PROCEEDINGS OF SCIENCE

Phenomenology of Excited Baryon in the 1/N_c expansion of QCD

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We discuss the strong decay widths and photoproduction helicity amplitudes of excited baryons in the context of the $1/N_c$ expansion of QCD. The results show that in order to get a satisfactory description of the empirical data the sub-leading corrections in $1/N_c$ are, in general, important. It is also found that, while one-body effective operators are dominant, there is some evidence for the need of two-body effects which, in general, are not included in quark model calculations.

International Workshop on Effective Field Theories: from the pion to the upsilon
February 2-6 2009
Valencia, Spain

*Speaker.

PoS(EFT09)045

1. Introduction

The $1/N_c$ expansion of QCD [1] has proven to be a useful and systematic framework for the analysis of various baryon properties. This is mostly due to the existence of a contracted spin-flavor symmetry in the large N_c limit[2, 3]. Although the larger symmetry $O(3) \times SU(2N_f)$ is not an exact symmetry in the excited baryon sector[4], its N_c^0 breaking is small as several analyses of the baryon spectrum have shown[5, 6, 7], and is therefore very useful to set the framework of the $1/N_c$ expansion. Here we report on the application of this approach to the study of the strong decays and the photoproduction amplitudes of the low-lying excited non-strange baryons.

2. Strong decays

The strong ℓ_P partial wave decay width of a resonance with total angular momentum, spin and isospin J^*, S^*, I^*, respectively, into a ground state baryon with quantum numbers J, I and a pseudo-scalar meson with isospin I_P can be expressed as

$$\Gamma^{[\ell_P, I_P]} = \frac{k_P}{8\pi^2} \frac{M_B}{M_B^*} \frac{|B(\ell_P, I_P, J, I, J^*, I^*, S^*)|^2}{(2J^*+1)(2I^*+1)}, \tag{2.1}$$

where $B(\ell_P, I_P, J, I, J^*, I^*, S^*)$ are the reduced matrix elements (RME's) of the baryonic operator. Such operator admits an expansion in $1/N_c$ that has the general form[8, 9]

$$B^{[\ell_P, I_P]}_{[\mu,\alpha]} = \left(\frac{k_P}{\Lambda}\right)^{\ell_P} \sum_{q,j} C_q^{[\ell_P, I_P]} \left(\xi^{(\ell)}\, \mathcal{G}_q^{[j, I_P]}\right)^{[\ell_P, I_P]}_{[\mu,\alpha]} \tag{2.2}$$

The factor $(k_P/\Lambda)^{\ell_P}$ is included to take into account the chief meson momentum dependence of the partial wave, where the scale Λ is arbitrary, and is taken to be equal to 200 MeV. The operator $\xi^{(\ell)}$ drives the transition from the $(2\ell+1)$-plet to the singlet $O(3)$ state. The operators $\mathcal{G}$ give transitions within the spin-flavor representations in which the excited and GS baryons reside. They can be written as products of the generators of the spin-flavor algebra acting on the excited quark state $\lambda = s_i, t_a, g_{ia}$ and on the core $\Lambda_c = (S_c)_i, (T_c)_a, (G_c)_{ia}$. The dynamics of the decays is encoded in the effective dimensionless coefficients $C_q^{[\ell_P, I_P]}$. Here, they are obtained by fitting the available empirical decay widths[10].

In the case of the baryons belonging to the $[70, 1^-]$-plet, we have two extra unknown parameters: the mixing angles θ_{2J} between the two sets of excited nucleon states N_J^*, where $J = 1/2, 3/2$. The corresponding operators and results of the fits for the non-strange members of such multiplet are given in Table 1. Details of the calculation can be found in Ref.[8]. We observe that the leading order (LO) fit already provides a reasonable description of the decay widths in the sense that it gives a χ^2 per degree of freedom $\chi^2_{\rm dof} = 1.5$. The next-to-leading (NLO) corrections play some role in improving the results, although the corresponding coefficients are not well determined due to the large error bars in the empirical widths. In any event, a clear dominance of the one-body operators is observed. Note that to NLO, although no degeneracy appears in θ_1, there is an *almost* two-fold ambiguity in θ_3.

The analysis of the strong decays corresponding to the positive parity resonances can be carried out in a similar fashion. Details are given in Ref.[9]. In the case of the non-strange members of

Table 1: Operators and fit parameters for strong decays of non-strange $[70,1^-]$ excited states. LO: There is a two-fold ambiguity for the angle θ_1. For θ_3 there is an *almost* two-fold ambiguity which leads to two slightly different values of $C_6^{[2,1]}$. NLO: Due to lack of empirical data, three-body operators and the subleading operator for η emission are not included. No degeneracy in θ_1 but *almost* two-fold ambiguity in θ_3. Values of coefficients which differ in the corresponding fits are indicated in parenthesis.

Operator	Fit parameters	
	LO	NLO
$\left(\xi^{(1)}\, g\right)^{[0,1]}$	31 ± 3	23 ± 3
$\frac{1}{N_c}\left(\xi^{(1)}\,(s\,T_c)^{[1,1]}\right)^{[0,1]}$	-	$(7.4, 32.5) \pm (27, 41)$
$\frac{1}{N_c}\left(\xi^{(1)}\,(t\,S_c)^{[1,1]}\right)^{[0,1]}$	-	$(20.7, 26.8) \pm (12, 14)$
$\frac{1}{N_c}\left(\xi^{(1)}\,(g\,S_c)^{[1,1]}\right)^{[0,1]}$	-	$(-26.3, -66.8) \pm (39, 65)$
$\left(\xi^{(1)}\, g\right)^{[2,1]}_{[i,a]}$	4.6 ± 0.5	3.4 ± 0.3
$\frac{1}{N_c}\left(\xi^{(1)}\,(s\,T_c)^{[1,1]}\right)^{[2,1]}$	-	-4.5 ± 2.4
$\frac{1}{N_c}\left(\xi^{(1)}\,(t\,S_c)^{[1,1]}\right)^{[2,1]}$	-	$(-0.01, 0.08) \pm 2$
$\frac{1}{N_c}\left(\xi^{(1)}\,(g\,S_c)^{[1,1]}\right)^{[2,1]}$	-	5.7 ± 4.0
$\frac{1}{N_c}\left(\xi^{(1)}\,(g\,S_c)^{[2,1]}\right)^{[2,1]}$	-	3.0 ± 2.2
$\frac{1}{N_c}\left(\xi^{(1)}\,(s\,G_c)^{[2,1]}\right)^{[2,1]}$	$(-1.86, -2.25) \pm 0.4$	-1.73 ± 0.26
$\left(\xi^{(1)}\, s\right)^{[0,0]}$	11 ± 4	17 ± 4
θ_1	1.56 ± 0.15 0.35 ± 0.14	0.39 ± 0.11
θ_3	$(3.00, 2.44) \pm 0.07$	$(2.82, 2.38) \pm 0.11$
χ^2_{dof}	1.5	0.9

the $[56', 0^+]$-plet the LO fit turns out to be rather poor. In fact, NLO operators are essential for improving the ratio between the two N(1440) decay widths. Concerning the non-strange resonances belonging to the $[56, 2^+]$-plet we find that for the P wave decays the LO analysis already provides an excellent fit while for the F wave decays, even at NLO, the only way to get a reasonably good result is by removing the $\pi\Delta$ decay of the N(1680) from the fit.

3. Photoproduction amplitudes

The helicity amplitudes of interest are defined in the standard form[10]

$$A_\delta = -\sqrt{\frac{2\pi\alpha}{\omega}}\; \eta(B^*)\langle B^*, \delta \mid \vec{\varepsilon}_{+1} \cdot \vec{J}(\omega\hat{z}) \mid N, \delta - 1\rangle, \tag{3.1}$$

where $\delta = 1/2$ or $3/2$ is the helicity defined along the $\hat{z}$-axis, which coincides with the photon momentum, $\vec{\varepsilon}_{+1}$ is the photon's polarization vector for helicity $+1$, α is the fine-structure constant, and N and B^* denote, respectively, the initial nucleon and the excited baryon. A sign factor $\eta(B^*)$ from the strong amplitude for $\pi N \to B^*$ is included. The electromagnetic current $\vec{J}$ can be represented as a linear combination of effective current operators which have the most general form $\left(k_\gamma^{[L']}\mathscr{B}^{[LI]}\right)^{[1I]}$ where the upper scripts display the angular momentum and isospin, and throughout the neutral component, i.e. $I_3 = 0$, is taken. $k_\gamma^{[L']}$ is an irreducible tensor built with the photon momentum, chosen here to be a spherical harmonic, and $\mathscr{B}^{[LI]} = \left(\xi^{(\ell)}\mathscr{G}^{[\ell' I]}\right)^{[LI]}$ are operators where, as in Sec.2, $\xi^{(\ell)}$ is the $O(3)$ tensor and $\mathscr{G}^{[\ell' I]}$ is a spin-flavor tensor operator with $I = 0$ or 1, which can be expressed in terms of products of the generators of spin-flavor algebra acting either on the excited quark or the $N_c - 1$ quark core. Since there is a one to one correspondence between L and the multipole to which an operator $\mathscr{B}^{[LI]}$ contributes to, we denote them accordingly, e.g., EL_n^S is the n^{th} EL isoscalar operator. The bases of operators are given in Table 2 for the non-strange members of negative parity multiplet $[70, 1^-]$. The E- and M-multipole components of a given helicity amplitude of isospin I can be expressed in terms of the RME of the operators $\mathscr{B}^{[LI]}$ as follows

$$A_\delta^{X[LI]}(I_3, J^*I^*) = \frac{(-1)^{J^*+I^*+I+1}\, w_X(L)\, \eta(B^*)}{\sqrt{(2J^*+1)(2I^*+1)}} \sqrt{\frac{3\alpha N_c}{4\omega}}\, \langle L,1;\frac{1}{2},\delta-1 \mid J^*,\delta\rangle \langle I,0;\frac{1}{2},I_3 \mid I^*,I_3\rangle$$
$$\times \sum_n g_{n,X}^{[LI]}(\omega)\, \langle J^*I^* \parallel \mathscr{B}_n^{[LI]} \parallel \frac{1}{2}\rangle \qquad (3.2)$$

where $X = M$ (E) and $w_X(L) = 1$ $(\sqrt{(L+1)/(2L+1)})$ with $(-1)^L \pi_{ex} =$ negative (positive). Here, π_{ex} is the parity of the excited multiplet. In Eq.(3.2) I_3 denotes the isospin projection of the initial nucleon. The RME in Eq.(3.2) can be evaluated using similar techniques to those in [5]. Details can be found in Ref.[11]. In our calculations the coefficients $g_{n,X}^{[LI]}(\omega)$ are expressed by including the barrier penetration factor: $g_{n,X}^{[L,I]} \times (\omega/\Gamma)^{L_X}$, where $L_X = L-1$ for EL operators, $L_X = L$ for ML operators. Throughout we choose the scale $\Gamma = m_\rho$. The coefficients $g_{n,X}^{[L,I]}$ are to be determined by fitting to the empirical helicity amplitudes[10]. The sign $\eta(B^*)$ can be fixed from the studies described in Sec.2. Those analyses determine the signs up to an overall sign for each pion partial wave.

We consider now the helicity amplitudes corresponding to the non-strange members of the $[70, 1^-]$-plet. Since the partial waves involved in the strong decay of these states are S and D waves, we have one extra relative sign, which we will call ξ. For the mixing angles θ_{2J} we use here the values obtained from the analysis of the strong decay widths discussed in Sec.2. As seen such analysis gives two consistent but different results for the mixing angle $\theta_3 = 2.82, 2.38$. One finds that some of the η signs are different for these two values. We take into account this with an extra sign factor κ, which is equal to $+1$ (-1) for $\theta_3 = 2.82$ (2.38). A first analysis concerns the choices left by the values of the mixing angle θ_3, and the signs ξ and κ. Using all the LO operators, the choices are made by considering the χ^2 for all possibilities. The sign $\xi = -1$ is strongly favored. This is in agreement with an old determination based on the single-quark-transition model [12, 13]. The second choice that is favored, although less markedly than the one for ξ, is $\theta_3 = 2.82$. The helicity amplitudes show here their importance by allowing to determine the relative sign ξ

between the strong S and D wave amplitudes, and by selecting between the two possible values of θ_3 consistent with the strong transitions. The results from the fits we have carried out are given in Table 2. In the fits we expand the operator matrix elements in powers of $1/N_c$ to the order corresponding to the fit. In the LO fits, we have set the errors in the input helicity amplitudes to be 30% of the value of the helicity amplitude or the experimental value if this is larger. The point of this is to test whether or not the LO analysis is consistent in the sense that it gives a $\chi^2_{\rm dof} \sim 1$. For the NLO fits, we use of course the empirical errors. The LO fit shows a $\chi^2_{\rm dof}$ of 2.42. This indicates that there are NLO effects to be taken into account for a satisfactory fit. The NLO order fit NLO1, involves all operators in the basis. It gives values for the coefficients of the LO operators which are, within the expected deviations from $1/N_c$ counting, consistent with the values obtained in the LO fits. Moreover, none of the coefficients of the NLO operators has a magnitude larger than that of the largest LO coefficients. This is a strong indication of the consistency of the $1/N_c$ expansion. From the magnitude of the coefficients, it is obvious that only a few NLO operators are needed for a consistent fit. In fact, as shown by the fit NLO2 in Table 2, a consistent fit is obtained with only five LO and one NLO operators. Of these dominant operators four are one-body and LO, and two are two-body with one of them LO and the other NLO. Note also that none of the two-body $E3$ operators is required. It is remarkable that out of eleven NLO operators only one is essential for obtaining consistent fits. At this point it is important to mention that many of the empirical amplitudes have errors that are larger than what is needed for an accurate NLO analysis. It is for this reason that one cannot draw a more precise NLO picture which could unveil the role of other operators.

The helicity amplitudes corresponding to the positive parity resonances can be analyzed in a similar way[14]. In the case of the $[56', 0^+]$-plet states one finds that to LO only the M1 operator $G^{[1,1]}$ contributes leading to $\chi^2_{\rm dof} \sim 2$. The inclusion of one-body NLO operators improves somewhat the fit, although the two-body isovector M2 operator $[S, G]^{[2,1]}$ is definitely required to get a good overall agreement with the empirical data. A similar situation happens for the $[56, 2^+]$-plet resonances in the sense that the LO fit leads to $\chi^2_{\rm dof} \sim 2.1$. In fact a minimum number of six operators, three of which are LO, is needed to get $\chi^2_{\rm dof} \sim 1$. One of the required NLO operators is a two-body operator generally not included in quark model calculations.

4. Conclusions

To conclude, the $1/N_c$ expansion of QCD provides a systematic approach to the properties of the excited baryons. The analysis of the masses shows that the N_c^0 breaking of the spin-flavor symmetry is small. For strong decays one finds, in general, a dominance of the one-body LO operators. In some cases, as e.g. the D wave decays of the negative parity excited baryons, the $1/N_c$ corrections are not well established due to the rather large uncertainties of the empirical data. In the particular case of the $[56', 0^+]$ baryons, two-body NLO operators seem to be required to obtain a good description of the empirical data. In the case of the photoproduction amplitudes, the present analysis indicates that only a reduced number of operators in the basis turn out to be relevant. Several of those operators can be easily identified with those in quark models, but there are also two-body operators usually not included in quark models which are necessary for an accurate description of the empirical helicity amplitudes.

Table 2: Basis operators and fit parameters $g_{n,X}^{[LI]}$ for the helicity amplitudes of non-strange $[70, 1^-]$ baryons. Errors are indicated in parenthesis. Results for the choice $\xi = -1$ are shown.

Operator	LO	NLO1	NLO2
$E1_1^S = \left(\xi^{[1,0]} s\right)^{[1,0]}$	−0.4(0.2)	−0.3(0.2)	−0.3(0.2)
$E1_2^S = \frac{1}{N_c}\left(\xi^{[1,0]}\,(s\,S_c)^{[0,0]}\right)^{[1,0]}$		0.5(0.6)	
$E1_3^S = \frac{1}{N_c}\left(\xi^{[1,0]}\,(s\,S_c)^{[1,0]}\right)^{[1,0]}$		1.0(0.9)	
$E1_4^S = \frac{1}{N_c}\left(\xi^{[1,0]}\,(s\,S_c)^{[2,0]}\right)^{[1,0]}$		0.5(0.6)	
$E1_1^V = \left(\xi^{[1,0]} t\right)^{[1,1]}$	2.3(0.3)	3.0(0.2)	3.5(0.1)
$E1_2^V = \left(\xi^{[1,0]} g\right)^{[1,1]}$	−0.7(0.4)	0.4(0.3)	
$E1_3^V = \frac{1}{N_c}\left(\xi^{[1,0]}\,(s\,G_c)^{[2,1]}\right)^{[1,1]}$	0.4(0.5)	−0.2(0.4)	
$E1_4^V = \frac{1}{N_c}\left(\xi^{[1,0]}\,(s\,T_c)^{[1,1]}\right)^{[1,1]}$		−1.9(1.4)	
$E1_5^V = \frac{1}{N_c}\left(\xi^{[1,0]}\,(s\,G_c)^{[0,1]}\right)^{[1,1]} + \frac{1}{4\sqrt{3}}\,E1_1^V$		−0.2(0.9)	
$E1_6^V = \frac{1}{N_c}\left(\xi^{[1,0]}\,(s\,G_c)^{[1,1]}\right)^{[1,1]} + \frac{1}{2\sqrt{2}}\,E1_2^V$		4.2(0.9)	3.9(0.8)
$M2_1^S = \left(\xi^{[1,0]} s\right)^{[2,0]}$	0.8(0.2)	1.5(0.3)	1.3(0.2)
$M2_2^S = \frac{1}{N_c}\left(\xi^{[1,0]}\,(s\,S_c)^{[1,0]}\right)^{[2,0]}$		−1.2(1.3)	
$M2_3^S = \frac{1}{N_c}\left(\xi^{[1,0]}\,(s\,S_c)^{[2,0]}\right)^{[2,0]}$		−1.2(1.7)	
$M2_1^V = \left(\xi^{[1,0]} g\right)^{[2,1]}$	3.0(0.6)	3.8(0.6)	3.9(0.4)
$M2_2^V = \frac{1}{N_c}\left(\xi^{[1,0]}\,(s\,G_c)^{[2,1]}\right)^{[2,1]}$	−3.1(1.0)	−2.3(1.1)	−2.7(0.6)
$M2_3^V = \frac{1}{N_c}\left(\xi^{[1,0]}\,(s\,T_c)^{[1,1]}\right)^{[2,1]}$		−0.1(1.1)	
$M2_4^V = \frac{1}{N_c}\left(\xi^{[1,0]}\,(s\,G_c)^{[1,1]}\right)^{[2,1]} + \frac{1}{2\sqrt{2}}\,M2_1^V$		−1.5(2.4)	
$E3_1^S = \frac{1}{N_c}\left(\xi^{[1,0]}\,(s\,S_c)^{[2,0]}\right)^{[3,0]}$		0.3(0.8)	
$E3_1^V = \frac{1}{N_c}\left(\xi^{[1,0]}\,(s\,G_c)^{[2,1]}\right)^{[3,1]}$	0.7(0.9)	0.3(0.5)	
χ^2_{dof}	2.42	–	0.94
dof	11	0	13

Acknowledgements

We are indebted to N. Matagne and C.L. Schat in collaboration with whom different parts of the work reported in this contribution has been done. NNS would also like to thank the organizers for their warm hospitality during the workshop. NNS is supported by CONICET (Argentina) grant # PIP 6084 and by ANPCyT (Argentina) grant # PICT04 03-25374. JLG is supported by DOE contract DE-AC05-84ER40150 and NSF grant PHY-0300185 (USA), and sabbatical partial support by MINCYT (Argentina).

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