B and D **Meson Decay Constants**

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We present an update of our calculations of the decay constants of the D, D_s, B, and B_s mesons in unquenched $2+1$ flavor QCD. We use the MILC library of improved staggered gauge ensembles at lattice spacings 0.09, 0.12, and 0.15 fm, clover heavy quarks with the Fermilab normalizations, and improved staggered light valence quarks.

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1. Introduction

In 2005, combined work by the Fermilab Lattice and MILC Collaborations [1] determined the value of the D_s decay constant f_{D_s} to around 10% before it had been determined to that accuracy by experiment. When the subsequent experimental determination agreed to within one sigma, we claimed that as a successful prediction. As lattice calculations become increasingly accurate, of course, at some point we do not expect perfect agreement between the Standard Model and experiment. With sufficient precision, the effects of Beyond-the-Standard-Model physics will start to show up in low energy measurements. We do not know what that precision will be, so we must be cautious in interpreting deviations between theory and experiment.

Since then, we have increased the precision of our calculations. Our result for f_{D_s} remains about 10% below the experimental result, and with the increased precision of theory and experiment, no longer agrees to within one sigma with experiment, as we describe in this paper. Further, earlier this year new results on the π, K, D, and D_s decay constants appeared from the HPQCD Collaboration [2]. They used a new lattice fermion method, Highly Improved Staggered Quarks, or "HISQ" fermions, which allowed them to calculate all four decay constants with nearly identical methods. They found very good agreement with experiment for the π and K decay constants. Their value for the D decay constant was subsequently confirmed by CLEO [3]. For f_{D_s}, they also found a result around 10% below experiment, but with improved precision. Instead of agreement between theory and experiment, there is now a greater than three sigma discrepancy. This is the only quantity in lattice QCD phenomenology with staggered fermions in which such a clear disagreement has arisen between theory and experiment, so a puzzle has developed. Precise calculations with other lattice methods are of great interest.

2. Calculations

We are finishing a reanalysis of the existing data for our calculations of f_D, f_{D_s}, f_B, and f_{B_s} that is reducing some of our largest uncertainties. We are also preparing for new runs this year with four times the statistics. Our calculations are done with improved staggered ("asqtad") light quarks [4, 5], and clover/Fermilab [6] $\mathcal{O}(a)$ improved heavy quarks. We use the MILC $2+1$ flavor library of unquenched gauge configurations [7], with lattice spacings of around 0.15, 0.12, and 0.09 fm (the so-called coarser, coarse, and fine ensembles). The masses of light sea-quarks range between $0.6m_s$ and $0.1m_s$. On each of the eleven ensembles, we use from eight to twelve partially quenched valence quark masses, ranging from around m_s to $0.1m_s$.

The decay constants are defined by

$$\langle 0|A_\mu|H_q(p)\rangle = if_{H_q}p_\mu. \tag{2.1}$$

The combination decay amplitude

$$\phi_{H_q} = f_{H_q}\sqrt{M_{H_q}} \tag{2.2}$$

can be obtained from the correlators

$$C_0(t) = \left\langle O_{H_q}^\dagger(t)O_{H_q}(0)\right\rangle, \tag{2.3}$$

$$C_{A_4}(t) = \left\langle A_4(t)O_{H_q}(0)\right\rangle. \tag{2.4}$$

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The current normalizations are obtained from

$$Z_{A_4}^{Qq} = \rho_{A_4}^{Qq} \sqrt{Z_{V_4}^{QQ} Z_{V_4}^{qq}}, \tag{2.5}$$

where $Z_{V_4}^{QQ}$ and $Z_{V_4}^{qq}$ are determined nonperturbatively and the remaining (perturbatively calculated) short distance corrections in the deviation of $\rho_{A_4}^{Qq}$ from 1 are no more than 0.6%.
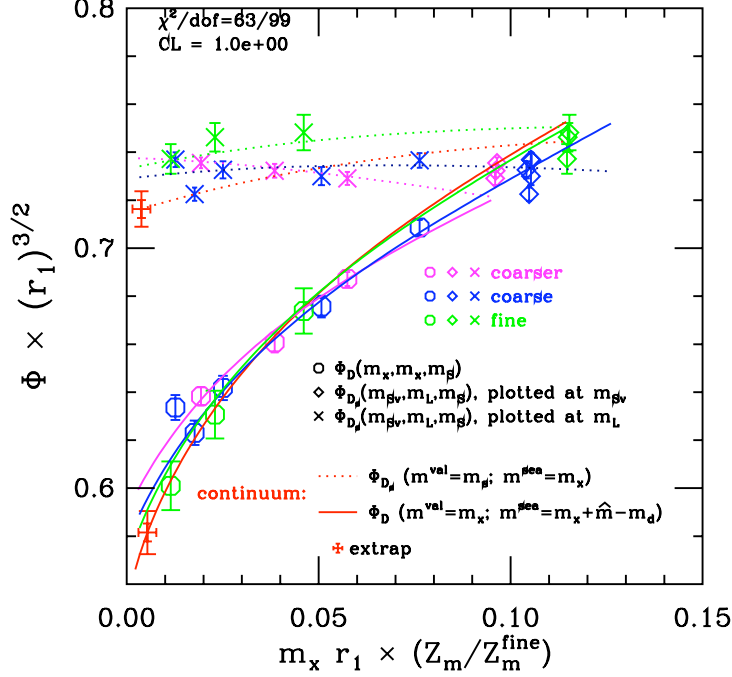

Figure 1: The leptonic decay amplitudes $f\sqrt{M}$ for the D and D_s mesons, extrapolated to the chiral limit. Units are in terms of the heavy-quark potential parameter, r_1.

Figure 1 shows the extrapolation of the D and D_s leptonic decay amplitudes to the physics light quark limit. (Units are in terms of the heavy-quark potential parameter r_1.) For ϕ_D (octagons), we show only those (fully unquenched) points for which the light valence and sea masses are equal to m_x, the mass on the abscissa. For ϕ_{D_s}, we keep both the strange sea mass (m_S) and the strange valence mass (m_{Sv}) fixed to their simulated values, and plot either as a function of the up/down sea mass m_L (crosses), or at m_{Sv} (diamonds). The chiral extrapolations make use of all the partially quenched data in addition to the points shown.

The m_q dependence is much stronger for the D than the D_s, as expected, since in the D_s it affects only the sea quarks and not the valence quarks. The slope is larger in the continuum limit, because taste breaking effects tend to suppress the dependence on the quark mass at finite a. Figure 2 shows the same thing for the B and B_s decay amplitudes, with a qualitatively similar picture.

Figure 3 shows the extrapolation of the ratio ϕ_D/ϕ_{D_s} to the chiral limit. The slope is strongest in the continuum limit (red line and cross), as expected.

3. Results

Table 1 shows the uncertainty budgets for the D_s, D, B_s, and B meson decay amplitudes ϕ_M, and for the ratios $R_D \equiv \phi_D/\phi_{D_s}$ and $R_B \equiv \phi_B/\phi_{B_s}$. The three largest uncertainties in our previous
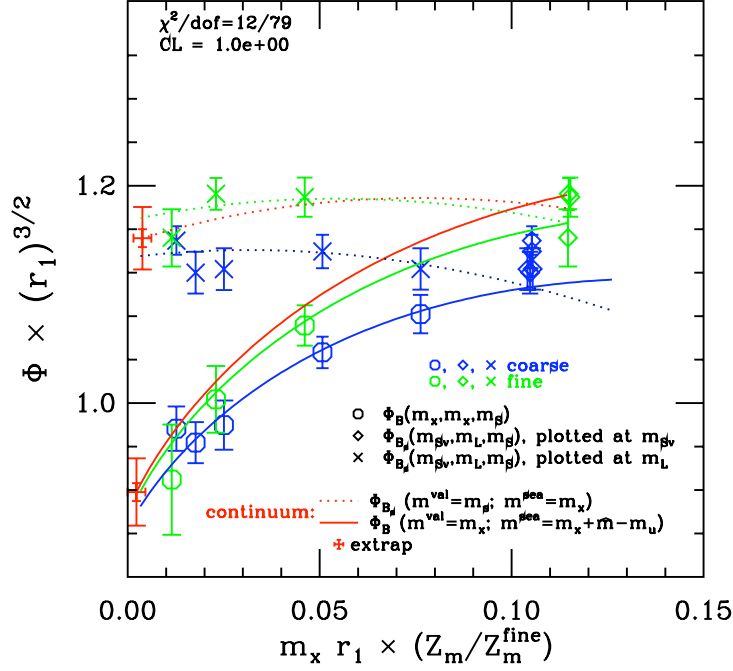
Figure 2: The same as for Fig. 1, but for the B and B_s mesons.

results were statistics, heavy quark discretization, and the heavy quark mass. The statistical error in the D and D_s decay amplitudes has been reduced this year through an improvement in analysis method, and without additional data. We are currently incorporating into the chiral and continuum extrapolation fits a term for the heavy quark discretization which we expect to substantially reduce the uncertainty from this source. The last uncertainty that is large is due to the input heavy quark mass, and will be removed with a more careful determination of this quantity.

We obtain for the decay constants

$$f_D = 207(11) \text{ MeV}, \tag{3.1}$$

Table 1: Uncertainty budgets in per cent for the leptonic decay amplitudes ϕ_{D_s}, ϕ_D, ϕ_{B_s}, and ϕ_B, and for the ratios $R_D \equiv \phi_D/\phi_{D_s}$ and $R_B \equiv \phi_B/\phi_{B_s}$.

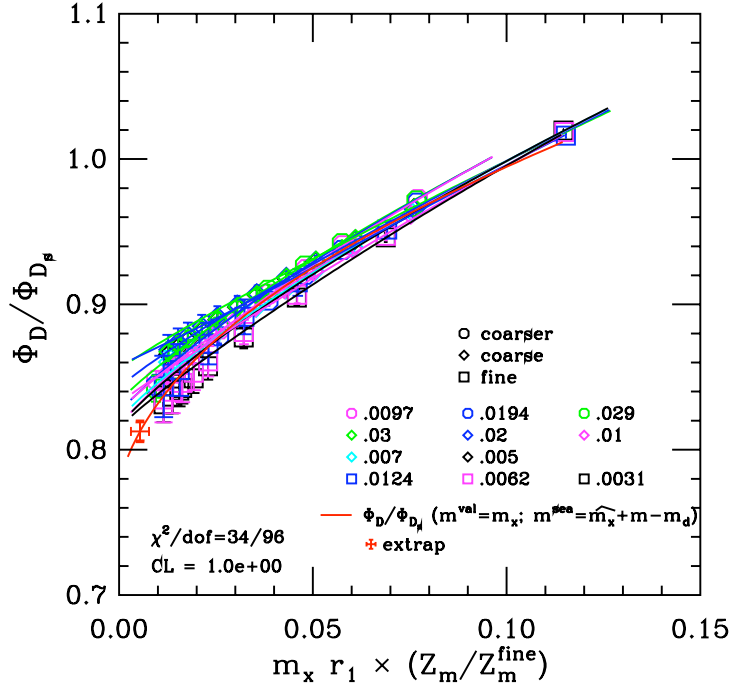|	ϕ_{D_s}	ϕ_{D_d}	R_D	ϕ_{B_s}	ϕ_{B_d}	R_B
Statistics	1.0	1.5	1.0	2.5	3.4	2.2
Inputs r_1, m_s, m_l	1.4	2.1	0.6	1.8	2.5	0.6
Inputs m_b or m_c	2.7	2.7	0.1	1.1	1.1	0.1
Z	1.4	1.4	<0.1	1.4	1.4	<0.1
Higher order ρ_{A_4}	0.1	0.1	<0.1	0.4	0.4	<0.1
Heavy q disc.	2.7	2.7	0.3	1.9	1.9	0.2
Light q disc. & χ extr.	1.2	2.6	1.6	2..0	2.4	2.4
V	0.2	0.6	0.6	0.2	0.6	0.6
Total systematic	4.5	5.3	1.8	3.8	4.4	2.6

Figure 3: The ratio of the D and D_s meson leptonic decay amplitudes, extrapolated to the chiral limit.

$$f_{D_s} = 249(11) \text{ MeV}, \tag{3.2}$$

$$f_B = 195(11) \text{ MeV}, \tag{3.3}$$

$$f_{B_s} = 243(11) \text{ MeV}, \tag{3.4}$$

and for the ratios

$$f_D/f_{D_s} = 0.833(19), \tag{3.5}$$

$$f_B/f_{B_s} = 0.803(28). \tag{3.6}$$

In Figure 4, we compare our results for f_D and f_{D_s} with the calculations of HPQCD [2] and with experiment [3, 8]. For f_D, there is very good agreement between experiment, HPQCD, and the Fermilab/MILC result. For f_{D_s}, there is

- agreement between HPQCD and Fermilab/MILC,

- 1.6 σ disagreement between Fermilab/MILC and experiment, and

- 3.5 σ disagreement between HPQCD and experiment.

Many uncertainties cancel in the ratio f_D/f_{D_s}, so we examined this quantity to see if it could enhance the significance of the discrepancy between our results and experiment. For now, looking at f_D/f_{D_s} doesn't sharpen the picture. In Figure 5, we show our results for f_D/f_{D_s} compared with the calculations of HPQCD and with experiment. There is a slight disagreement between HPQCD and FNAL/MILC in the ratio, even though f_D and f_{D_s} agree within one sigma. Further, the experimental uncertainties are independent. They add in quadrature, increasing the size of the experimental uncertainty and decreasing the significance of any discrepancy.

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New results for f_D and f_{D_s} recently appeared from ETMC using twisted-mass fermions [9]. They obtained $f_D = 205 \pm 10$ MeV and $f_{D_s} = 248 \pm 9$ MeV, which is in accord with the staggered determinations. They present a thorough uncertainty analysis, although we would quibble with their use of two rather than three light sea quarks without the inclusion of an uncertainty estimate for that approximation. Based on the difference between our unquenched and quenched calculations of f_{D_s} (249 MeV vs. 213 MeV) [10], we might have guessed a possible uncertainty of 5% from leaving out one of the three light sea quarks. (We see charm sea quarks as a different story, since $m_c \sim 1/a$ at our lattice spacings, and the dynamical effects of c quarks are for the most part above the cut-off.)

Three sigma discrepancies between experiment and the Standard Model have occasionally appeared and then disappeared before, but the discrepancy in f_{D_s} is hard to understand. The uncertainty is dominated by experimental statistical error, and three sigma statistical fluctuations are very rare. One can double the theory error, and still have a three sigma discrepancy. To explain the discrepancy as a theory error, one would have to find a mistake in the theory analysis of f_{D_s} whose correction would not affect the correct prediction of f_D. It is hard to imagine such a mistake. The calculations of f_D and f_{D_s} are almost identical. The only difference is that f_{D_s} should be somewhat easier, in that it doesn't require an extrapolation to the physical light quark masses. It may be premature to draw ultimate conclusions about the discrepancy, but the result is puzzling enough that Kronfeld and Dobrescu have investigated possible new-physics explanations for the discrepancy [11].

4. Outlook

We are in the process of reanalyzing our existing data, in which we hope to bring down several of our largest uncertainties. New runs are starting with quadruple the current statistics and at smaller lattice spacings, which we expect to help with several of the uncertainties. Comparison of f_{D_s} in theory and experiment remains a puzzle. This is the only known instance in which lattice
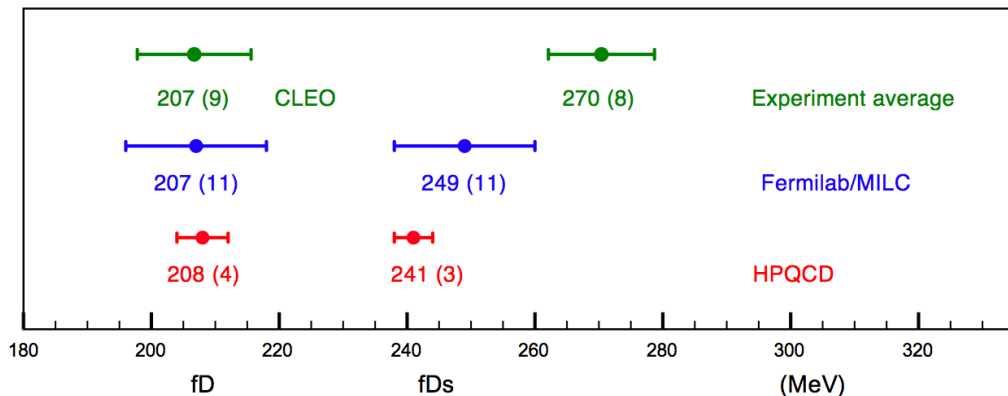

Figure 4: Comparison of f_D and f_{D_s} with the calculations of HPQCD and with experiment.
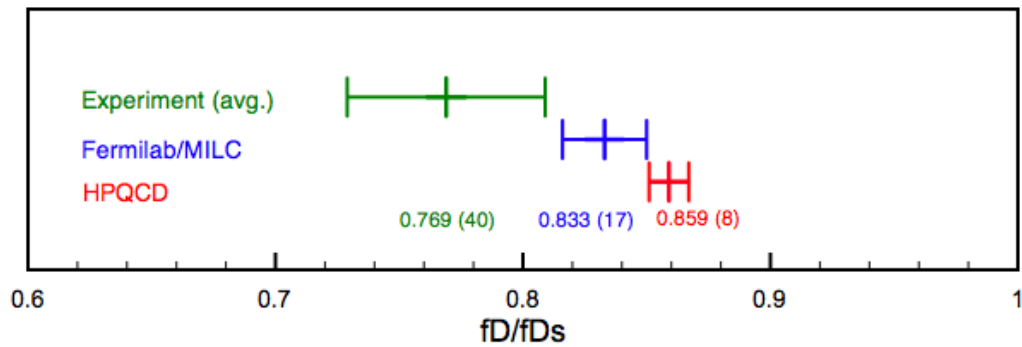
Figure 5: Results for f_D/f_{D_s} compared with the calculations of HPQCD and with experiment.

QCD with staggered fermions seems to clearly fail to reproduce the Standard Model. This provides a good target of opportunity for calculations with other lattice fermion methods.

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