Renormalization of B-meson distribution amplitudes

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We summarize a recent calculation of the evolution kernels of the two-particle $B$-meson distribution amplitudes $\phi_+$ and $\phi_-$ taking into account three-particle contributions. In addition to a few phenomenological comments, we give as a new result the evolution kernel of the combination of three-particle distribution amplitudes $\Psi_A - \Psi_V$ and confirm constraints on $\phi_+$ and $\phi_-$ derived from the light-quark equation of motion.
1. Introduction

Exclusive decays of $B$-mesons provide important tools to test the Standard Model and to search for physics beyond it. Hadronic inputs encoding soft physics are not only form factors but also light-cone distribution amplitudes (LCDAs). In particular the $B$-meson LCDAs enter the parametrization of the hard-scattering part of hadronic matrix elements of bilocal current operators where large momentum is transferred to the soft spectator quark [1-10]. Impressive progress has been made in the calculation of the hard scattering amplitudes entering factorization theorems, see e.g. [11-15] for the $B \rightarrow PP$ case, but one limiting factor for the extraction of fundamental parameters is the uncertainty coming from the hadronic input. Recent years have seen several analyses concerning the renormalization properties [16, 17, 18] and the shape of the $B$-meson LCDAs [4, 18, 19, 20, 21, 22, 23]. Up to now these analyses were restricted to the two-particle case or to leading order with the exception of [23]. Here we present the results of [24] for the renormalization of the two-particle $B$-meson LCDAs taking into account mixing with three-parton LCDAs and in section (2.3) the results of a new calculation for the combination of three-particle LCDAs $\Psi_A - \Psi_V$ entering the equations of motion.

2. One-loop calculation with three-parton external state

The relevant two- and three-parton distribution amplitudes are defined as $B$ to vacuum matrix-elements of a non-local heavy-to-light operator, which reads in the two-particle case [4]

$$
\langle 0|\bar{q}_\beta(z)[z,0](h_\nu)\alpha(0)|B(p)\rangle = -\frac{\hat{F}_{\beta}(\mu)}{4} \left[ (1 + \gamma^5) \left( \hat{\phi}^+(t) + \frac{i}{2m} [\hat{\phi}^-(t) - \hat{\phi}^+_+(t)] \right) \gamma_\nu \right]_{\alpha\beta} \tag{2.1}
$$

and in the three-particle case [21] (the most general decomposition without contraction with a light-like vector is given in [25]):

$$
\langle 0|\bar{q}_\beta(z)[z,uc]\gamma G_{\mu\nu}(uc)z^\nu[u\bar{c},0](h_\nu)\alpha(0)|B(p)\rangle = \frac{\hat{F}_{\beta}(\mu)M}{4} \left[ (1 + \gamma^5) \left( \gamma_\nu \left( v_{\mu}^2 - t\gamma_\mu \right) \langle \bar{\Psi}_A(t, u) - \bar{\Psi}_V(t, u) \rangle - i\sigma_{\mu\nu}z^\nu \langle \bar{\Psi}_V(t, u) \rangle 
\right. 
- \frac{z_{\mu}}{t} \langle \bar{y}_A(t, u) \rangle \right] \gamma_\nu \right]_{\alpha\beta}. \tag{2.2}
$$

We use light-like vectors $n_\pm$ so that every vector can be decomposed as

$$
q_\mu = (n_+ \cdot q) \frac{n_- - \mu}{2} + (n_- \cdot q) \frac{n_+ + \mu}{2} + q_{\pm \mu} = q_+ \frac{n_- - \mu}{2} + q_- \frac{n_+ + \mu}{2} + q_{\pm \mu},
$$

$$
n_+^2 = n_-^2 = 0 \quad n_+ \cdot n_- = 2 \quad v = (n_+ + n_-)/2. \tag{2.3}
$$

The computation of the renormalisation properties of the distribution amplitudes requires us to consider matrix elements of the relevant operators

$$
O^H_+(\omega) = \frac{1}{2\pi} \int dt e^{i\omega t} \langle 0|\bar{q}(z)[z,0]u_+ G h_\nu(0)|H\rangle \tag{2.4}
$$

$$
O^H_-(\omega) = \frac{1}{2\pi} \int dt e^{i\omega t} \langle 0|\bar{q}(z)[z,0]u_- G h_\nu(0)|H\rangle \tag{2.5}
$$

$$
O^H_3(\omega, \xi) = \frac{1}{(2\pi)^2} \int dt e^{i\omega t} \int du e^{i\xi u} \langle 0|\bar{q}(z)[z,uc]g_s G_{\mu\nu}(uc)z^\nu[u\bar{c},0]G h_\nu(0)|H\rangle \tag{2.6}
$$
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\[ A : -g_s \frac{\epsilon_+}{q_+} [\delta (\omega - k_+ - q_+) - \delta (\omega - k_+)] \bar{v}_H \Gamma \mu u \]

\[ B : -g_s \frac{v \cdot \epsilon}{v \cdot q} \delta (\omega - k_+) \bar{v}_H \Gamma \mu u \]

\[ C : g_s \frac{1}{(k + q)^2} \delta (\omega - k_+ - q_+) \bar{v}_H (k + q) \Gamma \mu u \]

\[ \text{Figure 1: The three leading-order contributions to the matrix element of } O_{\pm} \text{ with a three-parton external state.} \]

with \( z \) parallel to \( n_+ \), i.e. \( z_\mu = tn_+ \mu, t = v \cdot z = z_- / 2 \) and the path-ordered exponential in the \( n_+ \) direction:

\[ [z,0] = P \exp \left[ ig_s \int_0^z dy \mu A^\mu (y) \right] = 1 + ig_s \int_0^1 d\alpha z_\mu A^\mu (\alpha z) - g_s^2 \int_0^1 d\alpha \int_0^\alpha d\beta z_\mu z_\nu A^\mu (\alpha z) A^\nu (\beta z) + \ldots \]

The Fourier-transforms of the different distribution amplitudes are then defined via

\[ \phi_{\pm} (\omega) = \frac{1}{2\pi} \int dt e^{i\omega t} \bar{\phi}_{\pm} (t) \quad F (\omega, \xi) = \frac{1}{(2\pi)^2} \int dt \int du e^{i(\omega + u \xi) t} F (t, u) \]

where \( F = \Psi_V, \Psi_A, X_A, Y_A \). Since the renormalization of the operators is independent of the infrared properties of the matrix-elements, we can choose an on-shell partonic external state consisting of a light quark, a heavy quark and a gluon in equation (2.6). The resulting leading-order diagrams are shown in figures 1 and 2. Next-to-leading order (NLO) diagrams are obtained by adding a gluon or a quark loop (a ghost loop) in all possible places (for a complete list of diagrams, see [24]). Since the operators give rise to \( \delta \)-distributions in the + component of the momenta, we chose to proceed via the theorem of residues. To be more explicit, we decomposed the loop momentum \( l \) in light-cone components, picked up the poles in the \( l_- \)-integral and performed the \( l_+ \)-integration in dimensional regularization with \( D = 2 - 2\varepsilon \) dimensions. Additional \( \frac{1}{\varepsilon} \) poles arise through the \( l_+ \)-integration for diagrams where a gluon is exchanged between the Wilson-line from the operator and the heavy-quark field. These are related to the cusp anomalous dimension, see e.g [26, 27], stemming from the intersection of one light-like Wilson line from the path ordered exponential in the operator and one time-like Wilson line from the interaction of soft gluons with the heavy quark. The additional poles give rise to \( \frac{1}{\varepsilon} \) terms as well as Sudakov logarithms.
remains a genuine three-particle term. The renormalization group equation to order $\alpha_s$ is

$$
\frac{\partial \phi_-(\omega; \mu)}{\partial \log \mu} = -\frac{\alpha_s(\mu)}{4\pi} \left( \int d\omega' \gamma^{(1)}_-(\omega, \omega'; \mu) \phi_-(\omega'; \mu) \right) + \int d\omega' d\xi' \gamma^{(1)}_{-3}(\omega, \omega', \xi'; \mu) [\Psi_A - \Psi_V](\omega', \xi'; \mu)
$$

(2.11)

where $\gamma^{(1)}_-$ is the result from [17]

$$
\gamma^{(1)}_-(\omega, \omega'; \mu) = \gamma^{(1)}_+ - \Gamma^{(1)}_{cusp} \frac{\theta(\omega' - \omega)}{\omega'}
$$

(2.12)

and $\gamma^{(1)}_{-3}$ from [24]

$$
\gamma^{(1)}_{-3}(\omega, \omega', \xi') = 4 \left[ \frac{\Theta(\omega)}{\omega'} \left\{ (C_A - 2C_F) \left[ \frac{1}{\xi' \omega' + \xi' - \omega} \Theta(\xi' - \omega) + \frac{\Theta(\omega' + \xi' - \omega)}{(\omega' + \xi')^2} \right] 
- C_A \left[ \frac{\Theta(\omega' + \xi' - \omega)}{(\omega' + \xi')^2} - \frac{1}{\xi' \Theta(\omega - \omega') - \Theta(\omega - \omega' - \xi')} \right] \right\} \right]_+
$$

(2.13)

where we defined the $+$-distribution with three variables as

$$
\left[ f(\omega, \omega', \xi') \right]_+ = f(\omega, \omega', \xi') - \delta(\omega - \omega' - \xi') \int d\omega f(\omega, \omega', \xi'')
$$

(2.14)
2.3 $\Psi_A - \Psi_V$-renormalization and equation of motion constraints

Here we report on an up to now unpublished calculation of the renormalization of the three-particle LCDAs $\Psi_A - \Psi_V$. We project on the relevant distribution amplitudes in equation (2.3) using $\Gamma = \gamma^\mu_{\perp d_u d_u} \gamma_\perp$ (although a $\gamma^\mu$ instead of $\gamma^\perp_{\perp d_u d_u} \gamma_\perp$ yields the same result). The calculations go along the same lines as in the previous two cases, even though there is only one leading-order structure (shown in figure 2) and NLO diagrams must have one gluon attached to the vertex in order not to vanish trivially. For convenience the result is splitted into $C_F$- and $C_A$-colour structures.

\[
\gamma^{(1)}_{3,3,CA}(\omega, \xi, \omega', \xi') = 2 \left[ \delta(\omega - \omega') \left\{ \frac{\xi}{\xi^2} \Theta(\xi' - \xi) - \left[ \frac{\Theta(\xi - \xi')}{\xi - \xi'} \right]_+ - \left[ \frac{\xi' \Theta(\xi' - \xi)}{\xi' - \xi} \right]_+ \right\} \\
+ \delta(\xi - \xi') \left\{ \left[ \frac{\Theta(\omega - \omega')}{\omega - \omega'} \right]_+ + \left[ \frac{\omega \Theta(\omega' - \omega)}{\omega' - \omega} \right]_+ \right\} + \delta(\omega + \xi - \omega' - \xi') \times \left\{ \frac{1}{\xi} \Theta(\omega - \omega') - \left[ \frac{\Theta(\omega - \omega')}{\omega - \omega'} \right]_+ - \left[ \frac{\omega \Theta(\omega' - \omega)}{\omega' - \omega} \right]_+ \right\} \\
+ \delta(\omega + \xi - \omega' - \xi') \frac{1}{\xi' \omega + \xi' \omega} \left\{ \frac{\omega - \xi'}{\xi' \omega + \xi} (\omega' + \xi') \Theta(\omega') \right. \\
- \frac{\omega}{\omega'} (\omega' + 2 \xi' - \omega) \Theta(\omega') \Theta(\omega) + \frac{\omega}{\xi} (\omega - \xi') \Theta(\omega') \Theta(\omega) \\
\left. + \frac{\omega - \xi'}{\omega'} (\omega' + \xi') \Theta(\omega - \xi') \Theta(\omega) \right\} \\
\right\} \\
\gamma^{(1)}_{3,3,CF}(\omega, \xi, \omega', \xi'; \mu) = \gamma^{(1)}_{3}(\omega, \omega'; \mu) \delta(\xi - \xi') + \gamma^{(1)}_{3,3}(\omega, \xi, \omega', \xi') \\
\gamma^{(1)}_{R3,3}(\omega, \xi, \omega', \xi') = 4 \delta(\omega + \xi - \omega' - \xi') \times \left[ \frac{\xi^2}{\omega' (\omega + \xi)^2} \Theta(\xi) + \frac{\Theta(\omega - \omega')}{\xi'} + \frac{\Theta(\omega - \omega)}{\omega + \xi} \Theta(\omega) \left( \frac{\xi}{\omega + \xi} - \frac{\omega - \xi'}{\xi'} \right) \right] \\
\right\}
\]

(2.15)

(2.16)

with $\gamma^{(1)}_{3}$ the same as in equation (2.10) and $\gamma^{(1)}_{3,3}$ defined as in (2.11) with obvious changes. Part of this calculation, namely the light-quark-gluon part, has been calculated in a different context and a different scheme, e.g. in [29, 28].

In [21] two equations from the light- and heavy-quark equations of motion were derived

\[
\omega \phi_+^\prime(\omega; \mu) + \phi_+^\prime(\omega; \mu) = I(\omega; \mu), \quad (\omega - 2 \Lambda) \phi_+^\prime(\omega; \mu) + \omega \phi_+^\prime(\omega; \mu) = J(\omega; \mu),
\]

(2.17)

where $I(J)(\omega; \mu)$ are integro-differential expressions involving the three-particle LCDAs $\Psi_A - \Psi_V$ ($\Psi_A + X_A$ and $\Psi_V$) respectively. While the second equation was shown not to hold beyond leading order in [17, 23] we have checked that the first one is valid once renormalization is taken into account. Taking the derivative of the first equation with respect to log $\mu$, inserting

\[
I(\omega; \mu) = 2 \frac{d}{d \omega} \int_0^\omega d \rho \int_0^\infty \frac{d \xi}{\xi} \frac{d}{d \xi} \left[ \Psi_A(\rho, \xi; \mu) - \Psi_V(\rho, \xi; \mu) \right]
\]

(2.18)

and using the relation from [17]

\[
-\omega \frac{d}{d \omega} \int_0^\eta \frac{d \omega'}{\eta} \gamma^{(1)}_{-}(\omega, \omega'; \mu) = \gamma^{(1)}_{+}(\omega, \eta; \mu)
\]

(2.19)
one arrives at the following equation

\[
\omega \frac{d}{d\omega} \int d\omega' d\xi' \gamma^{(1)}_{\omega, \xi'}(\omega, \omega', \xi') \left( \Psi_A(\omega', \xi'; \mu) - \Psi_V(\omega', \xi'; \mu) \right) + 2 \left[ \int d\omega' \gamma^{(1)}_{\omega, \omega'; \mu} \frac{d}{d\omega'} \int_0^{\omega'} d\rho \int_{\omega' - \rho}^{\omega'} \frac{d\xi}{\xi} \frac{\partial}{\partial \xi} \left( \Psi_A(\rho, \xi; \mu) - \Psi_V(\rho, \xi; \mu) \right) \right]
\]

\[
= 2 \int d\omega' d\xi' \frac{d}{d\omega} \int_0^{\omega'} d\rho \int_{\omega' - \rho}^{\omega'} \frac{d\xi}{\xi} \frac{\partial}{\partial \xi} \gamma^{(1)}_{\omega, \xi'}(\rho, \xi, \omega'; \mu) \left( \Psi_A(\omega', \xi'; \mu) - \Psi_V(\omega', \xi'; \mu) \right),
\]

(2.20)

which can be proven to hold at order \( \alpha_s \) by simple insertion of the respective evolution kernels (2.10), (2.13), (2.15), (2.16). This non-trivial outcome gives us further confidence concerning the renormalization group properties of the LCDAs.

3. Conclusions

The presence of \( \delta(\omega - \omega') \log(\mu/\omega) \) in the renormalization matrices gives rise to a radiative tail falling off like \( (\log \omega)/\omega \) for large \( \omega \). Therefore non-negative moments of the LCDAs are not well defined and have to be considered with an ultraviolet cut-off.[16, 17, 22, 23]

\[
(\omega^N)_{\pm}(\mu) = \int_0^{\Lambda_{\text{UV}}} d\omega \omega^N \phi_{\pm}(\omega; \mu)
\]

(3.1)

For \( \phi_- \) it is interesting to examine the limit

\[
\lim_{\Lambda_{\text{UV}} \to \infty} \int_0^{\Lambda_{\text{UV}}} d\omega \omega^N z^{(1)}_{-3}(\omega, \omega', \xi') = 0 \quad N = 0, 1, \quad z^{(1)}_{-3} = \frac{1}{2e} \gamma^{(1)}_{-3},
\]

(3.2)

which is relevant for the calculation of the three-particle contributions to the moments:

\[
\int_0^{\Lambda_{\text{UV}}} d\omega \omega^N \phi_-(\omega; \mu) = 1 + \frac{\alpha_s}{4\pi} \left( \int d\omega' \phi_-(\omega') \int_0^{\Lambda_{\text{UV}}} d\omega \omega^N z^{(1)}_{-3}(\omega, \omega'; \mu) \right.
\]

\[
- \int d\omega' d\xi' (2 - D) \Psi_A - \Psi_V(\omega', \xi') \int_0^{\Lambda_{\text{UV}}} d\omega \omega^N z^{(1)}_{-3}(\omega, \omega'; \mu)
\]

(3.3)

Therefore as stated in [17] three-particle distribution amplitudes give only subleading contribution to the first two moments \( N = 0, 1 \) and we have explicitly checked that this statement cannot be extended to higher moments \( N \geq 2 \).

The next step consists in using the renormalization properties as a guide to go beyond the existing models derived from a leading-order sum-rule calculation resulting in \( \Psi_A = \Psi_V \) [20] and to analyze their influence on \( \phi_- \). Finally, for practical calculations involving three-particle contributions, one would need the evolution kernels for the all relevant LCDAs, which will be the subject of a future work.

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