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Using SCET to calculate electroweak corrections in gauge boson production

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We extend an effective theory framework developped in Refs. [1, 2] to sum electroweak Sudakov logarithms in high energy processes to also include massive gauge bosons in the final state. The calculations require an additional regulator on top of dimensional regularization to tame the collinear singularities. We propose to use the Δ regulator, which respects soft-collinear factorization.

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1. Introduction

The Large Hadron Collider (LHC) has a center-of-mass energy of $\sqrt{s} = 14$ TeV, and will be able to measure collisions with a partonic center-of-mass energy of several TeV, more than an order of magnitude larger than the masses of the electroweak gauge bosons. Radiative corrections to scattering processes depend on the ratio of mass scales, and radiative corrections at high energy depend on logarithms of the form $\log s/M_{W,Z}^2$. In high energy exclusive processes, radiative corrections are enhanced by two powers of a large logarithm for each order in perturbation theory, and the logarithms are often referred to as Sudakov (double) logarithms. Electroweak Sudakov corrections are not small at LHC energies, since $\alpha \log^2 s/M_{W,Z}^2/(4\pi \sin^2\theta_W) \sim 0.15$ at $\sqrt{s} = 4$ TeV. These Sudakov corrections lead to sizeable effects and might be summed to all orders.

The Sudakov logarithm $\log(s/M_{W,Z}^2)$ can be thought of as an infrared logarithm in the electroweak theory, since it diverges as $M_{W,Z} \to 0$. By using an effective field theory (EFT), these infrared logarithms in the original theory can be converted to ultraviolet logarithms in the effective theory, and summed using standard renormalization group techniques. The effective theory needed is soft-collinear effective theory (SCET) [3, 4], which has been used to study high energy processes in QCD, and to perform Sudakov resummations arising from radiative gluon corrections.

The summation of electroweak Sudakov logarithms using effective field theory methods has extensively been discussed in Ref. [1] for the Sudakov form factor and in Ref. [2] for four fermi scattering processes. Here, we extend the discussion to processes with gauge bosons in the final state. Only the Sudakov form factor will be considered in the following.

2. Exponentiation

We start by summarizing some known properties of the Sudakov form-factor [5] for the vector current. The Euclidean form-factor $F_E(Q^2)$ has the expansion ($\mathsf{L} = \log(Q^2/M^2)$)

$$F_E \;=\; 1+\alpha\left(k_{12}\mathsf{L}^2+k_{11}\mathsf{L}+k_{10}\right)+\alpha^2\left(k_{24}\mathsf{L}^4+k_{23}\mathsf{L}^3+k_{22}\mathsf{L}^2+k_{21}\mathsf{L}+k_{20}\right)+\ldots\,, \tag{2.1}$$

with the α^n term having powers of L up to L^{2n}. In the literature, the highest power of L is called the $\mathrm{LL_F}$ term, the next power is called the $\mathrm{NLL_F}$ term, etc. We have included the subscript F (for the form-factor) to distinguish it from the renormalization group counting described below.

The series for $\log F_E(Q^2)$ takes a simpler form

$$\log F_E \;=\; \alpha\left(\tilde{k}_{12}\mathsf{L}^2+\tilde{k}_{11}\mathsf{L}+\tilde{k}_{10}\right)+\alpha^2\left(\tilde{k}_{23}\mathsf{L}^3+\tilde{k}_{22}\mathsf{L}^2+\tilde{k}_{21}\mathsf{L}+\tilde{k}_{20}\right)+\ldots\,, \tag{2.2}$$

with the α^n term having powers of L up to L^{n+1}, and the expansion begins at order α. Note that Eq. (2.2) implies non-trivial relations among the coefficients k_{nm} in Eq. (2.1). At order n, there are $2n+1$ coefficients k_{nm}, $0 \le m \le 2n$ in Eq. (2.1), but only $n+2$ coefficients $\tilde{k}_{nm}$, $0 \le m \le n+1$ in Eq. (2.2).

The right-hand-side of Eq. (2.2) can be written in terms of the LL series $\mathsf{L} f_0(\alpha\mathsf{L}) = \tilde{k}_{12}\alpha\mathsf{L}^2 + \tilde{k}_{23}\alpha^2\mathsf{L}^3+\ldots$, the NLL series $f_1(\alpha\mathsf{L}) = \tilde{k}_{11}\alpha\mathsf{L}+\tilde{k}_{22}\alpha^2\mathsf{L}^2+\ldots$, the NNLL series $\alpha f_2(\alpha\mathsf{L}) = \tilde{k}_{10}\alpha + \tilde{k}_{21}\alpha^2\mathsf{L}+\ldots$ etc. as

$$\log F_E \;=\; \mathsf{L} f_0(\alpha\mathsf{L})+f_1(\alpha\mathsf{L})+\alpha f_2(\alpha\mathsf{L})+\ldots\,. \tag{2.3}$$

f_0 and f_1 begin at order α, and the remaining f_n begin at order one.

Here, LL, NLL, etc. (with no subscripts) will refer to the counting for $\log F_E$. This is also the counting appropriate for a renormalization group improved computation, and is different from the conventional counting discussed above. If one looks at the order α^2 terms, for example, the conventional counting is that the L^4 term is $\mathrm{LL_F}$, the L^3 term is $\mathrm{NLL_F}$, the L^2 term is $\mathrm{N^2LL_F}$, the L term is $\mathrm{N^3LL_F}$, and the L^0 term is $\mathrm{N^4LL_F}$. Using our counting, the terms are given by exponentiating $\log F_E$ to LL, NLL, $\mathrm{N^2LL}$, $\mathrm{N^2LL}$, and $\mathrm{N^3LL}$, respectively. At higher orders, the mismatch in powers of N between the two counting methods increases.

Since for electroweak corrections at the TeV scale $\alpha\mathsf{L}^2$ is quite sizeable, the LL series might be summed up to all orders.

3. SCET formalism

The theory we consider is a $SU(2)$ spontaneously broken gauge theory, with a Higgs in the fundamental representation, where all gauge bosons have a common mass, M. It is convenient to write the group theory factors using C_F, C_A, T_F[1]

The physical quantity of interest is the Sudakov form factor $F_O(Q^2)$ in the Euclidean region,

$$F_O(Q^2) = \mathscr{N}\langle p_2|O|p_1\rangle\,, \tag{3.1}$$

where $Q^2 = -(p_2-p_1)^2 \gg M^2$, O is a generic operator and $\mathscr{N}$ a normalization factor. In SCET, $F_O(Q^2)$ is computed in three steps: (i) matching from the full gauge theory to SCET at $\mu = Q$ (high-scale matching) (ii) running in SCET between Q and M and (iii) integrating out the gauge bosons at $\mu = M$ (low-scale matching). All computations are done to leading order in SCET power counting, i.e. neglecting M^2/Q^2 power corrections.

The SCET fields and Lagrangian depend on two null four-vectors n and $\bar{n}$, with $n = (1,\mathbf{n})$ and $\bar{n} = (1,-\mathbf{n})$, where $\mathbf{n}$ is a unit vector, so that $\bar{n}\cdot n = 2$. In the Sudakov problem, one works in the Breit frame, with n chosen to be along the p_2 direction, so that $\bar{n}$ is along the p_1 direction. In the Breit frame, the momentum transfer q has no time component, $q^0 = 0$, so that the particle is back-scattered. The light-cone components of a four-vector p are defined by $p^+ \equiv n\cdot p$, $p^- \equiv \bar{n}\cdot p$, and $p_\perp$, which is orthogonal to n and $\bar{n}$, so that

$$p^\mu \;=\; \frac{1}{2}n^\mu(\bar{n}\cdot p) + \frac{1}{2}\bar{n}^\mu(n\cdot p) + p_\perp^\mu. \tag{3.2}$$

In our problem, $p_1^- = p_{1\perp} = p_2^+ = p_{2\perp} = 0$, and $Q^2 = p_1^+ p_2^-$. A gauge boson moving in a direction close to n is described by the n-collinear SCET field $A_{n,p}(x)$, where p is a label momentum, and has components $\bar{n}\cdot p$ and $p_\perp$ [3, 4]. It describes particles (on- or off-shell) with energy $2E \sim \bar{n}\cdot p$, and $p^2 \ll Q^2$. The total momentum of the field $A_{n,p}(x)$ is $p+k$, where k is the residual momentum of order $Q\lambda^2$ contained in the Fourier transform of x. The scaling of the momenta is $\bar{n}\cdot p \sim Q$, $n\cdot p \sim Q\lambda^2$, $p_\perp \sim Q\lambda$. The $\bar{n}$-collinear field $A_{\bar{n},p}(x)$ contains massive gauge bosons moving near the $\bar{n}$-direction, with momentum scaling $n\cdot p \sim Q$, $\bar{n}\cdot p \sim Q\lambda^2$, $p_\perp \sim Q\lambda$. Here we have $\lambda \sim M/Q$, where $\lambda \ll 1$ is the power counting parameter used for the EFT expansion. The mass-mode field (see Ref. [6]) contains massive gauge bosons with all momentum components scaling as $Q\lambda \sim M$.

[1]Note that the results only hold for $C_A = 2$, since for an $SU(N)$ group with $N > 2$, a fundamental Higgs does not break the gauge symmetry completely.

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4. Wilson lines and regulator

Before we outline how to extend the framework of Refs. [1, 2] to also include final state gauge bosons, we elaborate on a technical issue encountered when calculating SCET diagrams with a massive gauge boson.

Consider a high energy scattering process with two or more particles, in the n_i direction, $i = 1,\dots,r$. n_i-collinear gauge bosons, which have momentum parallel to particle i can interact with particle i, or with the other particles $j \neq i$. The coupling of n_i-collinear gauge bosons to particle i is included explicitly in the SCET Lagrangian. The particle-gauge interactions are identical to those in the full theory, and there is no simplification on making the transition to SCET. However, if an n_i-collinear gauge boson interacts with a particle j not in the n_i-direction, then particle j becomes off-shell by an amount of order Q, and the intermediate particle j propagators can be integrated out, giving a Wilson line interaction in SCET. The form of these operators was derived in Ref. [4, 7], and gives the Wilson line interaction $W^\dagger_{n_i}\xi_{n_i}$, where W_{n_i} is a Wilson line in the $\bar{n}_i$ direction in the same representation as ξ_{n_i}. This is easy to see in processes with only two collinear particles. But even in complicated scattering processes with more than two collinear particles the Wilson line interaction still has the form $W^\dagger_{n_i}\xi_{n_i}$. To see that this statement also holds at one loop is not straightforward. The reason is that loop diagrams require an additional regulator on top of dimensional regularization. This introduces a dependence on all the other collinear directions n_j. Here, we use the Δ regulator introduced in Ref. [8], which amounts to modify the propagator denominators as

$$\frac{1}{(p_i+k)^2-m_i^2} \to \frac{1}{(p_i+k)^2-m_i^2-\Delta_i}. \tag{4.1}$$

In SCET, the collinear propagator denominators have the replacement of Eq. (4.1). Accordingly, Wilson lines become

$$\begin{aligned} \frac{\varepsilon\cdot n_j}{k\cdot n_j} &\to \frac{\varepsilon\cdot \bar{n}_i}{k\cdot\bar{n}_i-\delta_{j,n_i}}, \\ \delta_{j,n_i} &\equiv \frac{2\Delta_j}{(n_i\cdot n_j)(\bar{n}_j\cdot p_j)}. \end{aligned} \tag{4.2}$$

It turns out (see Ref. [8]) that after zero-bin subtraction [9] (see also Ref. [10]), the dependence on the other collinear directions drop out and n_i-collinear gauge boson emission can be combined into a single Wilson line. Note that in an intermediate step, regulator dependent regions are introduced into the calculation. However, they cancel between the soft- and collinear diagrams. The sum of all diagrams is of course independent of the Δ regulator.

5. Transversely polarized W bosons

Consider scattering of two gauge bosons via the operator $O_T = F^a_{\mu\nu}F^{\mu\nu,a}$ with $F_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g f^{abc}A^b_\mu A^c_\nu$ the non-abelian field strength tensor,

$$\langle p_2|O_T|p_1\rangle = 4F_{O_T}(Q^2)\left[p_1\cdot p_2\,\varepsilon(p_1)\cdot\varepsilon^*(p_2) - \varepsilon^*(p_2)\cdot p_1\,\varepsilon(p_1)\cdot p_2\right]. \tag{5.1}$$

Following the steps outlined in Sect. 3, one first matches the full theory matrix element at the high scale $\mu \sim Q$ onto the matrix element in the effective theory. The corresponding operator to O_T in

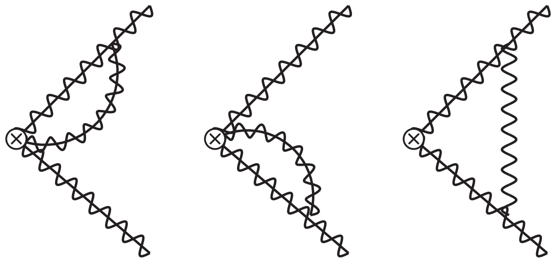

Figure 1: One loop diagrams in the effective theory. The wiggly line denotes a mass mode gauge boson and a wiggly line with an overlaid straight line indicates a collinear field. Wave function diagrams are not shown.

the effective theory reads $\tilde{O}_T = B^{\dagger\perp}_{n,p_2} B^{\perp}_{\bar{n},p_1}$, with $B^{\mu}_{n,p}$ defined as [11, 12]

$$B^{\mu}_{n,p} = \frac{1}{g}\left[W_n^{\dagger} iD_n^{\mu} W_n\right], \qquad iD_n^{\mu} = i\partial^{\mu} - gA^{\mu}_{n,p}. \tag{5.2}$$

The matching coefficient $C_T(\mu)$ of the effective theory operator up to $\mathscr{O}(\alpha)$ reads

$$C_T(\mu) = 2Q^2\left[1 + \frac{\alpha}{4\pi}C_A\left(-\mathsf{L_Q}^2 + \frac{\pi^2}{6}\right)\right], \tag{5.3}$$

where we use the notation $\mathsf{L_X} \equiv \ln\left(X^2/\mu^2\right)$. This is just the finite part of the one loop amplitude in the full theory, normalized according to Eq. (5.1). Note that since we are in the high energy limit, the $SU(2)$ gauge theory is in the unbroken phase and all the masses have been set to zero. The effective theory one loop diagrams shown in Fig. 1 are all scaleless in dimensional regularization and vanish. The anomalous dimension of the effective theory operator $\tilde{O}_T$ follows from the $\frac{1}{\varepsilon}$ poles from the one loop matrix element,

$$\gamma_T = \frac{\alpha}{4\pi}C_A\left[4\mathsf{L_Q} - 4\right] + \frac{\alpha}{2\pi}\left[-\frac{10}{3}C_A + \frac{4}{3}T_F n_f + \frac{2}{3}T_F n_s + \frac{1}{3}\right]. \tag{5.4}$$

The second term in the square brackets is the contribution from the wave function renormalization of the gauge bosons. The matching coefficient $C_T(\mu)$ can be evolved down to a scale $\mu \sim M$ with the renormalization group equation

$$C_T(\mu_2) = C_T(\mu_1)\exp\left[\int_{\mu_1}^{\mu_2}\frac{d\mu}{\mu}\gamma_T(\mu)\right]. \tag{5.5}$$

At the low scale $\mu \sim M$, the gauge bosons are integrated out by matching the effective theory with dynamical massive gauge bosons onto an effective theory where the gauge bosons are treated as heavy background fields. The matching coefficient at the low scale, $D_T(\mu)$, up to order $\mathscr{O}(\alpha)$ is again obtained by comparing the finite parts of the one loop matrix elements. Since the gauge boson has to be treated as massive, the one loop diagrams do not vanish anymore. The calculation of the individual diagrams shown in Fig. 1 requires an additional regulator, as described in Sect. 4. In the effective theory below the scale M, there are no dynamical interacting degrees of freedom left and therefore, no quantum corrections appear. Also, there is no need to evolve $D_T(\mu)$. One obtains

$$D_T(\mu) = 1+\frac{\alpha}{4\pi}C_A\left[2\mathsf{L_M}\mathsf{L_Q}-\mathsf{L_M}^2-2\mathsf{L_M}+2-\frac{5\pi^2}{6}+2f_s(1,1)\right]+\delta R_W\,,$$
$$f_s(v,w) = \int_0^1 dx\frac{2-x}{x}\ln\frac{1-x+wx-vx(1-x)}{1-x}\,. \tag{5.6}$$

The quantity δR_χ denotes the finite part of the residue of the full propagator of the field χ. The result for the resummed Sudakov form factor at high energies reads

$$F_{O_T}(Q^2) = C_T(Q)\exp\left[\int_Q^M\frac{d\mu}{\mu}\gamma_T(\mu)\right]D_T(M)\,. \tag{5.7}$$

Note that it was proven in Ref. [1] that there can appear at most one logarithm of the high scale, $\mathsf{L_Q}$, in the low scale matching coefficient D_T.

6. Longitudinally polarized W bosons

The emission of longitudinally polarized gauge bosons at high energies is related to a truncated matrix element (indicated by the subscript 'tr') of unphysical Goldstone bosons by virtue of the Goldstone boson equivalence theorem (see Ref. [13]),

$$\varepsilon^\mu(p_1)\varepsilon^{\nu *}(p_2)R_W\left\langle 0|TA^a_\mu A^b_\nu|0\right\rangle_{\mathrm{tr}} = i^2\mathscr{E}^2R_\phi\left\langle 0|T\phi^a\phi^b|0\right\rangle_{\mathrm{tr}}+\mathscr{O}\left(\frac{M}{E}\right)\,. \tag{6.1}$$

The quantity $\mathscr{E}$ is a nontrivial correction factor arising from the truncation of the Greens functions,

$$\mathscr{E} = 1+\frac{\alpha}{4\pi}\left[-\frac{z^2}{4}+\frac{47\pi}{12\sqrt{3}}-\frac{73}{12}+\frac{2z^4-7z^2+5}{8}\ln(z)+\frac{-2z^5+7z^3+z}{4\sqrt{4-z^2}}\arctan\left(\frac{\sqrt{2-z}}{\sqrt{2+z}}\right)\right] \tag{6.2}$$

with $z = M_h/M$ the Higgs-Goldstone boson mass ratio. Note that $\mathscr{E}$ does not run, however, it compensates the gauge dependence of the unphysical Goldstone bosons.

The effective theory calculation proceeds in the same manner as described in the previous section. The full theory operator is the square of the Higgs doublet operator

$$O_L = H^\dagger H\,, \qquad H = \frac{1}{\sqrt{2}}\begin{pmatrix}\phi^2+i\phi^1\\ h-i\phi^3\end{pmatrix}\,. \tag{6.3}$$

Since the Higgs doublet field in the effective theory is a scalar collinear field $\phi_{n,p}$ in the same representation as H, the high scale matching coefficient $C_L(\mu)$ of the operator in the effective theory, $\tilde{O}_L = [\phi^\dagger_{n,p_2}W_n][W^\dagger_{\bar{n}}\phi_{\bar{n},p_1}]$, and its anomalous dimension γ_L have already been calculated in Ref. [1]. The result is

$$C_L(\mu) = 1+\frac{\alpha}{4\pi}C_F\left[-\mathsf{L_Q}^2+4\mathsf{L_Q}-2+\frac{\pi^2}{6}\right]\,, \qquad \gamma_L = \frac{\alpha}{4\pi}C_F\left[4\mathsf{L_Q}-8\right]\,. \tag{6.4}$$

At the low scale, the $SU(2)$ invariant operator splits up into invariants of the remaining $SO(3)$ custodial symmetry. Again, in the theory below $\mu\sim M$, the Higgs and the Goldstone boson are treated as heavy fields $h_v^{(h)}$ and $h_v^{(\phi)}$ and the theory has no quantum corrections. Matching onto the operators $O_{hh} = h_{v_2}^{(h)} h_{v_1}^{(h)}$ and $O_{\phi\phi} = h_{v_2}^{(\phi)} h_{v_1}^{(\phi)}$, one finds

$$\begin{aligned} D_L^{(\phi\phi)}(\mu) &= 1 + \frac{\alpha}{4\pi} C_F \left[2\mathsf{L_M}\mathsf{L_Q} - \mathsf{L_M}^2 - 2\mathsf{L_M} + 2 - \frac{5\pi^2}{6} + \frac{4}{3} f_s(1,1) + \frac{2}{3} f_s(1,z^2) \right] + \delta R_\phi \,, \\ D_L^{(hh)}(\mu) &= 1 + \frac{\alpha}{4\pi} C_F \left[2\mathsf{L_M}\mathsf{L_Q} - \mathsf{L_M}^2 - 2\mathsf{L_M} + 2 - \frac{5\pi^2}{6} + 2 f_s(z^2,1) \right] + \delta R_h \,. \end{aligned} \tag{6.5}$$

Having resummed the large logarithms in the Goldstone boson scattering off the operator O_L, one only needs to correct with the factor $\mathscr{E}$ of Eq. (6.2) to obtain the result for the scattering of longitudinally polarized gauge bosons.

7. Summary and conclusions

We discuss a new regulator to tame the collinear singularities in the effective theory with massive gauge bosons. This regulator respects soft-collinear factorization. Furthermore, the framework of Refs. [1, 2] is extended to also include gauge bosons in the final state. We restrict the discussion to a spontaneously broken $SU(2)$ gauge symmetry. Results for scattering of gauge bosons off an external operator are presented. We consider massive gauge bosons with transverse as well as longitudinal polarization. The latter relies on the Goldstone boson equivalence theorem. A generalization to the full standard model gauge group is straightforward.

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