

Effective Theories for Flavour Physics beyond the Standard Model

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We discuss the role of flavour physics in building effective theories at the TeV scale. Particular attention is devoted to the Minimal Flavour Violation hypothesis, both in the quark and in the lepton sector. Alternative flavour-protection mechanisms, such as the hierarchical fermion profiles of models with a warped fifth dimension, are also briefly discussed.

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1. Introduction: the SM as an Effective Field Theory

Most of this conference has been devoted to discuss Effective Field Theories (EFT) describing the low-energy limit of the Standard Model (SM); however, the SM itself should be considered as the low-energy limit of a more fundamental theory. In this perspective, three key issues need to be addressed: (1) which is the cut-off, (2) which are the light degrees of freedom, and (3) which are the symmetries of the SM viewed as an EFT.

The first two questions are intimately related to the breaking to the $SU(2)_L \times U(1)_Y$ electroweak gauge symmetry. Experiments provide a clear indication of a spontaneous breaking of $SU(2)_L \times U(1)_Y$, characterised by the order parameter $v = (\sqrt{2}G_F)^{-1/2} \simeq 246$ GeV. If we do not include a Higgs field among the light degrees of freedom of the theory, assuming that the electroweak symmetry breaking occurs because of some new strong dynamics at high energies, the cut-off of the EFT cannot be larger than $4\pi v \approx 3$ TeV, in close analogy to what happens in Chiral Perturbation Theory. If we do assume that there is a Higgs field, with renormalisable potential and non-trivial electroweak vacuum, the EFT becomes renormalisable and it is not obvious how to determine its cut-off. However, also in this case it is *natural* to assume that the EFT breaks down around or below the TeV, because of the instability of the Higgs mass term under quantum corrections.

What I will try to address in this talk in the last of the three questions listed above: which are the symmetries, and in particular which is the flavour symmetry and the related symmetry-breaking pattern, of the SM viewed as an effective theory with a few TeV cut-off. The assumption that the cut-off of the theory is around a few TeV is consistent with the measurements of flavour-conserving electroweak precision observables (EWPO). More precisely, the bounds on gauge-invariant higher-dimensional operators constructed in terms of light degrees of freedom (with or without the Higgs boson) are consistent with an effective cut-off in the TeV range.¹ The situation is very different if we look at operators contributing to flavour-changing observables. Here the good agreement between SM expectations and data leads to bounds on the EFT cut-off which are orders of magnitude higher, unless some protective flavour symmetry is invoked to suppress the corresponding couplings.

The need of additional global symmetries does not come only from the flavour sector: the most stringent constraints on the EFT arise by baryon- and lepton-number violating processes. However, since B and L are exact symmetries of the SM Lagrangian, in this case the problem can easily be solved promoting B and L to be exact symmetries of the new dynamics at the TeV scale. The peculiar aspects of flavour physics is that there is no exact flavour symmetry in the low-energy theory. In this case it is not sufficient to invoke a flavour symmetry for the EFT. We also need to specify how this symmetry is broken in order to describe the observed low-energy spectrum and, at the same time, be in agreement with the precise experimental tests of flavour-changing processes.

2. The flavour problem

Assuming for simplicity that there is a single elementary Higgs field, responsible for the

¹ In the Higgsless case this statement is true only if we impose the so-called $SU(2)$ *custodial symmetry* on the leading operators responsible for the electroweak symmetry breaking.

$SU(2)_L \times U(1)_Y \rightarrow U(1)_Q$ spontaneous breaking, the Lagrangian of our EFT can be written as follows

$$\mathcal{L}_{\text{eff}} = \mathcal{L}_{\text{gauge}}^{\text{SM}} + \mathcal{L}_{\text{Higgs}}^{\text{SM}} + \mathcal{L}_{\text{Yukawa}}^{\text{SM}} + \Delta\mathcal{L}_{d>4}, \quad (2.1)$$

where $\Delta\mathcal{L}_{d>4}$ denotes the series of higher-dimensional operators constructed in terms of SM fields and invariant under the SM gauge group:

$$\Delta\mathcal{L}_{d>4} = \sum_{d>4} \sum_{n=1}^{N_d} \frac{c_n^{(d)}}{\Lambda^{d-4}} \mathcal{O}_n^{(d)} \quad (2.2)$$

As discussed in the introduction, we should expect $\Lambda = O(\text{few TeV})$ and $c_i^{(d)} = O(1)$, if the corresponding operator is not suppressed by some protective symmetry (such a B or L conservation). The observation that that this expectation is *not* fulfilled by several dimension-six operators contributing to flavour-changing processes is often denoted as the *flavour problem*.

Let's consider for instance the following set of $\Delta F = 2$ dimensions-six operators

$$\mathcal{O}_{ij}^{\Delta F=2} = (\bar{Q}_L^i \gamma^\mu Q_L^j)^2, \quad \mathcal{Q}_L^i = \begin{pmatrix} u_L^i \\ d_L^i \end{pmatrix}, \quad (2.3)$$

where $i, j = 1 \dots 3$ are flavour indices and, for reasons that will become clear in the following, we work in mass-eigenstate basis of down-type quarks. These operators contribute at the tree-level to the meson-antimeson mixing amplitudes, which in the SM are generated only at the one-loop level. The case of $K^0 - \bar{K}^0$, $B_d - \bar{B}_d$, $B_s - \bar{B}_s$ mixing is particularly interesting: here the SM contribution is dominated by the short-distance contribution of top-quark loops

$$\mathcal{A}_{\text{SM}}^{\Delta F=2} \approx \frac{m_t^2}{16\pi^2 v^4} (V_{3i}^* V_{3j})^2 \times \langle \bar{M} | (\bar{d}_L^i \gamma^\mu d_L^j)^2 | M \rangle \quad [M = K^0, B_d, B_s], \quad (2.4)$$

(V_{ij} denote the elements of the CKM matrix) and can be computed with high accuracy. Moreover, both moduli and phases of these three mixing amplitudes have been determined with good accuracy (with the exception of the CP-violating case in $B_s - \bar{B}_s$ mixing) from experiments. In all cases the magnitude of the new-physics amplitude (or the term generated by the dimension-six operators) cannot exceed, in size, the SM short-distance contribution. Denoting c_{ij} the couplings of the non-standard operators in (2.3), the condition $|\mathcal{A}_{\text{NP}}^{\Delta F=2}| < |\mathcal{A}_{\text{SM}}^{\Delta F=2}|$ implies

$$\Lambda < \frac{3.4 \text{ TeV}}{|V_{3i}^* V_{3j}| / |c_{ij}|^{1/2}} < \begin{cases} 9 \times 10^3 \text{ TeV} \times |c_{21}|^{1/2} & \text{from } K^0 - \bar{K}^0 \\ 4 \times 10^2 \text{ TeV} \times |c_{31}|^{1/2} & \text{from } B_d - \bar{B}_d \\ 7 \times 10^1 \text{ TeV} \times |c_{32}|^{1/2} & \text{from } B_s - \bar{B}_s \end{cases} \quad (2.5)$$

The message of these bounds is quite clear: if we want to keep Λ in the TeV range, the EFT must have a highly non-generic flavour structure. In the specific case of the $\Delta F = 2$ operators in (2.3), we must find a symmetry argument such that $|c_{ij}| \lesssim |V_{3i}^* V_{3j}|$. A similar problem is found also in the case of $\Delta F = 1$ operators contributing to flavour-changing neutral-current (FCNC) processes. In this case the bounds are different, but the main problem is the same: FCNC and $\Delta F = 2$ amplitudes are suppressed in the SM not only by the typical $1/(4\pi)^2$ of loop amplitudes, but also by the GIM mechanisms and the hierarchy of the CKM matrix ($|V_{3i}| \ll 1$, for $i \neq 3$).

The most reasonable (but also most *pessimistic*) solution to the flavour problem is the so-called Minimal Flavour Violation hypothesis that will be discussed below.

3. Minimal Flavour Violation in the quark sector

The main idea of MFV is that flavour-violating interactions are linked to the known structure of Yukawa couplings also beyond the SM. In a more quantitative way, the MFV construction consists in identifying the flavour symmetry and symmetry-breaking structure of the SM and enforce it the EFT.

The largest group of flavour-changing field transformations commuting with the SM gauge group is $\mathcal{G}_f = \mathcal{G}_q \otimes \mathcal{G}_\ell \otimes U(1)^5$, where

$$\mathcal{G}_q = \text{SU}(3)_{Q_L} \otimes \text{SU}(3)_{U_R} \otimes \text{SU}(3)_{D_R}, \quad \mathcal{G}_\ell = \text{SU}(3)_{L_L} \otimes \text{SU}(3)_{E_R} \quad (3.1)$$

and three of the five $U(1)$ charges can be identified with baryon number, lepton number and hypercharge [1].² This large group and, particularly the $\text{SU}(3)$ subgroups controlling flavour-changing transitions, is explicitly broken by the Yukawa interaction

$$\mathcal{L}_Y = \bar{Q}_L \lambda_d D_R H + \bar{Q}_L \lambda_u U_R H_c + \bar{L}_L \lambda_e E_R H + \text{h.c.} \quad (3.2)$$

Since \mathcal{G}_f is broken already within the SM, it would not be consistent to impose it as an exact symmetry of the additional degrees of freedom present in SM extensions: even if absent at the tree-level, the breaking of \mathcal{G}_f would reappear at the quantum level because of the Yukawa interaction. The most restrictive hypothesis we can make to *protect* the breaking of \mathcal{G}_f in a consistent way, is to assume that λ_d , λ_u and λ_e are the only source of \mathcal{G}_f -breaking also beyond the SM.

To derive the phenomenological consequences of this hypothesis, it is convenient to treat \mathcal{G}_f as an unbroken symmetry of the underlying theory, promoting the λ_i to be non-dynamical fields (spurions) with non-trivial transformation properties under \mathcal{G}_f

$$\lambda_u \sim (3, \bar{3}, 1)_{\text{SU}(3)_q^3}, \quad \lambda_d \sim (3, 1, \bar{3})_{\text{SU}(3)_q^3}, \quad \lambda_e \sim (3, \bar{3})_{\text{SU}(3)_\ell^2}. \quad (3.3)$$

Employing the EFT language, we then define that an effective theory satisfies the criterion of Minimal Flavour Violation if all higher-dimensional operators, constructed from SM fields and λ spurions, are invariant under the flavour group \mathcal{G}_f [2].

According to this criterion, one should in principle consider operators with arbitrary powers of the (adimensional) Yukawa spurions. However, a strong simplification arises by the observation that all the eigenvalues of the Yukawa matrices are small, but for the top one, and that the off-diagonal elements of the CKM matrix (V_{ij}) are very suppressed. In the mass-eigenstate basis of down-type quarks, the two quark Yukawa coupling assume the following form:

$$\lambda_d = \text{diag}(y_d, y_s, y_b), \quad \lambda_u = V^\dagger \times \text{diag}(y_u, y_c, y_t), \quad y_i = \frac{\sqrt{2}m_i}{v}. \quad (3.4)$$

It is then easy to realize that, similarly to the pure SM case, the leading coupling ruling all FCNC transitions with external down-type quarks is [2]

$$(\Delta_{LL}^q)_{i \neq j} = (\lambda_u \lambda_u^\dagger)_{ij} \approx y_t^2 V_{3i}^* V_{3j}, \quad y_t = m_t/v \approx 1. \quad (3.5)$$

²Since hypercharge is gauged and involves also the Higgs field, it may be more convenient not to include it in the flavour group, which would then be defined as $\mathcal{G}_{\text{SM}} = \mathcal{G}_\ell \otimes U(1)^4$ [3].

Higher-order spurion combinations contributing to FCNCs are either negligible or proportional to Δ_{LL}^q .

The suppression of the off-diagonal entries of Δ_{LL}^q implies that, within the MFV framework, the bounds on the scale of dimension-six FCNC effective operators are in the few TeV range (detailed bounds for $\Delta F = 2$ and $\Delta F = 1$ operators can be found in [4] and [5], respectively). Moreover, the flavour structure of Δ_{LL}^q implies a well-defined link among possible deviations from the SM in FCNC transitions of the type $s \rightarrow d$, $b \rightarrow d$, and $b \rightarrow s$ (the only quark-level transitions where observable deviations from the SM are expected).

3.1 General considerations

The idea that the CKM matrix rules the strength of FCNC transitions also beyond the SM is a concept that has been implemented and discussed in several works, especially after the first results of the B factories (see e.g. Ref. [6, 7]). However, it is worth stressing that the CKM matrix represents only one part of the problem: a key role in determining the structure of FCNCs is also played by quark masses (via the GIM mechanism), or by the Yukawa eigenvalues. In this respect, the above MFV criterion provides the maximal protection of FCNCs (or the minimal violation of flavour symmetry), since the full structure of Yukawa matrices is preserved. Moreover, contrary to other approaches, the above MFV criterion is based on a renormalization-group-invariant symmetry argument, which can easily be extended to TeV-scale effective theories where new degrees of freedoms, such as extra Higgs doublets or SUSY partners of the SM fields are included.

In particular, it is worth stressing that the MFV hypothesis can be implemented also if the EFT does not include a light Higgs boson in the spectrum. In this case Eq. (3.2) is replaced by an effective interaction between fermion fields and the Goldstone bosons associated to the spontaneous breaking of the gauge symmetry. From the point of view of the flavour symmetry, this interaction is identical to the one in (3.2), and this allows us to proceed as in the case with the explicit Higgs field (see e.g. Ref. [8]). The only difference between weakly- and strongly-interacting theories at the TeV scale is that in the latter case the expansion in powers of the Yukawa spurions is not necessarily a rapidly convergent series. If this is the case, then a resummation of the terms involving the top-quark Yukawa coupling needs to be performed [9, 10].

As shown in Fig. 1, the MFV hypothesis provides a natural (a posteriori) justification of why no NP effects have been observed in the quark sector: by construction, most of the clean observables measured at B factories are insensitive to NP effects in this framework. However, it should be stressed that we are still very far from having proved the validity of this hypothesis from data. A proof of the MFV hypothesis can be achieved only with a positive evidence of physics beyond the SM exhibiting the flavour pattern predicted by the MFV assumption. This proof could come from the $B_{s,d} \rightarrow \ell^+ \ell^-$ or the $K \rightarrow \pi \nu \bar{\nu}$ systems: in both cases there is still room for sizable non-standard contributions (even within the MFV framework), and we can identify two observables unambiguously correlated by the MFV hypothesis [5, 11]

This model-independent structure does not hold in most of the alternative definitions of MFV models that can be found in the literature. For instance, the definition of Ref. [7] (denoted constrained MFV, or CMFV) contains the additional requirement that the effective FCNC operators playing a significant role within the SM are the only relevant ones also beyond the SM. This condition is realized only in weakly coupled theories at the TeV scale with only one light Higgs doublet,

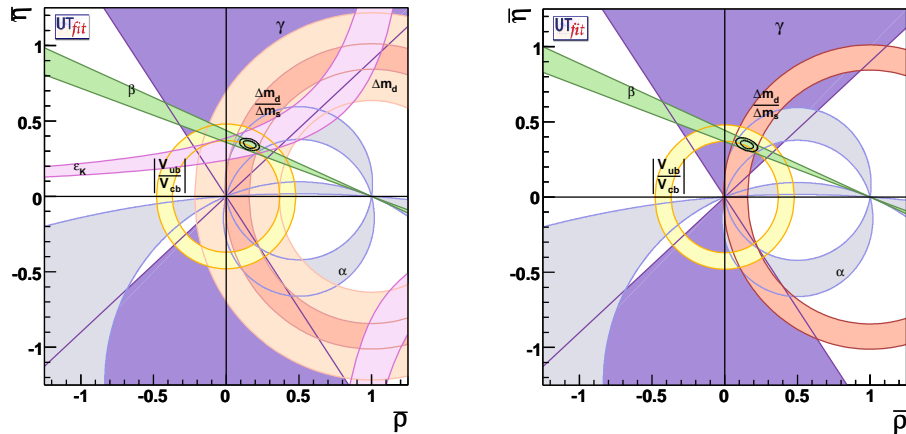


Figure 1: Fit of the CKM unitarity triangle (in 2008) within the SM (left) and in generic extensions of the SM satisfying the MFV hypothesis (right) [4].

such as the MSSM with small $\tan\beta$. It does not hold in several other frameworks, such as Higgsless models, or the MSSM with large $\tan\beta$.

3.2 MFV at large $\tan\beta$.

If the Yukawa Lagrangian contains more than one Higgs field, we can still assume that the Yukawa couplings are the only irreducible breaking sources of \mathcal{G}_q , but we can change their overall normalization. A particularly interesting scenario is the two-Higgs-doublet model where the two Higgses are coupled separately to up- and down-type quarks:

$$\mathcal{L}_Y^{2HDM} = \bar{Q}_L \lambda_d D_R H_D + \bar{Q}_L \lambda_u U_R H_U + \bar{L}_L \lambda_e E_R H_D + \text{h.c.} \quad (3.6)$$

This Lagrangian is invariant under an extra $U(1)$ symmetry with respect to the one-Higgs Lagrangian in Eq. (3.2): a symmetry under which the only charged fields are D_R and E_R (charge +1) and H_D (charge -1). This symmetry, denoted U_{PQ} , prevents tree-level FCNCs and implies that $\lambda_{u,d}$ are the only sources of \mathcal{G}_q breaking appearing in the Yukawa interaction (similar to the one-Higgs-doublet scenario). Coherently with the MFV hypothesis, we can then assume that $\lambda_{u,d}$ are the only relevant sources of \mathcal{G}_q breaking appearing in all the low-energy effective operators. This is sufficient to ensure that flavour-mixing is still governed by the CKM matrix, and naturally guarantees a good agreement with present data in the $\Delta F = 2$ sector. However, the extra symmetry of the Yukawa interaction allows us to change the overall normalization of $\lambda_{u,d}$ with interesting phenomenological consequences in specific rare modes.

The normalization of the Yukawa couplings is controlled by the ratio of the vacuum expectation values of the two Higgs fields, or by the parameter $\tan\beta = \langle H_U \rangle / \langle H_D \rangle$. For $\tan\beta \gg 1$ the smallness of the b quark and τ lepton masses can be attributed to the smallness of $1/\tan\beta$ rather than to the corresponding Yukawa couplings. As a result, for $\tan\beta \gg 1$ we cannot anymore neglect the down-type Yukawa coupling. Moreover, the $U(1)_{PQ}$ symmetry cannot be exact: it has to be broken at least in the scalar potential in order to avoid the presence of a massless pseudoscalar

Higgs. Even if the breaking of $U(1)_{PQ}$ and \mathcal{G}_q are decoupled, the presence of $U(1)_{PQ}$ breaking sources can have important implications on the structure of the Yukawa interaction, especially if $\tan\beta$ is large [12, 13, 14, 15]. We can indeed consider new dimension-four operators such as

$$\varepsilon \bar{Q}_L \lambda_d D_R (H_U)^c \quad \text{or} \quad \varepsilon \bar{Q}_L \lambda_u \lambda_u^\dagger \lambda_d D_R (H_U)^c, \quad (3.7)$$

where ε denotes a generic MFV-invariant $U(1)_{PQ}$ -breaking source. Even if $\varepsilon \ll 1$, the product $\varepsilon \times \tan\beta$ can be $\mathcal{O}(1)$, inducing large corrections to the down-type Yukawa sector:

$$\varepsilon \bar{Q}_L \lambda_d D_R (H_U)^c \xrightarrow{vev} \varepsilon \bar{Q}_L \lambda_d D_R \langle H_U \rangle = (\varepsilon \times \tan\beta) \bar{Q}_L \lambda_d D_R \langle H_D \rangle. \quad (3.8)$$

Since the b -quark Yukawa coupling becomes $\mathcal{O}(1)$, the large- $\tan\beta$ regime is particularly interesting for helicity-suppressed observables in B physics.

One of the clearest phenomenological consequences is a suppression (typically in the 10–50% range) of the $B \rightarrow \ell\nu$ decay rate with respect to its SM expectation [16, 17, 18]. But the most striking signature could arise from the rare decays $B_{s,d} \rightarrow \ell^+ \ell^-$ whose rates could be enhanced over the SM expectations by more than one order of magnitude [19, 20, 14]. An enhancement of both $B_s \rightarrow \ell^+ \ell^-$ and $B_d \rightarrow \ell^+ \ell^-$ respecting the MFV relation $\Gamma(B_s \rightarrow \ell^+ \ell^-)/\Gamma(B_d \rightarrow \ell^+ \ell^-) \approx |V_{ts}/V_{td}|^2$ would be an unambiguous signature of MFV at large $\tan\beta$ [5].

3.3 MFV in supersymmetry

One of the explicit new-physics framework where the MFV hypothesis could be more easily be implemented is the minimal supersymmetric extension of the SM (MSSM). Since the squark fields have well-defined transformation properties under the SM quark-flavour group \mathcal{G}_q , the MFV hypothesis can easily be implemented in the MSSM framework following the general rules outlined above.

We need to consider all possible interactions compatible with i) softly-broken supersymmetry; ii) the breaking of \mathcal{G}_q via the spurion fields $\lambda_{u,d}$. This allows to express the squark mass terms and the trilinear quark-squark-Higgs couplings as follows [21, 2]:

$$\begin{aligned} \tilde{m}_{Q_L}^2 &= \tilde{m}^2 \left(a_1 \mathbb{1} + b_1 \lambda_u \lambda_u^\dagger + b_2 \lambda_d \lambda_d^\dagger + b_3 \lambda_d \lambda_d^\dagger \lambda_u \lambda_u^\dagger + b_4 \lambda_u \lambda_u^\dagger \lambda_d \lambda_d^\dagger + \dots \right), \\ \tilde{m}_{U_R}^2 &= \tilde{m}^2 \left(a_2 \mathbb{1} + b_5 \lambda_u^\dagger \lambda_u + \dots \right), \quad A_U = A \left(a_3 \mathbb{1} + b_6 \lambda_d \lambda_d^\dagger + \dots \right) \lambda_u, \end{aligned} \quad (3.9)$$

and similarly for the down-type terms. The dimensionful parameters \tilde{m} and A , expected to be in the range few 100 GeV – 1 TeV, set the overall scale of the soft-breaking terms. In Eq. (3.9) we have explicitly shown all independent flavour structures which cannot be absorbed into a redefinition of the leading terms (up to tiny contributions quadratic in the Yukawas of the first two families), when $\tan\beta$ is not too large and the bottom Yukawa coupling is small, the terms quadratic in λ_d can be dropped.

In a bottom-up approach, the dimensionless coefficients a_i and b_i should be considered as free parameters of the model. Note that this structure is renormalization-group invariant: the values of a_i and b_i change according to the Renormalization Group (RG) flow, but the general structure of Eq. (3.9) is unchanged. This is not the case if the b_i are set to zero, corresponding to the so-called hypothesis of *flavour universality*. In several explicit mechanism of supersymmetry breaking, the

condition of flavour universality holds at some high scale M , such as the scale of Grand Unification in the CMSSM (see below) or the mass-scale of the messenger particles in gauge mediation (see Ref. [22]). In this case non-vanishing $b_i \sim (1/4\pi)^2 \ln M^2/\tilde{m}^2$ are generated by the RG evolution. As recently pointed out in Ref. [23, 24], the RG flow in the MSSM-MFV framework exhibit quasi infra-red fixed points: even if we start with all the $b_i = \mathcal{O}(1)$ at some high scale, the only non-negligible terms at the TeV scale are those associated to the $\lambda_u \lambda_u^\dagger$ structures.

While MFV can easily be implemented in the MSSM, it is worth to stress that the flavour problem could have a different solution in supersymmetric extensions of the SM. For instance, one interesting possibility is that there is no special flavor symmetry, but the first two generations of squarks are well above the TeV scale (see Ref. [25] for a recent analysis). Keeping only the third generation light is sufficient to stabilise the Higgs sector. On the other hand, with heavy squarks for the first two generations the severe bound from the kaon system in (2.5) are less problematic.

4. MFV in the lepton sector

Apart from arguments based on the analogy between quarks and leptons, the introduction of a MFV hypothesis for the lepton sector (MLFV) is demanded by a severe fine-tuning problem also in the lepton sector: within a generic EFT approach, the non-observation of $\mu \rightarrow e\gamma$ implies an effective NP scale above 10^5 TeV unless the coupling of the corresponding effective operator is suppressed by some symmetry principle.

Since the observed neutrino mass parameters are not described by the SM Yukawa interaction in Eq. (3.2), the formulation of a MLFV hypothesis is not straightforward, and some additional hypothesis is needed. A first natural assumption is that the breaking of the total lepton number (L) and the lepton flavour –the \mathcal{G}_ℓ group in (3.1)– are decoupled in the underlying theory. Following Ref. [26] we can then consider two main scenarios. They are characterized by the different status assigned to the effective Majorana mass matrix g_ν appearing as coefficient of the L -violating dimension-five operator in the low energy effective theory [27]:

$$\mathcal{L}_{\text{eff}}^\nu = -\frac{1}{\Lambda_{\text{LN}}} g_\nu^{ij} (\bar{L}_L^i \tau_2 H) (H^T \tau_2 L_L^j) + \text{h.c.} \quad \longrightarrow \quad m_\nu = \frac{g_\nu v^2}{\Lambda_{\text{LN}}} \quad (4.1)$$

In the truly minimal scenario (dubbed *minimal field content*), g_ν and the charged-lepton Yukawa coupling (λ_e) are assumed to be the only irreducible sources of breaking of \mathcal{G}_ℓ , the lepton-flavour symmetry of the low-energy theory.

The irreducible character of g_ν does not hold in many realistic underlying theories with heavy right-handed neutrinos. For this reason, a second scenario (dubbed *extended field content*), with heavy right-handed neutrinos and a larger lepton-flavour symmetry group, $\mathcal{G}_\ell \times \text{O}(3)_{\nu_R}$, has also been considered. In this extended scenario, a natural and economical choice about the symmetry-breaking terms is the identification of the two Yukawa couplings, λ_ν and λ_e , as the only irreducible symmetry-breaking structures. In this context, $g_\nu \sim \lambda_\nu^T \lambda_\nu$ and the L -violating mass term of the heavy right-handed neutrinos is flavour-blind (up to Yukawa-induced corrections):

$$\begin{aligned} \mathcal{L}_{\text{heavy}} &= -\frac{1}{2} M_\nu^{ij} \bar{\nu}_R^i \nu_R^j + \text{h.c.} & M_\nu^{ij} &= M_\nu \delta^{ij} \\ \mathcal{L}_Y^{\text{ext}} &= \mathcal{L}_Y + i\lambda_\nu^{ij} \bar{\nu}_R^i (H^T \tau_2 L_L^j) + \text{h.c.} \end{aligned} \quad (4.2)$$

In this scenario the flavour changing coupling relevant to $l_i \rightarrow l_j \gamma$ decays reads

$$(\Delta_{LR}^\ell)_{\text{MLFV}} \propto \lambda_e \lambda_\nu^\dagger \lambda_\nu \rightarrow \frac{m_e}{v} \frac{M_\nu}{v^2} U_{\text{PMNS}} (m_\nu^{1/2})_{\text{diag}} H^2 (m_\nu^{1/2})_{\text{diag}} U_{\text{PMNS}}^\dagger \quad (4.3)$$

where H is an Hermitian-orthogonal matrix which can be parametrized in terms of three real parameters (ϕ_i) which control the amount of CP-violation in the right-handed sector [28]. In the CP-conserving limit, $H \rightarrow I$ and the phenomenological predictions for lepton FCNC decays turns out to be quite similar to the minimal field content scenario [26].

Once the field content of model is extended, there are in principle many alternative options to define the irreducible sources of lepton flavour symmetry breaking (see Ref. [29, 30] for an extensive discussion and alternative scenarios). The specific choice discussed above has two important advantages: it is predictive and closely resemble the MFV hypothesis in the quark sector. The ν_R 's are the counterpart of right-handed up quarks and, similarly to the quark sector, the symmetry-breaking sources are two Yukawa couplings.

The basic assumptions of the MLFV hypotheses are definitely less data-driven with respect to the quark sector. Nonetheless, the formulation of an EFT based on these assumptions is still very useful. As I will briefly illustrate in the following, it allows us to address in a very general way the following fundamental question: how can we detect the presence of new irreducible (fundamental) sources of LF symmetry breaking?

4.1 Phenomenological consequences

Using the MLFV-EFT approach, one can easily demonstrate that –in absence of new sources of LF violation– visible FCNC decays of μ and τ can occur only if there is a large hierarchy between Λ (the scale of new degrees of freedoms carrying LF) and $\Lambda_{\text{LN}} \sim M_\nu$ (the scale of total LN violation) [26]. More interestingly, the EFT allows us to draw unambiguous predictions about the relative size of LF violating decays of charged leptons (in terms of neutrino masses and mixing angles). At present, the uncertainty in the predictions for such ratios is limited from the poorly constrained value of the 1–3 mixing angle in the neutrino mass matrix (s_{13}) and, to a lesser extent, from the neutrino spectrum ordering and the CP violating phase δ . One of the clearest consequences from the phenomenological point of view is the observation that if $s_{13} \gtrsim 0.1$ there is no hope to observe $\tau \rightarrow \mu \gamma$ at future accelerators (see Fig. 2). This happens because the stringent constraints from $\mu \rightarrow e \gamma$ already forbid too low values for the effective scale of LF violation. In other words, in absence of new sources of LF violation the most sensitive FCNC probe in the lepton sector is $\mu \rightarrow e \gamma$. This process should indeed be observed at MEG [31] for very realistic values of the new-physics scales Λ and Λ_{LN} . Interestingly enough, this conclusion holds both in the minimal- and in the extended-field-content formulation of the MLFV framework.

The expectation of a higher NP sensitivity of $\mu \rightarrow \mu \gamma$ with respect to $\tau \rightarrow \mu \gamma$ (taking into account the corresponding experimental resolutions) is confirmed in several realistic NP frameworks. This happens for instance in the MSSM scenarios analysed in Ref. [32, 33, 34] with the exception of specific corners of the parameter space [32].

In the MLFV scenario with extended field content we can generate the observed matter-antimatter asymmetry of the Universe by means of leptogenesis [35]. The viability of leptogenesis

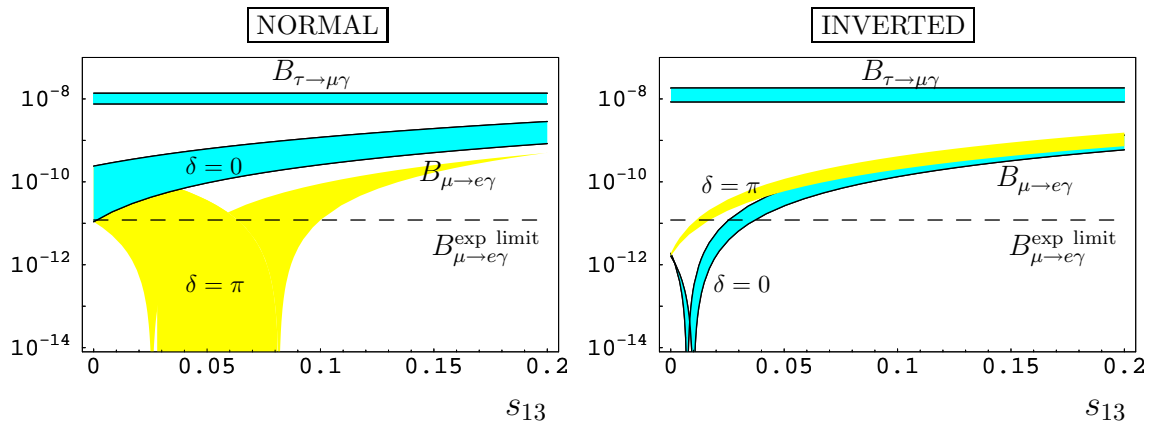


Figure 2: $B_{\tau \rightarrow \mu \gamma} = \Gamma(\tau \rightarrow \mu \gamma) / \Gamma(\tau \rightarrow \mu \nu \bar{\nu})$ compared to the $\mu \rightarrow e \gamma$ constraint within MLFV (minimal field content), as a function of the neutrino mixing angle s_{13} [26]. The shading corresponds to different values of the phase δ and the normal/inverted spectrum. The NP scales have been set to $\Lambda_{LN} / \Lambda = 10^{10}$; their variation affects only the overall vertical scale.

within the MLFV framework, which has been demonstrated in Ref. [36, 37, 38], is an interesting conceptual point: it implies that there are no phenomenological motivations to introduce new sources of flavour symmetry breaking in addition to the four λ_i (the three SM Yukawa couplings and λ_ν). A necessary condition for leptogenesis to occur is a non-degenerate heavy-neutrino spectrum. Within the MLFV framework, the tree-level degeneracy of heavy neutrinos is lifted only by radiative corrections, which implies a rather predictive/constrained scenario. From the phenomenological point of view, an important difference with respect to the CP-conserving case is the fact that non-vanishing ϕ_i change the predictions of the LFV decays, typically producing a further enhancement of the $\mathcal{B}(\mu \rightarrow e \gamma) / \mathcal{B}(\tau \rightarrow \mu \gamma)$ ratio [37].

5. Beyond Minimal Flavour Violation

Despite its phenomenological success, there are various reasons to expect some deviations from the MFV hypothesis. In the following we illustrate two well-motivated examples where such deviations are indeed expected.

Interestingly, in both cases the phenomenological signatures of such deviations are expected in the kaon system rather than in the B system. This is not surprising given the strong suppression of short-distance $s \rightarrow d$ FCNC transitions in the SM.

5.1 Grand Unified Theories

Once we accept the idea that flavour dynamics obeys a MFV principle, both in the quark and in the lepton sector, it is interesting to ask if and how this is compatible with a grand-unified theory (GUT), where quarks and leptons sit in the same representations of a unified gauge group. This question has been addressed in Ref. [39], considering the exemplifying case of $SU(5)_{\text{gauge}}$.

Within $SU(5)_{\text{gauge}}$, the down-type singlet quarks (d_{iR}^c) and the lepton doublets (L_{iL}) belong to the $\bar{\mathbf{5}}$ representation; the quark doublet (Q_{iL}), the up-type (u_{iR}^c) and lepton singlets (e_{iR}^c) belong

to the **10** representation, and finally the right-handed neutrinos (ν_{iR}) are singlet. In this framework the largest group of flavour transformation commuting with the gauge group is $\mathcal{G}_{\text{GUT}} = \text{SU}(3)_{\bar{3}} \times \text{SU}(3)_{10} \times \text{SU}(3)_1$, which is smaller than the direct product of the quark and lepton groups discussed before ($\mathcal{G}_q \times \mathcal{G}_l$). We should therefore expect some violations of the MFV+MLFV predictions, either in the quark sector, or in the lepton sector, or in both.

A phenomenologically acceptable description of the low-energy fermion mass matrices requires the introduction of at least four irreducible sources of \mathcal{G}_{GUT} breaking. From this point of view the situation is apparently similar to the non-unified case: the four \mathcal{G}_{GUT} spurions can be put in one-to-one correspondence with the low-energy spurions $\lambda_u, \lambda_d, \lambda_e$, and λ_ν . However, the smaller flavour group does not allow the diagonalization of λ_d and λ_e (which transform in the same way under \mathcal{G}_{GUT}) in the same basis. As a result, two additional mixing matrices can appear in the expressions for flavour changing rates [39]. The hierarchical texture of the new mixing matrices is known since they reduce to the identity matrix in the limit $\lambda_e^T = \lambda_d$. Taking into account this fact, and analysing the structure of the allowed higher-dimensional operators, a number of reasonably firm phenomenological consequences can be deduced [39]:

- There is a well defined limit in which the standard MFV scenario for the quark sector is fully recovered: $M_\nu \ll 10^{12}$ GeV and small $\tan\beta$ (in a two-Higgs doublet case). For $M_\nu \sim 10^{12}$ GeV and small $\tan\beta$, deviations from the standard MFV pattern can be expected in rare K decays but not in B physics. Ignoring fine-tuned scenarios, $M_\nu \gg 10^{12}$ GeV is excluded by the present constraints on quark FCNC transitions. Independently from the value of M_ν , deviations from the standard MFV pattern can appear both in K and in B physics for $\tan\beta \gtrsim m_t/m_b$ (see the next section).
- Contrary to the non-GUT MFV framework, the rate for $\mu \rightarrow e\gamma$ (and other LFV decays) cannot be arbitrarily suppressed by lowering the average mass M_ν of the heavy ν_R . This fact can easily be understood by looking at the flavour structure of the relevant effective couplings, which now assume the following form:

$$(\Delta_{\text{LR}}^\ell)_{\text{MFV-GUT}} = c_1 \lambda_e \lambda_\nu^\dagger \lambda_\nu + c_2 \lambda_u \lambda_u^\dagger \lambda_e + c_3 \lambda_u \lambda_u^\dagger \lambda_d^T + \dots \quad (5.1)$$

In addition to the terms involving $\lambda_\nu \sim \sqrt{M_\nu}$ already present in the non-unified case, the GUT group allows also M_ν -independent terms involving the quark Yukawa couplings. The latter become competitive for $M_\nu \lesssim 10^{12}$ GeV and their contribution is such that for $\Lambda \lesssim 10$ TeV the $\mu \rightarrow e\gamma$ rate is above 10^{-13} (i.e. within the reach of MEG [31]).

- Improved experimental information on $\tau \rightarrow \mu\gamma$ and $\tau \rightarrow e\gamma$ are now a key tool: the best observables to discriminate the relative size of the MLFV contributions with respect to the GUT-MFV ones. In particular, if the quark-induced terms turn out to be dominant, the $\mathcal{B}(\tau \rightarrow \mu\gamma)/\mathcal{B}(\mu \rightarrow e\gamma)$ ratio could reach values of $\mathcal{O}(10^{-4})$, allowing $\tau \rightarrow \mu\gamma$ to be just below the present exclusion bounds.

5.2 Flavour protection from hierarchical fermion profiles

So far we have assumed that the suppression of flavour-changing transitions beyond the SM can be attributed to a flavour symmetry, and a specific form of the symmetry-breaking terms. An

interesting alternative is the possibility of a generic *dynamical suppression* of flavour-changing interactions, related to the weak mixing of the light SM fermions with the new dynamics at the TeV scale. A mechanism of this type is the so-called RS-GIM mechanism occurring in models with a warped extra dimension. In this framework the hierarchy of fermion masses, which is attributed to the different localization of the fermions in the bulk [40, 41], implies that the lightest fermions are those localised far from the infra-red (SM) brane. As a result, the suppression of FCNCs involving light quarks is a consequence of the small overlap of the light fermions with the lightest Kaluza-Klein excitations [42, 44].

As pointed out in [45], also the general features of this class of models can be described by means of a general EFT approach. The two ingredients of this EFT are the following: i) there exists a (non-canonical) basis for the SM fermions where their kinetic terms exhibit a rather hierarchical form:

$$\mathcal{L}_{\text{kin}}^{\text{quarks}} = \sum_{\Psi=Q_L, U_R, D_R} \bar{\Psi} Z_{\Psi}^{-2} \not{D} \Psi, \quad Z_{\Psi} = \text{diag}(z_{\Psi}^{(1)}, z_{\Psi}^{(2)}, z_{\Psi}^{(3)}), \quad z_{\Psi}^{(1)} \ll z_{\Psi}^{(2)} \ll z_{\Psi}^{(3)} \lesssim 1,$$

ii) in such basis there is no flavour symmetry and all the flavour-violating interactions, including the Yukawa couplings, are $\mathcal{O}(1)$. Once the fields are transformed into the canonical basis, the hierarchical kinetic terms act as a distorting lens, through which all interactions are seen as approximately aligned on the magnification axes of the lens. As anticipated, this construction provide an effective four-dimensional description of a wide class of models with a with a warped extra spacial dimension. However, it should be stresses that this mechanism is not a general feature of models with extra dimensions: as discussed in [46, 47], also in extra-dimensional models is possible to postulate the existence of additional symmetries and, for instance, recover a MFV structure. On the other hand, hierarchical fermion profiles can be generated also in different theoretical frameworks: they could arise from Renormalisation Group running, with large, positive, and distinct anomalous dimensions for the different generations of fermions [48].

This dynamical mechanism is quite effective in suppressing FCNCs beyond the SM. In particular, it can be shown that all the dimensions-six FCNC left-left operators (such as the $\Delta F = 2$ terms in (2.3)), have the same suppression as in MFV [45]. However, a residual problem is present in the left-right operators contributing to CP-violating observables in the kaon system: ε_K [4, 49, 50] (see also [51]) and $\varepsilon'/\varepsilon_K$ [45, 52] (with potentially visible effects also in rare B and K decays [53]). To suppress the latter, either the effective scale of new-physics is push up to ~ 10 TeV, or some form of alignment (of MFV type) must be invoked (see e.g. Ref. [54, 55] for a possible implementation of the alignment in the context of models with warped extra dimensions).

6. Conclusions

The absence of significant deviations from the SM in quark flavour physics is a key information about any extension of the SM. Only models with a highly non generic flavour structure can both stabilise the electroweak sector and, at the same time, be compatible with flavour observables.

A useful tool to identify the flavour structure of physics beyond the SM is provided by the construction of effective theories at the TeV scale, based on specific flavour symmetries and symmetry-breaking hypotheses, and compare them with data. As we have seen, the MFV hypothesis emerges

as a natural candidate to *protect* flavour physics in extensions of the SM at the TeV scale. However, as discussed in the last chapter, MFV is not the only allowed possibility and is unlikely to be an exact symmetry principle.

The identification of the flavour structure of physics beyond the SM remains an open issue. Shedding more light on this problem require a twofold effort. On the one hand, we need to identify the TeV scale dynamics responsible for the breaking of the electroweak symmetry. On the other hand, we need more accurate measurements of clean FCNCs at low energies (such $B_{s,d} \rightarrow \ell^+ \ell^-$, $K \rightarrow \pi \nu \bar{\nu}$, and the CP-violating phase of B_s mixing) to probe the flavour symmetry-breaking pattern of the new degrees of freedom.

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