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Scalar radius of the pion and $\gamma\gamma \to \pi\pi$

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We make an improvement of the dispersion relation calculation of the quadratic scalar radius of the pion, $\langle r^2\rangle_s^\pi$, and the reaction $\gamma\gamma \to \pi^0\pi^0$. We solve a previous discrepancy between the the solution of the Muskhelishvili-Omnès equations for the non-strange null isospin (I) pion scalar form factor and the Indurain's calculation using an Omnès representation of this form factor. We show that Ynduráin's method is indeed compatible with the determinations from the Muskhelishvili-Omnès equations once a possible zero in the scalar form factor is considered. Once this is accounted for, the resulting value is $\langle r^2\rangle_s^\pi = 0.63 \pm 0.05\,\mathrm{fm}^2$.
Regarding the reaction $\gamma\gamma \to \pi^0\pi^0$ we emphasize how the $f_0(980)$ signal emerges in $\gamma\gamma \to \pi\pi$ within the dispersive approach and how this fixes to a large extent the phase of the isoscalar S-wave $\gamma\gamma \to \pi\pi$ amplitude above the $K\bar{K}$ threshold. This allows us to make sharper predictions for the cross section at lower energies and our results could then be used to distinguish between different $\pi\pi$ isoscalar S-wave parameterizations with the advent of new precise data on $\gamma\gamma \to \pi^0\pi^0$. We also pay special attention to the role played by the σ resonance in $\gamma\gamma \to \pi\pi$ and calculate its coupling and width to $\gamma\gamma$, for which we obtain $\Gamma(\sigma \to \gamma\gamma) = (1.68 \pm 0.15)$ KeV.

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1. Introduction

In the present contribution we summarize the three papers [1, 2, 3] that mainly deal with the strong influence of the $I = 0$ S-wave meson-meson final state interactions in the non-strange $I = 0$ scalar form factor of the pion [1] and $\gamma\gamma \to \pi^0\pi^0$ [2]. Both processes can be formulated in a way that share in common the same basic function in order to take care of the strong final state interactions in the $I = 0$ S-wave. This function has been recently the origin of large uncertainties in its implementation in the literature, both for the scalar form factor of the pion [4, 5, 6] and for $\gamma\gamma \to \pi^0\pi^0$ [7].

Performing a Taylor expansion around $t = 0$ of the scalar form factor of the pion, $\Gamma_\pi(t)$, $\Gamma_\pi(t) = \Gamma_\pi(0)\left\{1 + \frac{1}{6}t\langle r^2\rangle_s^\pi + \mathscr{O}(t^2)\right\}$, the coeffcient of the linear term defines the quadratic scalar radius of the pion, $\langle r^2\rangle_s^\pi$. The quantity $\langle r^2\rangle_s^\pi$ contributes around 10% to the values of the S-wave $\pi\pi$ scattering lengths as determined in Ref. [8] by solving the Roy equations with constraints from two loop Chiral Perturbation Theory (CHPT). Related to that, $\langle r^2\rangle_s^\pi$ is also important in $SU(2) \times SU(2)$ CHPT since it gives the low energy constant $\overline{\ell}_4$ that controls the departure of F_π from its value in the chiral limit [9, 10] at next-to-leading order. Based on one loop χPT, Gasser and Leutwyler [9] obtained $\langle r^2\rangle_s^\pi = 0.55 \pm 0.15$ fm^2. This calculation was improved later on by the same authors together with Donoghue [11], who solved the corresponding Muskhelishvili-Omnès equations with the coupled channels of $\pi\pi$ and $K\bar{K}$. The update of this calculation, performed in Ref. [8], gives $\langle r^2\rangle_s^\pi = 0.61 \pm 0.04$ fm^2. One should notice that solutions of the Muskhelishvili-Omnès equations for the scalar form factor consider only two coupled channels ($\pi\pi$, KK) and have systematic uncertainties, specially for energies between 1-1.5 GeV coming from the not consideration of the 4π, 6π, 2η and $\eta\eta'$ states. Furthermore it relies on non-measured T-matrix elements or on assumptions about which are the channels that matter. Therefore, other independent approaches are then required. In this respect we quote the works [12, 13, 14], and Ynduráin's ones [4, 5, 6]. These latter works have challenged the previous value for $\langle r^2\rangle_s^\pi$, shifting it to the larger $\langle r^2\rangle_s^\pi = 0.75 \pm 0.07$ fm^2. If this is translated to the scattering lengths, it implies a shift of slightly more than one sigma. Refs. [4, 5] emphasize that one should have a precise knowledge of the $I = 0$ S-wave phase shits, $\delta_0(s)$, for $s \geq 4M_K^2$ GeV^2, M_K is the kaon mass, to disentangle which of the values, either that of Ref. [8] or [4], is the right one. However, this point is based on an unstable behaviour of the solution of Ref. [4] with respect to the value of $\delta_0(4M_K^2)$. Once this instability is cured, as shown below, the resulting $\langle r^2\rangle_s^\pi$ only depends weakly on $\delta_0(s)$, $s \geq 4M_K^2$, and is compatible with the value of Ref. [8].

Regarding the reaction $\gamma\gamma \to \pi^0\pi^0$ one has to emphasize that due to the absence of the Born term (since the π^0 is neutral), this reaction is specially sensitive to final state interactions. For energies below 0.6 GeV or so, only the S-waves matter, which have $I = 0$ or 2. It is in this point where both the study of this reaction and the scalar form factor match. Recently, Ref. [7] updated the dispersive approach of Ref. [15] to calculate $\sigma(\gamma\gamma \to \pi^0\pi^0)$. Here one finds a large uncertainty in the results for $\sqrt{s} \geq 0.5$ GeV that at around 0.6 GeV is already almost 200%. This is due to the lack of a precise knowledge of the phase of the $\gamma\gamma \to \pi\pi$ $I = 0$ S-wave amplitude above $4m_K^2$. We showed in Refs. [2, 3] that one can largely remove the sensitivity for lower energies, $\sqrt{s} \lesssim 0.8$ GeV, on the uncertainty in the not precisely known phase of the $I = 0$ S-wave $\gamma\gamma \to \pi\pi$ amplitude above the $K\bar{K}$ threshold. The novelty was to include a further subtraction in the dispersion relation for the

$I = 0$ S-wave $\gamma\gamma \to \pi\pi$, together with an extra constraint to fix the additional subtraction constant. This was motivated by the use of an improved $I = 0$ S-wave Omnès function. In Ref. [2, 3] it is discussed in detail how the $f_0(980)$ peak, clearly seen recently in $\gamma\gamma \to \pi^+\pi^-$ [16], can be generated within the dispersive method. As a result, the remaining ambiguity in this phase by π in ref.[2] is removed and this allows to sharpen the prediction of the $\gamma\gamma \to \pi^0\pi^0$ cross section for $\sqrt{s} \lesssim 0.8$ GeV, up to the onset of D-waves, as compared with ref.[2]. This could then be used to constraint further different parameterizations of the low energy $\pi\pi$ $I = 0$ S-wave.

2. The scalar form factor of the pion

Ref.[4] makes use of an Omnès representation for the pion scalar form factor,

$$\Gamma_\pi(t) = P(t) \exp\left[\frac{t}{\pi}\int_{4M_\pi^2}^{\infty} ds' \frac{\phi_0(s')}{s'(s'-t-i\varepsilon)}\right] . \tag{2.1}$$

Here, $P(t)$ is a polynomial in t normalized such that $P(0) = \Gamma_\pi(0)$ and whose zeroes are those of $\Gamma_\pi(t)$. On the other hand, $\phi_0(t)$ is the continuous phase of $\Gamma_\pi(t)/P(t)$. Refs. [4, 5] make an assumption that is not always necessarily fulfilled. Namely, to identify $\phi_0(t)$ with the phase of $\Gamma_\pi(t)$, that we denote in the following as $\rho(t)$. If this identification is done it follows that $P(t)$ must be a constant. One must be aware that in Eq. (2.1) $\phi_0(t)$ is the phase of $\Gamma_\pi(t)/P(t)$. Notice that the phase of $\Gamma_\pi(t)$ is not continuous when crossing a zero located at $t_1 \in \mathbb{R}$, since there is a flip in the sign when passing through. However, the phase of $\Gamma_\pi(t)/P(t)$ *is* continuous, since the zero is removed. This is the phase one should use in the Omnès representation, Eq. (2.1), because it results from a dispersion relation of $\log\Gamma_\pi(t)/P(t)$, and then $\phi(t)$ must be continuous (but not necessarily $\rho(t)$).

As stated, Ref. [4] took $\Gamma_\pi(t) = \Gamma_\pi(0)\exp\left[\frac{t}{\pi}\int_{4M_\pi^2}^{\infty} ds' \frac{\rho(s')}{s'(s'-t-i\varepsilon)}\right]$. So that the scalar form factor is given by, $\langle r^2\rangle_s^\pi = \frac{6}{\pi}\int_{4M_\pi^2}^{+\infty} \frac{\rho(s)}{s^2} ds$. The phase $\rho(s)$ is fixed in Refs. [4, 5] by invoking Watson's final state theorem. For $s < s_K$, $s_K = 4M_K^2$, it implies that $\rho(s) = \delta_0(s)$, where neglecting inelasticity due to multipion states, an experimental fact. For $1.42 > \sqrt{s} \gtrsim 1.1$ GeV, Ref. [4] stressed the interesting fact that experimentally the inelasticity turns out to be small and hence Watson's final state theorem can be applied approximately again. In the narrow region between $2M_K$ and 1.1 GeV inelasticity cannot be neglected but Ref. [4] argues that, as it is so narrow, its contribution to Eq. (2) is small anyhow and, furthermore, that the elasticity parameter η is not so small, so that one could still apply Watson's final state theorem with corrections. Finally, for $s > s_0 = 2$ GeV^2 Ref. [4] takes a linear extrapolation from $\delta_0(s_0)$ to π. One should here criticize that it is still a long way to run from values of $\delta_0(s_0) \lesssim 2\pi$ up to π at $s \to +\infty$. With all these ingredients, and some error estimates, the value $\langle r^2\rangle_s^\pi = 0.75 \pm 0.07$ fm^2 results [4, 5].

The steps performed in Ref. [4] are not always compatible. In Ref. [1] we took as granted the assumption that Watson's final state theorem can be approximately applied for 1.5 GeV $> \sqrt{s} > 2M_K$. Our assumption is in agreement with any explicit calculation of the pion non-strange $I = 0$ scalar form factor. Now, Watson's final state theorem implies that $\phi(s) = \varphi(s)$ (modulo π), with $\varphi(s)$ the phase of the $I = 0$ S-wave $\pi\pi$ amplitude, $t_{\pi\pi} = (\eta e^{2i\delta_0} - 1)/2i$. It occurs, as stressed in Refs. [17, 5], that $\varphi(s)$ can be either $\sim \delta_0(s)$ or $\sim \delta_0(s) - \pi$ depending on whether $\delta_0(s_K) > \pi$ or $< \pi$, respectively, for $s_K < s < 2$ GeV^2. The latter case corresponds to the calculation in Ref. [8], while the former is the preferred one in Ref. [5] and arguments are put forward for this preference in this reference. Let us evolve continuously from one situation ($\delta_0(s_K) < \pi$) to the other ($\delta_0(s_K) > \pi$). In the first case $\varphi(s)$ has an abrupt drop for $s > s_K$ simply because then $\eta < 1$ and while the real part of $t_{\pi\pi}$ rapidly changes sign, its imaginary part is positive (> 0). The rapid movement in the real part is due to the swift one in $\delta_0(s)$ in the $K\bar{K}$ threshold due to the $f_0(980)$ resonance. As a result for $s \lesssim s_K$, $\varphi(s) = \delta_0(s) \simeq \pi$ and for $s \gtrsim s_K$ then $\varphi(s) < \pi/2$. This rapid movement gives rise to a rapid drop in the Omnés function, Eq. (2), so that the modulus of the form factor has a deep minimum around s_K. Here, one is using Watson's final state theorem with $\phi_0(s) = \varphi(s)$ and the form factor of Ref. [11] is reproduced. Notice as well that in this case the function $\phi(s)$ approaches π from below for asymptotic s and then $P(t) = \Gamma_0(0)$ in Eq. (2.1). Now, we consider the limit $\delta_0(s) \to \pi^-$ for $s \to s_K^-$. The superscript $-(+)$ indicates that the limit is approached from below(above). In the limit, the change in sign in the real part of $t_{\pi\pi}$ occurs precisely at s_K, so that for $s = s_K^-$, $\varphi(s) = \pi$ and for $s = s_K^+$ then $\varphi(s) < \pi/2$. As a result one has a drop by $-\pi$ in $\varphi(s)$ which gives rise to a zero in the Omnès representation of the scalar form factor. Thus, the deep has evolved to a zero when $\delta_0(s_K) \to \pi^-$. Because of this zero the proper Omnès representation now involves a $P(t) = \Gamma_\pi(0)(1 - t/s_K)$ and $\phi(s)$ is no longer $\varphi(s)$ but $\simeq \varphi(s) + \pi \simeq \delta_0(s)$ for $2.25\ \mathrm{GeV}^2 > s > s_K$. This follows simply because $\phi(s)$ is continuous. Thus, we go into a new realm where $\phi(s) \simeq \delta_0(s)$ and the degree of $P(t)$ is 1, so that $\Gamma_\pi(t)$ has a zero at the point s_1 where $\delta_0(s_1) = \pi$ and $s_1 < s_K$. Note that only at s_1 the imaginary part of $\Gamma_\pi(t)$ is zero and this fixes the position of the zero [1].

Hence for $\delta_0(s_K) \geq \pi$ one has to use

$$\Gamma_\pi(t) = \Gamma_\pi(0)\left(1 - \frac{t}{s_K}\right) \exp\left[\frac{t}{\pi}\int_{4M_\pi^2}^{\infty} ds' \frac{\phi(s')}{s'(s'-t-i\varepsilon)}\right] , \tag{2.2}$$

with $\phi(s) \simeq \delta_0(s)$ for $s < 2.25\ \mathrm{GeV}^2$. The uncertainties in this approximation for $s > s_K$ are discussed in Ref. [1] and included in the final error in $\langle r^2\rangle_s^\pi$.

Our final result is

$$\langle r^2\rangle_s^\pi = 0.63 \pm 0.05\ \mathrm{fm}^2. \tag{2.3}$$

The error takes into account different $\pi\pi$ $I = 0$ S-wave parameterizations, namely those of Refs. [8] and [18], the error in the application of Watson's final state theorem above 1 GeV and up to 1.5 GeV, and the uncertainties in $\phi(s)$ given by asymptotic QCD for $s > 2.25\ \mathrm{GeV}^2$. This value is compatible with that of Ref. [8], $\langle r^2\rangle_s^\pi = 0.61 \pm 0.04\ \mathrm{fm}^2$, and also with $\langle r^2\rangle_s^\pi = 0.64 \pm 0.06\ \mathrm{fm}^2$ of Ref. [13] calculated from Unitary CHPT.

3. $\gamma\gamma \to \pi\pi$

In this section we report on the results of refs.[2, 3], where a more detailed account can be found. Let us consider the S-wave amplitude $\gamma\gamma \to (\pi\pi)_I$, $F_I(s)$, where the two pions have definite $I = 0$ or 2. The function $F_I(s)$ on the complex $s-$plane is analytic except for two cuts along the real $s-$axis, the unitarity one for $s \geq 4m_\pi^2$ and the left hand cut for $s \leq 0$, with m_π the pion mass. Let us denote by $L_I(s)$ the complete left hand cut contribution to $F_I(s)$. Then, the function $F_I(s) - L_I(s)$, by construction, has only right hand cut. Let $\phi_I(s)$ be the phase of $F_I(s)$ modulo π, chosen in such a way that $\phi_I(s)$ is *continuous* and $\phi_I(4m_\pi^2) = 0$. For the exotic $I = 2$ S-wave one can invoke Watson's final state theorem[#1] so that $\phi_2(s) = \delta_\pi(s)_2$. For $I = 0$ the same theorem guarantees that $\phi_0(s) = \delta_\pi(s)_0$ for $s \leq 4m_K^2$, where we denote by $\delta_\pi(s)_I$ the isospin I S-wave $\pi\pi$ phase shifts. Here one neglects the inelasticity due to the 4π and 6π states below the two kaon threshold. Above the two kaon threshold $s_K = 4m_K^2$, the phase function $\phi_0(s)$ cannot be fixed *a priori* due the onset of inelasticity. However, as remarked in refs.[4, 1], inelasticity is again small for $\sqrt{s} \gtrsim 1.1$ GeV , and one can then apply approximately Watson's final state theorem which implies that $\phi_0(s) \simeq \delta^{(+)}(s)$ modulo π. Here $\delta^{(+)}(s)$ is the eigenphase of the $\pi\pi$, $K\bar{K}$ $I = 0$ S-wave S-matrix such that it is continuous and $\delta^{(+)}(s_K) = \delta_\pi(s_K)_0$. In refs.[5, 1] it is shown that $\delta^{(+)}(s) \simeq \delta_\pi(s)_0$ or $\delta_\pi(s)_0 - \pi$, depending on whether $\delta_\pi(s_K)_0 \geq \pi$ or $< \pi$, respectively. In order to fix the integer factor in front of π in the relation $\phi_0(s) \simeq \delta^{(+)}(s)$ modulo π, it is necessary to follow the possible trajectories of $\phi_0(s)$ in the *narrow* region $1 \lesssim \sqrt{s} \lesssim 1.1$ GeV. The remarkable physical effects happening there are the appearance of the $f_0(980)$ resonance on top of the $K\bar{K}$ threshold and the cusp effect of the latter that induces a discontinuity at s_K in the derivative of observables. Between 1.05 to 1.1 GeV there are no further narrow structures and observables evolve smoothly. Approximately half of the region between 0.95 and 1.05 GeV is elastic and $\phi_0(s) = \delta_\pi(s)_0$ (Watson's theorem), so that it raises rapidly. Above $2m_K \simeq 1$ GeV and up to 1.05 GeV the function $\phi_0(s)$ can keep increasing with energy, like $\delta_\pi(s)_0$. The other possibility is a change of sign in the slope at s_K due to the $K\bar{K}$ cusp effect such that $\phi_0(s)$ starts a rapid decrease in energy. Above $\sqrt{s} = 1.05$ GeV, $\phi_0(s)$ matches smoothly with the behaviour for $\sqrt{s} \gtrsim 1.1$ GeV, which is constraint by Watson's final state theorem. As a result, for $\sqrt{s} \gtrsim 1$ GeV *either* $\phi_0(s) \simeq \delta_\pi(s)_0$ *or* $\phi_0(s) \simeq \delta_\pi(s) - \pi$, corresponding to an increasing or decreasing $\phi_0(s)$ above s_K, in order.

Let us define the switch z to characterize the behaviour of $\phi_0(s)$ for $s > s_K$, and close to s_K, such that $z = +1$ if $\phi_0(s)$ rises with energy and $z = -1$ if it decreases. Let s_1 be the value of s at which $\phi_0(s_1) = \pi$. Following ref.[1] we introduce the Omnès function,

$$\Omega_0(s) = \left(1 - \theta(z)\frac{s}{s_1}\right) \exp\left[\frac{s}{\pi}\int_{4m_\pi^2}^{\infty} \frac{\phi_0(s')}{s'(s'-s)}ds'\right] , \tag{3.1}$$

with $\theta(z) = 1$ for $z = +1$ and 0 for $z = -1$. Given the definition of the phase function $\phi_I(s)$ the function $F_I(s)/\Omega_I(s)$ has no right hand cut. Next, we perform a twice subtracted dispersion relation for $(F_0(s) - L_0(s))/\Omega_0(s)$

$$F_0(s) = L_0(s) + c_0 s\Omega_0(s) + \frac{s^2}{\pi}\Omega_0(s)\int_{4m_\pi^2}^{\infty} \frac{L_0(s')\sin\bar{\phi}_0(s')}{s'^2(s'-s)|\Omega_0(s')|}ds' + \theta(z)\frac{\omega_0(s)}{\omega_0(s_1)}\frac{s^2}{s_1^2}(F_0(s_1) - L_0(s_1)) , \tag{3.2}$$

where $\omega_0(s) = \exp\left[\frac{s}{\pi}\int_{4m_\pi^2}^{\infty}\frac{\phi_0(s')}{s'(s'-s)}ds'\right]$. In the previous equation we introduce $\bar{\phi}_0(s)$ that is defined as the phase of $\Omega_0(s)$. Proceeding similarly for $I = 2$ one has

$$F_2(s) = L_2(s) + c_I s\Omega_2(s) + \frac{s^2}{\pi}\Omega_2(s)\int_{4m_\pi^2}^{\infty} \frac{L_2(s')\sin\phi_2(s')}{s'^2(s'-s)|\Omega_2(s')|}ds' . \tag{3.3}$$

It is worth mentioning that eq.(3.2) for $I = 0$ and $z = +1$ is equivalent to perform a three times subtracted dispersion relation for $(F_0(s) - L_0(s))/\omega_0(s)$. Let us denote by $F_N(s)$ the S-wave

[#1] This theorem implies that the phase of $F_I(s)$ when there is no inelasticity is the same, modulo π, as the one of the isospin I S-wave $\pi\pi$ elastic strong amplitude.

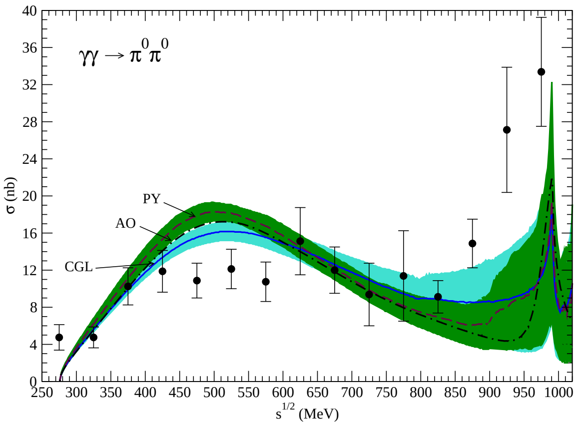

Figure 1: Final result for the $\gamma\gamma \to \pi^0\pi^0$ cross for $\sqrt{s} \leq 1.05$ GeV. The experimental data are from the Crystal Ball Collaboration [20].

$\gamma\gamma \to \pi^0\pi^0$ amplitude and by $F_C(s)$ the $\gamma\gamma \to \pi^+\pi^-$ one. We are still left with the unknown subtraction constants c_0, c_2 for $I = 0$ and 2, respectively, and $F_0(s_1) - L_0(s_1)$ for $I = 0$ and $z = +1$. The c_0, c_2, constants can be obtained from low energy theorems and matching to one loop χPT. The value of $F_0(s_1) - L_0(s_1)$ can be restricted because the cross section $\sigma(\gamma\gamma \to \pi^0\pi^0)$ around the $f_0(980)$ resonance is quite sensitive to this constant. We impose that $\sigma(\gamma\gamma \to \pi^0\pi^0) \leq 40$ nb at s_1. This upper bound for the peak of the $f_0(980)$ in $\gamma\gamma \to \pi^0\pi^0$ is equivalent to impose that the $\gamma\gamma$ width of the $f_0(980)$ lies in the range $205^{+95}_{-83}(stat)^{+147}_{-117}(sys)$ eV as determined in ref.[16]. We shall see that the effect of this rather large uncertainty allowed at 1 GeV, see fig.1, is very mild at lower energies. As the $f_0(980)$ resonance gives rise to a small *peak* in the precise data on $\gamma\gamma \to \pi^+\pi^-$ [16], then $\phi_0(s)$ must increase with energy above s_K and the case with $z = +1$ is the one realized in nature. Note that for $z = -1$ in eq.(3.2), there is no a local maximum associated with this resonance in $|F_0(s)|$ but a minimum, because $|\omega_0(s)|$ has a dip around the $f_0(980)$ mass.

The source of uncertainty in the approximate relation $\phi_0(s) \simeq \delta_\pi(s)_0$ for $4m_K^2 \lesssim \sqrt{s} \lesssim 1.5$ GeV and its functional dependence for $s > s_H = 2.25$ GeV^2 is estimated similarly as in ref.[3, 1]. In fig.1 we show our final results for the $\gamma\gamma \to \pi^0\pi^0$, where the band around each line corresponds to the estimated error. The error band for the dot-dashed line is not shown because it is similar to the ones of the other two curves. In this figure PY refers to using the $I = 0$ S-wave $\pi\pi$ of ref.[18], CGL that of ref.[8] and AO the one of ref.[19]. One observes that for $\sqrt{s} \lesssim 0.8$ GeV the uncertainty in the loose bound for the $f_0(980)$ greatly disappears. For such energies the main source of uncertainty originates from the uncertainties in the $\pi\pi$ phase parameterizations used.

The previous model allows dor an evaluation of the $\sigma \to \gamma\gamma$ width. The coupling $\sigma \to \gamma\gamma$, $g_{\sigma\gamma\gamma}$, can be evaluated from the residue of the amplitude $F_N(s)$ at the second Riemann sheet. It can be easily obtained that

$$\frac{g^2_{\sigma\gamma\gamma}}{g^2_{\sigma\pi\pi}} = -\frac{1}{2}\left(\frac{\sigma_\pi(s_\sigma)}{8\pi}\right)^2 F_0(s_\sigma)^2 \,, \tag{3.4}$$

where $g_{\sigma\pi\pi}$ is the coupling of the σ to $\pi\pi$.

We denote by $s_\sigma = (M_\sigma - i\Gamma_\sigma/2)^2$. Ref.[21] provides $M_\sigma^{CCL} = 441^{+16}_{-8}$ MeV and $\Gamma_\sigma^{CCL} = 544^{+18}_{-25}$ MeV, while from ref.[19] one has $M_\sigma^{AO} = (456 \pm 6)$ MeV and $\Gamma_\sigma^{AO} = (482 \pm 20)$ MeV. In the following the superscripts *AO* and *CCL* refer to those results obtained by employing s_σ from ref.[19] or [21], respectively. From eq.(3.4) we obtain $|g_{\sigma\gamma\gamma}/g_{\sigma\pi\pi}| = 2.01 \pm 0.11$ for s_σ^{CCL} and 1.85 ± 0.09 for s_σ^{AO}. Given s_σ, this ratio of residua is the well defined prediction that follow from our $F_0(s)$. We employ the standard narrow resonance width formula in terms of $g_{\sigma\gamma\gamma}$ to calculate $\Gamma(\sigma \to \gamma\gamma) = \frac{|g_{\sigma\gamma\gamma}|^2}{16\pi M_\sigma}$. One needs to provide numbers for $|g_{\sigma\pi\pi}|$ in order to apply the previous equation and the determined $|g_{\sigma\gamma\gamma}/g_{\sigma\pi\pi}|$. We first consider the value $|g^{AO}_{\sigma\pi\pi}| = (3.17 \pm 0.10)$ GeV from the approach of ref.[19]. The calculated width is $\Gamma^{AO}(\sigma \to \pi\pi) = (1.50 \pm 0.18)$ KeV. Not only the position of the pole in the partial wave amplitude, but also its residue can be calculated in the framework of the dispersive analysis described in ref.[21]. Expressed in terms of the complex coefficient $g_{\sigma\pi\pi}$, the preliminary result for the residue amounts to $|g^{CCL}_{\sigma\pi\pi}| = (3.31^{+0.17}_{-0.08})$ GeV, $\Gamma^{CCL}(\sigma \to \gamma\gamma) = (1.98^{+0.30}_{-0.24})$ KeV. Taking the average between these two values for $\Gamma(\sigma \to \gamma\gamma)$ we end with,

$$\Gamma(\sigma \to \gamma\gamma) = (1.68 \pm 0.15) \text{ KeV} . \tag{3.5}$$

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