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$\pi\pi$ scattering lengths at $\mathcal{O}(p^6)$: resonance estimates

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In a previous paper, some deviations were found in the $\mathcal{O}(p^6)$ low-energy constants that contribute to the $\pi\pi$–scattering lengths. This work completes the study of all the relevant couplings ($r_1, ...r_6, r_{S_2}$). We also perform a reanalysis of the hadronical inputs used for the estimation (resonance masses, widths...), checking the impact of the input uncertainties on the determinations of the chiral couplings and the scattering lengths a^I_J. A good agreement is found with respect to former works, though our detailed analysis produces a more solid estimate of these couplings and slightly larger errors. The effect in the final values of the a^I_J is negligible after combining them with the other uncertainties, being the previous scattering length determinations sound and reliable. Nevertheless, the uncertainties derived here for the $\mathcal{O}(p^6)$ contributions to the scattering lengths point out the limitation on further improvements unless the precision of the $\mathcal{O}(p^6)$ low-energy couplings is properly increased.

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1. Introduction

This talk presents the culmination [1] of a former work [2], where some of the $\mathscr{O}(p^6)$ Chiral Perturbation Theory low energy constants ($r_2, \dots r_6$) that describe the $\pi\pi$–scattering were calculated. Some shifts were found with respect to former estimates [3], inducing slight modifications on the corresponding predictions for the scattering lengths a_J^I and effective ranges b_J^I [3, 4]. These constants provide the partial wave amplitudes for isospin I and angular momentum J near threshold $T_J^I(s) = k^{2J}\left(a_J^I + b_J^I k^2 + \dots\right)$, with $k = \sqrt{s/4 - m_\pi^2}$ the pion three-momentum in the dipion rest-frame [3]. However, our previous article [2] lacked of revised predictions for the r_1 low-energy constant (LEC), which also enters into the $\pi^+(p_1)\pi^-(p_2) \to \pi^0(p_3)\pi^0(p_4)$ amplitude at $\mathscr{O}(P^6)$ [3]:

$$A(s,t,u)|_{r_i} = \frac{m_\pi^4 s}{F^6}(r_2 - 2r_F) + \frac{m_\pi^2 s^2}{F^6} r_3 + \frac{m_\pi^2 (t-u)^2}{F^6} r_4 + \frac{s^3}{F^6} r_5 + \frac{s(t-u)^2}{F^6} r_6 + \frac{m_\pi^6}{F^6}(r_1 + 2r_F)\,.$$

Likewise, the dispersive method considered by Colangelo *et al.* [4] required the $\mathscr{O}(p^6)$ LEC r_{S_2} instead of r_5 and r_6. Here we complete the study of these last LECs and perform a full reanalysis of the different hadronic inputs and their uncertainties.

2. Resonance estimates of $\mathscr{O}(p^6)$ LECs

- **Set A:**

  This is the group of estimates commonly employed in nowadays calculations [3, 5]. The χPT couplings are assumed to be determined by the resonance exchanges provided by the phenomenological lagrangian

$$\begin{aligned}\mathscr{L} =\ & \frac{F^2}{4}\langle u_\mu u^\mu + \chi_+\rangle + \frac{1}{2}\langle \nabla^\mu S \nabla_\mu S\rangle - \frac{1}{2}M_S^2\langle SS\rangle + c_d\langle S u_\mu u^\mu\rangle + c_m\langle S\chi_+\rangle \\ & - \frac{1}{4}\langle \hat{V}_{\mu\nu}\hat{V}^{\mu\nu}\rangle + \frac{1}{2}M_V^2\langle \hat{V}_\mu \hat{V}^\mu\rangle - \frac{ig_V}{2\sqrt{2}}\langle \hat{V}_{\mu\nu}[u^\mu, u^\nu]\rangle + f_\chi\langle \hat{V}_\mu[u^\mu, \chi_-]\rangle\,, \qquad (2.1)\end{aligned}$$

  where $\langle \dots \rangle$ stands for trace in flavour space, S and $\hat{V}^\mu$ account respectively for the scalar and vector multiplets. The tensor u^μ contains the chiral pseudo-Goldstone and $\chi_\pm$ is, in addition, proportional to the light quark masses. Their precise definitions can be found in Refs. [3, 5, 6]. From the comparison of the $\rho \to \pi\pi$ and $K^* \to K\pi$ decays and other processes, Ref. [3] obtained the set of parameters

$$\begin{aligned} M_V = 770\,\text{MeV}\,, \qquad g_V = 0.09\,, \qquad f_\chi = -0.03\,, \\ M_S = 983\,\text{MeV}\,, \qquad c_m = 42\,\text{MeV}\,, \qquad c_d = 32\,\text{MeV}\,. \qquad (2.2)\end{aligned}$$

  Taking this inputs and the phenomenological lagrangian (2.1), Ref. [3] provided

$$\begin{aligned} & r_1^A = -0.6\times 10^{-4}\,, \qquad r_2^A = 1.3\times 10^{-4}\,, \qquad r_3^A = -1.7\times 10^{-4}\,, \qquad (2.3) \\ & r_4^A = -1.0\times 10^{-4}\,, \qquad r_5^A = 1.1\times 10^{-4}\,, \qquad r_6^A = 0.3\times 10^{-4}\,, \qquad r_{S_2}^A = -0.3\times 10^{-4}\,. \end{aligned}$$

- **Set B:**

  However, some scalar meson contributions were found to be missing in previous estimates of the $\mathscr{O}(p^6)$ LECs [3, 4, 5]. The couplings $r_2, \dots r_6$ were fully calculated at large N_C [2], being expressed in terms of the ratios

$$\frac{\Gamma_R}{M_R^3} = \frac{\overline{\Gamma}_R}{\overline{M}_R^3}\left[1 + \alpha_R \frac{m_\pi^2}{\overline{M}_R^2} + \gamma_R \frac{m_\pi^4}{\overline{M}_R^4} + \mathscr{O}(m_\pi^6)\right]\,, \qquad \frac{\Gamma_R}{M_R^5} = \frac{\overline{\Gamma}_R}{\overline{M}_R^5}\left[1 + \beta_R \frac{m_\pi^2}{\overline{M}_R^2} + \mathscr{O}(m_\pi^4)\right]\,, (2.4)$$

where $\overline{M}_R$ and $\overline{\Gamma}_R$ stand for the chiral limit of M_R and Γ_R, respectively. The constants α_R, β_R, γ_R are quark mass independent and rule the m_π corrections in the ratios. The resonance masses and widths were computed at large N_C by means of the resonance lagrangian (2.1). Using exactly the same inputs (2.2) of set A, we found that r_5 and r_6 remained unchanged, r_3 and r_4 varied slightly ($r_3^B = 0.9\times 10^{-4}$, $r_4^B = -1.9\times 10^{-4}$) and r_2 suffered a big variation ($r_2^B = 18\times 10^{-4}$) [1, 2].

- **Set C:**
  As relevant variations were found in some of the r_i, in addition to performing the full large–N_C estimate (without dropping any possible resonance contribution), a detailed analysis of the experimental inputs and their uncertainties was also in order. It was found that although the vector sector is quite under control, our knowledge on the scalar resonance properties is rather poor. This work is devoted to this analysis.

3. Phenomenology of the resonance parameters

3.1 Mass splitting up to $\mathcal{O}(m_P^2)$

In the large–N_C limit, the mass splitting of the resonance multiplets can be described at leading order by one single operator e_m^R [7]

$$-\frac{\overline{M}_R^{\,2}}{2}\langle RR\rangle + e_m^R\langle RR\chi_+\rangle\,, \tag{3.1}$$

which leads at large N_C to the mass eigenstates

$$\begin{aligned} M_{I=1}^2 &= \overline{M}_R^{\,2} - 4e_m^R m_\pi^2 \;+\; \mathcal{O}(m_P^4) \;=\; M_{I=0}^{(\bar uu+\bar dd)}\,,\\ M_{I=\frac12}^2 &= \overline{M}_R^{\,2} - 4e_m^R m_K^2 \;+\; \mathcal{O}(m_P^4)\,,\\ M_{I=0}^{(\bar ss)\,2} &= \overline{M}_R^{\,2} - 4e_m^R\,(2m_K^2 - m_\pi^2) \;+\; \mathcal{O}(m_P^4)\,. \end{aligned} \tag{3.2}$$

The combined study of the $\rho(770)$, $K^*(892)$ and $\phi(1020)$ masses leads to the values [1]

$$\overline{M}_V \;=\; 764.3\pm 1.1\,\mathrm{MeV}\,,\qquad e_m^V \;=\; -0.228\pm 0.015\,. \tag{3.3}$$

In the case of the scalars, the lightest $I=1$ resonance is identified with the $a_0(980)$: $M_{I=1} = 984.7\pm 1.2$ MeV [8]. In order to avoid the problem of the mixing of iso-singlet scalars, the analysis is performed with the $I=1/2$ state. The broad $\kappa(800)$ seems to be a possible candidate although the first clear $I=1/2$ scalar resonance signal is provided by the $K_0^*(1430)$ [8]. Hence, we take the conservative estimate $M_{I=1/2} = 1050\pm 400$ MeV, which ranges from the κ up to the $K_0^*(1430)$ mass. This leads then to the values

$$\overline{M}_S \;=\; 980\pm 40\,\mathrm{MeV}\,,\qquad e_m^S \;=\; -0.1\pm 0.9\,. \tag{3.4}$$

3.2 The splitting of the vector resonance decay width up to $\mathcal{O}(m_P^2)$

The vector decay width into two light pseudo-scalars, $V\to\phi_1\phi_2$, shows the general structure

$$\Gamma_{V\to\phi_1\phi_2} \;=\; C_{V12}\;\times\;\frac{M_V^{\,n}\,\rho_{V12}^3}{48\,\pi F_1^2 F_2^2}\;\lambda_{V\pi\pi}^2\;\left[1+\varepsilon_V\frac{m_1^2+m_2^2}{2\overline{M}_V^2}\;+\;\mathcal{O}(m_P^4)\right]^2\,, \tag{3.5}$$

with the phase-space factor $\rho_{V12} = M_V^{-2}\sqrt{(M_V^2-(m_1+m_2)^2)\,(M_V^2-(m_1-m_2)^2)}$. The F_i are the physical decay constants for the ϕ_i pseudo-Goldstones ($F_\pi \simeq 92.4$ MeV and $F_K \simeq 113$ MeV) and they appear due to the large–N_C wave function renormalization of the light pseudo-scalars [2, 9]. M_V and m_i correspond, respectively, to the physical vector and pseudo-scalar mass. Depending on the channel, one has the Clebsch-Gordan $C_{\rho\pi\pi}=1$, $C_{K^*K\pi}=3/4$ and $C_{\phi K\overline{K}}=1$. The $V\phi_1\phi_2$ coupling and the mass scaling M_V^n depend on the considered lagrangian realization, either Proca fourvector (n=5) or Antisymmetric tensor formalism (n=3) [6, 5, 11]. The combination of the experimental K^* and ρ widths yields [1]:

$$\begin{array}{lll} \lambda_{V\pi\pi} = g_V = 0.0846\pm 0.0008\,, & \varepsilon_V = 0.01\pm 0.09\,, & (\text{ Proca [5, 11] })\,, \\ \lambda_{V\pi\pi} = G_V = 63.9\pm 0.6\,\mathrm{MeV}\,, & \varepsilon_V = 0.82\pm 0.10\,, & (\text{ Antisym. [6, 11] })\,. \end{array} \tag{3.6}$$

3.3 The decay width for the scalar resonance

In the case of the scalar mesons the current knowledge nowadays is still very poor. We had then to rely on the phenomenological lagrangian (2.1) for the description of the $a_0(980)\to\pi\eta$ width [1, 6], and on the theoretical scalar form-factor constraint $4c_dc_m = F^2$ [13]:

$$c_d \,=\, 26\pm 7\,\mathrm{MeV}\,, \qquad c_m \,=\, 80\pm 21\,\mathrm{MeV}\,, \tag{3.7}$$

where their large errors stems essentially from the wide range we considered for the $a_0(980)$ partial width, $\Gamma_{a_0\to\pi\eta} = 75\pm 25$ MeV [1].

3.4 Chiral corrections to F_π

At large–N_C, the wave-function renormalization of the π field is related to the decay constant in the way $F_\pi = F\,Z_\pi^{-1/2}$ [9, 10]. The m_π^2 corrections to F_π can be parametrized in the form

$$F_\pi \;=\; F\;\left[1\,+\,\delta F_{(2)}\,\frac{m_\pi^2}{\overline{M}_S^2}\,+\,\delta F_{(4)}\,\frac{m_\pi^4}{\overline{M}_S^4}\,+\,\mathcal{O}(m_\pi^6)\right]\,. \tag{3.8}$$

The scalar lagrangian (2.1), the mass splitting (3.2) and the former inputs produce the predictions

$$\delta F_{(2)} = \frac{4c_dc_m}{F^2} = 1\,, \qquad \delta F_{(4)} = \frac{8c_dc_m}{F^2}\left(\frac{3c_dc_m}{F^2}-\frac{4c_m^2}{F^2}\right)+\frac{16c_dc_me_m^S}{F^2} = -5\pm 5\,. \tag{3.9}$$

3.5 Next-to-next-to-leading order chiral corrections to M_V, Γ_V and Γ_S

The next-to-next-to-leading order corrections (NNLO) to the vector mass are also needed in order to extract the LEC r_2 [1]. At large N_C, the quark mass corrections are given at NNLO by $M_{I=1}^2 = \overline{M}_R^2 - 4e_m^R m_\pi^2 - 4\widetilde{e}_m^R m_\pi^4/\overline{M}_R^2$. Demanding that the NNLO terms never overcome the NLO corrections in the vector multiplet sets the range $\left|\widetilde{e}_m^V\right| \;\leq\; \dfrac{\overline{M}_V^2}{2m_K^2-m_\pi^2}\,\left|e_m^V\right| \;\simeq\; 0.3$ [1].

The determination of r_2 also requires the NNLO chiral corrections $\widetilde{\varepsilon}_R$ to the resonance widths

$$\begin{aligned} \Gamma_{\rho\to\pi\pi} \;&=\; \frac{M_\rho^n\,\rho_{\rho\pi\pi}^3}{48\,\pi\,F_\pi^4}\;\lambda_{V\pi\pi}^2\;\left[1+\varepsilon_V\frac{m_\pi^2}{\overline{M}_V^2}+\widetilde{\varepsilon}_V\frac{m_\pi^4}{\overline{M}_V^4}\;+\;\mathcal{O}(m_\pi^6)\right]^2\,, \\ \Gamma_{\sigma\to\pi\pi} \;&=\; \frac{3M_\sigma^3\rho_{\sigma\pi\pi}}{16\pi F_\pi^4}\;c_d^2\;\left[1+\varepsilon_S\frac{m_\pi^2}{\overline{M}_S^2}+\widetilde{\varepsilon}_S\frac{m_\pi^4}{\overline{M}_S^4}+\mathcal{O}(m_\pi^6)\right]^2\,. \end{aligned} \tag{3.10}$$

The phenomenological lagrangian (2.1) [6, 5] yields the predictions

$$\widetilde{\varepsilon}_V = \varepsilon_V \left[\frac{8c_m(c_d-c_m)}{F^2}\frac{\overline{M}_V^2}{\overline{M}_S^2} + 4e_m^V \right] = \begin{cases} -0.03\pm 0.18\,, & \text{(Proca)} \\ -1.6\pm 0.9\,, & \text{(Antisym.)} \end{cases}$$
$$\widetilde{\varepsilon}_S = \frac{16c_m^2(c_d-c_m)}{c_dF^2} + \frac{8(c_m-c_d)e_m^S}{c_d} = -7\pm 12\,. \tag{3.11}$$

4. Low-energy constant determination at $\mathscr{O}(p^6)$

Based on the partial-wave dispersion relations developed in Refs. [2, 12], it is possible to extract the large–N_C values of $r_2,\dots r_6$ from the $I=1$ vector and $I=0$ scalar $(\bar{u}u+\bar{d}d)$ width and mass ratios Γ_R/M_R^3 and Γ_R/M_R^5: the couplings r_5 and r_6 are determined by the Γ_R/M_R^5 ratio in the chiral limit; r_3 and r_4 also require its first m_π^2 correction β_R; those and the NNLO m_π^2 contribution to Γ_R/M_R^3 are needed in order to obtain r_2. All these LECs have been found to be dominated by the vector resonance exchanges. The $\mathscr{O}(p^6)$ couplings r_1 [5] and r_{S_2} [5] could not be computed through the partial-wave dispersion relations in [2, 12]. They were calculated directly from the phenomenological lagrangian (2.1):

$$\begin{aligned} r_1^{\text{Proca}} = &-\frac{16c_dc_m(8c_d^2-17c_dc_m+12c_m^2)}{\overline{M}_S^4} + \frac{32(c_d-c_m)^2F^2}{\overline{M}_S^4}e_m^S \\ &-\frac{16g_V^2F^2}{\overline{M}_V^2}\left[1+\varepsilon_V+\frac{1}{4}\varepsilon_V^2 - \frac{8c_dc_m}{F^2}\frac{\overline{M}_V^2}{\overline{M}_S^2}\right]\,, \end{aligned} \tag{4.1}$$
$$r_{S2} = \frac{8c_m(c_m-c_d)F^2}{\overline{M}_S^4} - \frac{32c_d^2c_m^2}{\overline{M}_S^4} + \frac{16c_dc_mF^2}{\overline{M}_S^4}e_m^S\,. \tag{4.2}$$

The expression for r_1 in the Antisymmetric tensor formalism is similar to (4.1) but with the second line replaced by $-\frac{16G_V^2F^2}{\overline{M}_V^4}\left[1+\varepsilon_V-\frac{8c_dc_m}{F^2}\frac{\overline{M}_V^2}{\overline{M}_S^2}+2e_m^V\right]$. All this leads to the values of the low-energy constants shown in Table 1. The first error derives from the phenomenological inputs and the second one stems from the uncertainty on the saturation scale μ_s where $r_i^r(\mu_s)=r_i^{N_C\to\infty}$ [1].

	ND est. [14]	set A [3, 5]	set C (Proca)	set C (Antisym.)
$10^4\cdot r_1^r$	± 80	-0.6	$-14\pm 17\pm 3$	$-20\pm 17\pm 3$
$10^4\cdot r_2^r$	± 40	1.3	$22\pm 16\pm 4$	$7\pm 10\pm 4$
$10^4\cdot r_3^r$	± 20	-1.7	$-3\pm 1\pm 3$	$-4\pm 1\pm 3$
$10^4\cdot r_4^r$	± 3	-1.0	$-0.22\pm 0.13\pm 0.05$	$0.13\pm 0.13\pm 0.05$
$10^4\cdot r_5^r$	± 6	1.1	$0.9\pm 0.1\pm 0.5$	$0.9\pm 0.1\pm 0.5$
$10^4\cdot r_6^r$	± 2	0.3	$0.25\pm 0.01\pm 0.05$	$0.25\pm 0.01\pm 0.05$
$10^4\cdot r_{S_2}^r$	± 1	-0.3	$1\pm 4\pm 1$	$1\pm 4\pm 1$

Table 1: Different predictions for the $\mathscr{O}(p^6)$ LECs $r_i^r(\mu)$ for $\mu=770$ MeV: The first column presents the order of magnitude estimate based on naive dimensional analysis [14]; In the set A column we show former estimates from Refs. [3, 5]; in the last two columns, one can find the values for the present reanalysis.

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5. Scattering lengths

The χPT expression for the scattering lengths contains at $\mathcal{O}(p^6)$ a series of logarithmic terms together with analytical $\mathcal{O}(p^6)$ contributions [3]. Although the first ones are the most complicate contributions to compute, their value is nevertheless rather sound and under control. On the other hand, the local terms are determined by the $\mathcal{O}(p^6)$ LECs r_i and, although they can be easily computed, these couplings are badly known and their estimation is pretty cumbersome.

In Ref. [4], Colangelo *et al.* combined the NNLO chiral perturbation theory computation of the scattering lengths [3] with a phenomenological dispersive representation. This allowed them to produce one of the most precise determinations of the scattering lengths. They were expressed in terms of some dispersive integrals, the pion quadratic scalar radius $\langle r^2\rangle^\pi_S$, the $\mathcal{O}(p^4)$ coupling ℓ_3 and a set of $\mathcal{O}(p^6)$ LECs ($r_1,\ r_2, r_3, r_4, r_{S_2}$). Following the work of Ref.[4], we extracted the part of their scattering lengths that depended on the inputs $r_i^r(\mu)$ [1, 4]:

$$
\begin{aligned}
a_0^0|_{r_i} = & \ \tfrac{7m_\pi^2}{32\pi F_\pi^2} C_0|_{r_i} &= \frac{m_\pi^6}{32\pi F_\pi^6}\left[5r_1^r+12r_2^r+28r_3^r-28r_4^r-14r_{S_2}\right], \\
a_0^2|_{r_i} = & -\tfrac{m_\pi^2}{16\pi F_\pi^2} C_2|_{r_i} &= \frac{m_\pi^6}{16\pi F_\pi^6}\left[r_1^r-4r_3+4r_4+2r_{S_2}\right].
\end{aligned}
\qquad (5.1)
$$

The largest contributions to the a_0^0 and a_0^2 errors are found to be produced in similar terms by r_1, r_2, r_3 and r_{S_2}, being the impact of r_4 negligible.

	Total: Ref. [4] ($\times 10^{-3}$)	$a_J^I\vert_{r_i}$ [4] ($\times 10^{-3}$)	$a_J^I\vert_{r_i}$ Set C (Proca) ($\times 10^{-3}$)	$a_J^I\vert_{r_i}$ Set C (Antisym.) ($\times 10^{-3}$)
a_0^0	220 ± 5	0.0 ± 1.0	$1.0\pm 1.5\pm 1.0$	$-1.6\pm 1.5\pm 1.0$
$10\,a_0^2$	-444 ± 10	0.4 ± 2.0	$0\pm 4\pm 2$	$0\pm 4\pm 2$

Table 2: The first and second columns show, respectively, the total scattering lengths and the r_i contribution to them in the dispersive method from Colangelo *et al.* [4], where the authors used the r_i in Eq. (2.3), $F_\pi = 92.4$ MeV and $m_\pi = 139.57$ MeV. The last two columns show the reanalyzed quantities $a_J^I|_{r_i}$ (set C) for the Proca and antisymmetric tensor formalisms for the usual scale $\mu = 770$ MeV. There, the first error derives from the inputs and the second one from the saturation scale uncertainty.

6. Summary and conclusion

The combination of the Proca and antisymmetric results yields for our prediction of the LECs the final numbers (for $\mu = 770$ MeV),

$$
\begin{aligned}
& r_1^r = (-17\pm 20)\times 10^{-4}, \qquad r_2^r = (17\pm 21)\times 10^{-4}, \qquad r_3^r = (-4\pm 4)\times 10^{-4}, \\
& r_4^r = (0.0\pm 0.3)\times 10^{-4}, \qquad r_5^r = (0.9\pm 0.5)\times 10^{-4}, \qquad r_6^r = (0.25\pm 0.05)\times 10^{-4}, \\
& r_{S_2}^r = (1\pm 4)\times 10^{-4}.
\end{aligned}
\qquad (6.1)
$$

The r_i^r contributions to the scattering lengths with the Colangelo *et al.*'s method [4] can be summarized in the predictions

$$
10^3\, a_0^0|_{r_i} = 0\pm 3, \qquad 10^4 a_0^2|_{r_i} = 0\pm 5. \qquad (6.2)
$$

Following the analysis of global uncertainties of Ref. [4] leads to the updated values $a_0^0 = 0.220 \pm 0.005$ and $10\,a_0^2 = -0.444 \pm 0.011$. These values leave essentially unchanged the previous determinations $a_0^0 = 0.220 \pm 0.005$ and $10\,a_0^2 = -0.444 \pm 0.010$ [4].

This calculation shows that the determinations of the scattering lengths through the dispersive method and r_i resonance saturation estimates are rather solid [4]. Our detailed analysis shows that the error stemming from the r_i does not modify the final numbers quoted in Ref. [4]. Nonetheless, unless the the precision in the $\mathscr{O}(p^6)$ low-energy constants is conveniently increased, it will be difficult to carry on further relevant improvements in the scattering length determinations.

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