addition, we need the masses of the sea-quark $\bar{\ell}\ell$ and $\bar{s}s$ flavor non-singlet pions with taste V, A and I, and of the flavor singlet η_B and η'_B for $B = V$ and A. These we take from the results of the MILC collaboration (including the axial and vector hairpin vertices δ'^{ss}_B) [11]. The final inputs we need are a (we use the MILC values) and f (which, for the moment, we simply set to $f = 132\,\text{MeV}$).

Although we have 10 degenerate and 45 non-degenerate data points on each lattice, a direct fit to eq. (3.1) is difficult, since many of the fit functions are similar. As a first stage, therefore, we use only a few representative "lattice" contributions, while keeping all the continuum terms. We also use Bayesian priors to constrain some terms, since we know their orders of magnitude.

The fit functions we use are, firstly,

$$f_1 = 1 + \frac{3}{8f^2G}[\mathscr{M}_{conn} + \mathscr{M}_{disc}]^{\text{continuum}}\,, \tag{3.2}$$

where $G = m^2_{xy,P}$, $\mathscr{M}_{conn,disc}$ come from quark connected/disconnected 1-loop diagrams and are given in Ref. [3], and the superscript indicates that we keep only the contributions from chiral operators present in the continuum. We do, however, include taste breaking in the pion masses in these contributions. We set the scale in the chiral logarithm to $\mu = 1\text{GeV}$. Next we include the continuum analytic terms (with the χPT scale set to $\Lambda = 1\,\text{GeV}$):

$$f_2 = G/\Lambda^2\,, \qquad f_3 = (G/\Lambda^2)^2\,, \qquad f_5 = (X_P - Y_P)^2/(G\Lambda^2)\,,$$

Note that f_3 is a NNLO contribution, but is needed to fit our data up to the highest quark masses.

Finally, we include three representative discretization terms. We use the contribution to $\mathscr{M}_{conn}$ containing taste-T pions, which contains two parts with independent coefficients [3]:

$$f_6 = \frac{3}{8fG}\left(\ell(X_T) + \ell(Y_T) - 2\ell(m^2_{xy,T})\right) \tag{3.3}$$

and the remainder which we call f_4. We also include the likely dominant contribution to $\mathscr{M}_{disc}$, which is that proportional to δ_A^{MA1}. We set $c_7 = (2C_\chi^{2A} + C_\chi^{3A})a^2\delta_A^{MA1}/(\pi^2 f^4)$, and f_7 is the coefficient of this term in eq. (50) of Ref. [3]. Note that $f_{5,6,7}$ contribute only for non-degenerate quarks.

Based on the power-counting of Ref. [3], the coefficients should have the magnitudes:

$$\begin{aligned} &c_1 \approx c_2 \approx c_3 \approx c_5 \sim \mathscr{O}(1) \\ &c_4 \approx c_6 \approx \Lambda^2_{QCD}(a\Lambda_{QCD})^2 = 0.003 \quad \text{to} \quad \Lambda^2_{QCD}\alpha_s^2 = 0.01\ \text{GeV}^2 \text{ on the coarse lattices.} \\ &c_7 \approx \Lambda^4_{QCD}(a\Lambda_{QCD})^2 = 0.0003 \quad \text{to} \quad \Lambda^4_{QCD}\alpha_s^2 = 0.001\ \text{GeV}^4 \text{ on the coarse lattices.} \end{aligned} \tag{3.4}$$

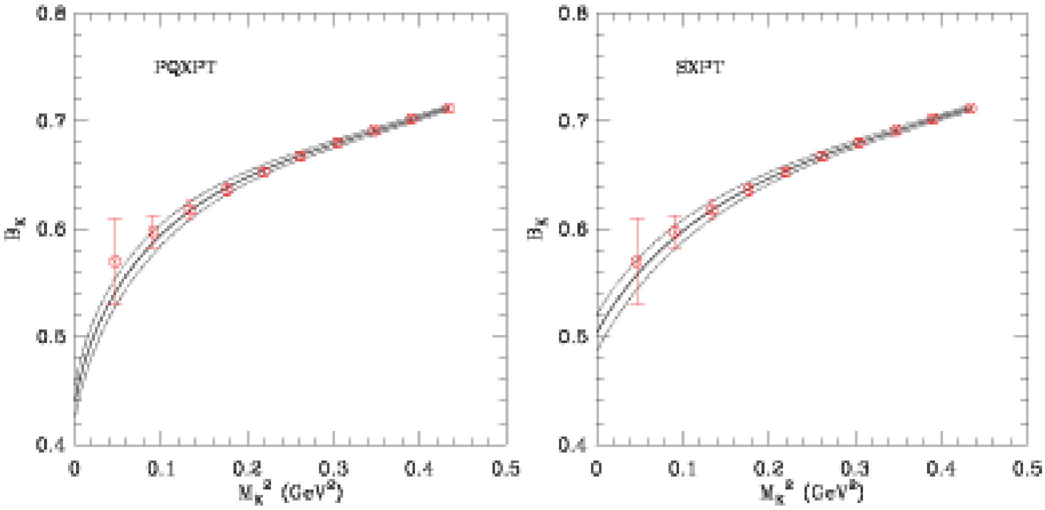

Figure 1: Fits of B_K (with degenerate quarks) vs. m_K^2 on the MILC coarse lattices with $am_\ell = 0.01$ and $am_s = 0.05$. The fits are to continuum PQχPT (left) and SχPT (right), as described in the text.

Here, we use $\Lambda_{QCD} \approx 0.3$ GeV and $\alpha_s = \alpha_{\overline{MS}}(\mu = 1.6GeV) \approx 0.36$. The smallness of $c_{4,6,7}$ is somewhat offset by the fact that $f_{4,6,7}$ are logarithmically divergent in the chiral limit.

4. Examples of fits

We first try fitting to the degenerate data without the lattice terms, so that only c_{1-3} are non-zero. We compare using only the Goldstone-kaon mass in loops ("PQχPT" fit) to using the appropriate combination of all tastes ("SχPT"). The resulting fits are shown in Fig. 1.[1] The SχPT fit is significantly better, because its smaller curvature at small quark masses more accurately represents our data. This is the familiar "softening" of the chiral logarithms caused by the heavier masses of non-Goldstone taste pions.

fit type	c_1	c_2	c_3	c_4	c_5	c_6	c_7	χ^2/dof
D-T4	0.39(1)	$-.01(1)$	0.89(17)					.05(11)
D-BT4	0.390(3)	$-.011(7)$	0.90(4)	.0001(4)				.07(16)
ND-BT4	0.390(2)	$-.011(1)$	0.90(2)	.0001(2)	.13(7)	$-.011(7)$	.0018(8)	.06(7)
ND-T2	0.31(7)	.6(5)	0.23(50)	.002(2)	.04(4)	$-.005(4)$	.0010(5)	.03(2)

Table 1: Fitting parameters. Fits are described in the text.

The degenerate data itself shows no indication of a logarithmically divergent contribution (as would be produced by f_4). In a first attempt to quantify this, we try and make use of the fact that the dominant contribution from f_4 is for the lightest few kaon masses. Thus we drop the lightest mass point from the SχPT degenerate fit, giving a fit we name "D-T4", whose parameters we list in Table 1. They have the expected magnitudes (except perhaps for c_2, but this is scale dependent,

[1] We fit using an uncorrelated χ^2, with errors in fit parameters determined by jackknife. Thus only the relative goodness of fit can be estimated, but not the absolute goodness of fit.

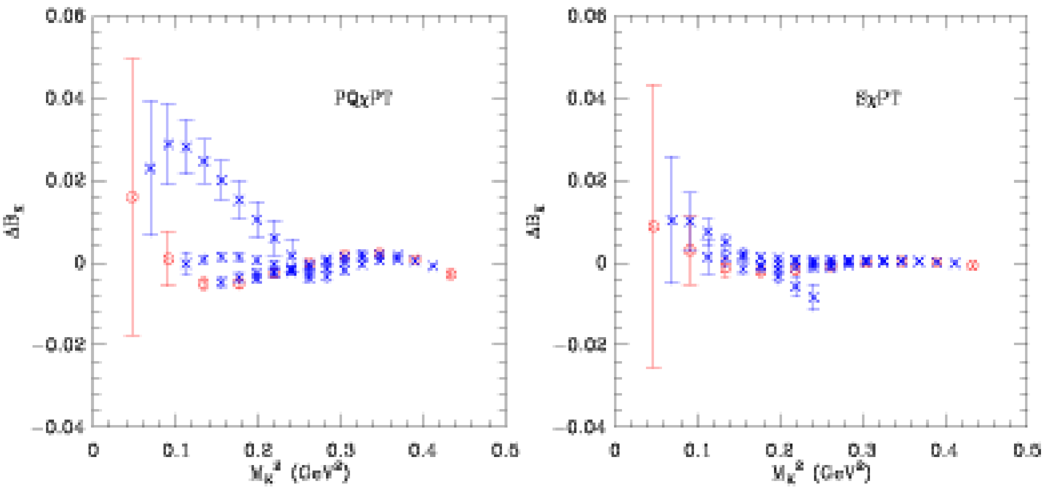

Figure 2: ΔB_K vs. m_K^2, including degenerate (red) and non-degenerate (blue) data.

and becomes ≈ 0.4 if $\mu = 0.77\,\text{GeV}$ instead of $\mu = 1\,\text{GeV}$). We then do a fit to all 10 degenerate points including f_4, but with c_{1-3} constrained, using the values and errors from D-T4 as Bayesian priors [12]. This fit ("D-BT4" in Table 1), has a very small c_4, with the f_4 term making no more than a 1% contribution. Finally, we fit to the full data set (55 points) now including f_{5-7}, but with c_{1-4} constrained using the results from fit D-BT4. The resulting fit we call "ND-BT4". We have also done an unconstrained fit using all 7 f_i to all 55 data points—fit "ND-T2" in the Table.

The difference between fits ND-BT4 and ND-T2 indicates the size of the present uncertainty in the coefficients. While this is substantial, it is encouraging that the coefficients have sizes roughly consistent with the estimates (3.4).[2] Furthermore, when we use the fit form to determine our best estimate for the continuum B_K for physical kaon masses, we obtain consistent values: 0.76(6) and 0.67(4).[3] It is also encouraging that the overall fits looks reasonable. We illustrate this in Fig. 2, where we compare the residuals, $\Delta B_K(x) = B_K(x) - f(x)$ for a continuum PQχPT fit ($c_{4,6,7} = 0$, Goldstone kaon masses only) to the SχPT fit (ND-BT4). We notice a significant improvement in the fitting quality using SχPT.

We have repeated this analysis on four other coarse MILC ensembles and one fine ensemble. Some results are collected in Table 2. All we can conclude at this stage is that there is a rough consistency between different coarse ensembles, and that discretization errors are not enormous. We stress again that these results use tree-level matching and so are very preliminary.

5. Conclusion

We have taken the first stab at fitting our mixed-action results for B_K. We clearly have a lot of work to do to control the systematic errors (and at this stage cannot quote a continuum result with all errors estimated). It is important to keep in mind, however, that the main goal of

[2] The relatively large size of c_7 in fit ND-BT4 is possible, in part, because of a cancellation with the c_6 contribution, and it may be better to directly constrain both these coefficients to have smaller magnitudes.

[3] These values are obtained by setting $c_{4,6,7} = 0$. We stress, however, that they still contain those discretization errors that are absorbed into c_1, since they are based on fits at a single lattice spacing.

a (fm)	am_l/am_s	geometry	ens	B_K(tree, ND-BT4)	B_K(tree, ND-T2)
0.12	0.03/0.05	$20^3 \times 64$	564	0.83(6)	0.63(4)
0.12	0.02/0.05	$20^3 \times 64$	486	0.72(6)	0.71(4)
0.12	0.01/0.05	$20^3 \times 64$	671	0.76(7)	0.67(4)
0.12	0.007/0.05	$20^3 \times 64$	651	0.87(5)	0.62(3)
0.12	0.005/0.05	$24^3 \times 64$	509	0.80(4)	0.66(3)
0.09	0.0062/0.031	$28^3 \times 96$	995	0.72(4)	0.62(3)

Table 2: Comparison of tree-level B_K from different ensembles (preliminary)

the fitting is to provide a reasonable extrapolation formula to the physical quark masses. We are less interested in the coefficients themselves (except for c_0, which can be compared to results from large N_c approaches). We are also investigating fits based on (mixed-action staggered) $SU(2)$ chiral perturbation theory, which appear to be much simplified because discretization terms are of NNLO. We also expect that the more extensive data on the fine lattices which we are presently collecting should be more straightforward to fit. Finally, we are improving our statistics on several of the coarse and fine ensembles.

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