Old nuclear symmetries and large N_c as long distance symmetries in the two nucleon system

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Wigner and Serber symmetries for the two-nucleon system provide unique examples of long distance symmetries in Nuclear Physics, i.e. symmetries of the meson exchange forces broken only at arbitrarily small distances. We analyze the large N_c picture as a key ingredient to understand these, so far accidental, symmetries from a more fundamental viewpoint. A set of sum rules for NN phase-shifts, NN potentials and coarse grained V_{lowk} NN potentials can be derived showing Wigner SU(4) and Serber symmetries not to be fully compatible everywhere. The symmetry breaking pattern found from the partial wave analysis data, high quality potentials in coordinate space at long distances and their V_{lowk} relatives is analyzed on the light of large N_c contracted $SU(4)_C$ symmetry. Our results suggest using large N_c potentials as long distance ones for the two-nucleon system where the meson exchange potential picture is justified and known to be consistent with large N_c counting rules. We also show that potentials based on chiral expansions do not embody the Wigner and Serber symmetries nor do they scale properly with N_c. We implement the One Boson Exchange potential realization saturated with their leading N_c contributions due to π, σ, ρ and ω mesons. The short distance $1/r^3$ singularities stemming from the tensor force can be handled by renormalization of the Schrödinger equation. A good description of deuteron properties and deuteron electromagnetic form factors in the impulse approximation for realistic values of the meson-nucleon couplings is achieved.

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1. Introduction

The standard point of view in Particle Physics has often been that increasing the energy implies a higher degree of symmetry. In QCD, for instance, scale invariance roughly sets in for momenta much higher than the quark masses. In Nuclear Physics the situation may be exactly the opposite; some symmetries such as those introduced by Wigner [1] and Serber [1] are unveiled at low energies where the wavelength becomes larger than a certain scale. For obvious reasons we call them *Long Distance Symmetries* (LDS) [3, 4]. In the meson exchange picture this implies the presence of arbitrarily large symmetry breaking counterterms. We analyze these, so far accidental, LDS in the two-nucleon system below pion production threshold corresponding to CM momenta $p \leq 400$MeV.

2. Wigner symmetry

The Wigner SU(4) spin-flavour symmetry corresponds to the algebra of isospin T^a, spin S^i and Gamow-Teller G^{ia} generators in terms of the one particle spin σ_A^i and isospin τ_A^a Pauli matrices,

$$T^a = \frac{1}{2}\sum_A \tau_A^a, \quad S^i = \frac{1}{2}\sum_A \sigma_A^i, \quad G^{ia} = \frac{1}{2}\sum_A \sigma_A^i \tau_A^a. \tag{2.1}$$

The two-body Casimir operator is $C_{SU(4)} = T^a T_a + S^i S_i + G^{ia} G_{ia}$. The one-nucleon irreducible representations is a quartet made of a spin and isospin doublet

$$\mathbf{4} = (p\uparrow, p\downarrow, n\uparrow, n\downarrow) = (S = 1/2, T = 1/2).$$

Two nucleon states with relative angular momentum L and total spin S and isospin T fulfilling $(-1)^{S+L+T} = -1$ due to Fermi statistics correspond to an antisymmetric sextet and a symmetric decuplet which, in terms of (S, T) representations of the $SU_S(2) \otimes SU_T(2)$ subgroup, are

$$\mathbf{6}_A = (1,0)\oplus(1,0) \quad L = 0,2,\dots \quad \rightarrow ({}^1S_0, {}^3S_1), ({}^1D_2, {}^3D_{1,2,3}), ({}^1G_2, {}^3G_{1,2,3}), \dots \tag{2.2}$$

$$\mathbf{10}_S = (0,0)\oplus(1,1) \quad L = 1,3,\dots \quad \rightarrow ({}^1P_1, {}^3P_{0,1,2}), ({}^1F_1, {}^3F_{0,1,2}), \dots \tag{2.3}$$

In particular, one obtains $V_{{}^3S_1}(r) = V_{{}^1S_0}(r)$ which seems verified for $r > 2$fm (see Fig. 1, left) for high quality potentials [5], i.e. having $\chi^2/\text{DOF} < 1$ for 6000 data !. However, one might think that since a symmetry of the potential implies a symmetry of the S-matrix one should also have $\delta_{{}^1S_0}(p) = \delta_{{}^3S_1}(p)$ at low energies, in total contradiction to the data in Fig. 1. (see Sect. 4).

3. Serber symmetry

A vivid demonstration of Serber symmetry is demonstrated in Fig. 2 (left) where the pn differential cross section at low CM momenta, $p \leq 250$MeV, fulfills to a good approximation

$$\frac{d\sigma_{pn}}{d\Omega} = |f_{pn}(\pi - \theta)|^2 = |f_{pn}(\theta)|^2, \tag{3.1}$$

suggesting no interaction in odd L-waves as $P_L(\theta) = (-)^L P_L(\pi - \theta)$, a fact verified by NN potentials in the spin-triplet states for $r > 1.2$fm, see Fig. 2 (middle) for the P-wave case. This assumption

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[1]There is no reference. According to R. Serber [2] the name "Serber force" was coined by E. Wigner around 1947.
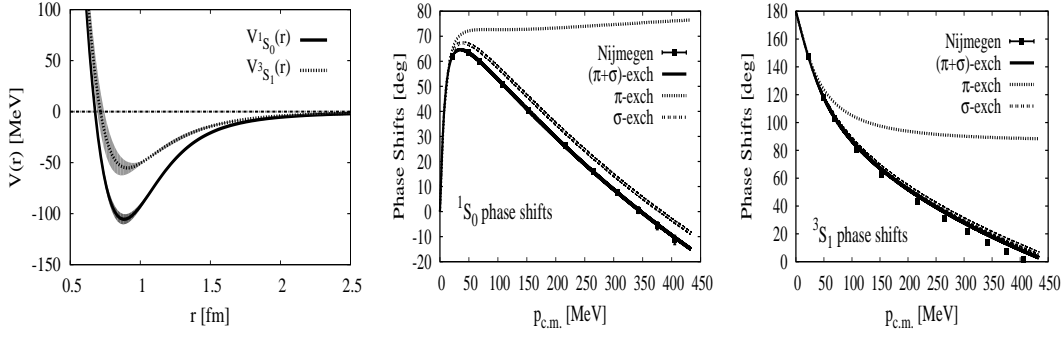
Figure 1: Wigner symmetry. Left: NN potentials in the $^1S_0$ and $^3S_1$ channel. Middle: $^1S_0$ NN phase shift. Right: $^3S_1$ NN phase shift. Potentials are similar for $r > 2$fm but phase shifts are different *everywhere*.
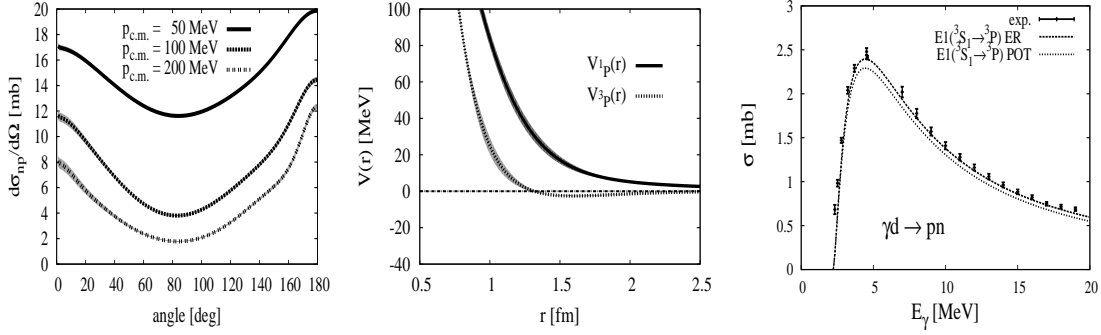

Figure 2: Serber symmetry. Left: pn differential cross section taken from an average of the Nijmegen database (nn.online.org). Middle: P-wave spin singlet and spin triplet potentials. Right: Deuteron photodisintegration as a function of the photon energy based on vanishing ^3P-wave interactions.

can also be tested by looking at Deuteron photodisintegration, $\gamma d \to pn$, dominated above threshold by the E_1 transition $^3S_1 \to ^3P$. Neglecting tensor force and meson exchange currents (MEC) the cross section for a normalized deuteron state $u_d(r)$ with binding energy B_d reads [6]

$$\sigma_{E1}(\gamma d \to pn) = \frac{\alpha\pi}{3p}(p^2 + 2\mu_{pn}B_d)\left| \int_0^\infty dr u_d(r)\, r\, u_{3P}(r) \right|^2 \qquad (3.2)$$

with $E_\gamma = B_d + p^2/(2\mu_{pn})$. For a free spherical P-wave $u_{3P}(r) = pr j_1(pr)$, the agreement is good using $u_d(r)$ from effective range (ER) theory [6] or from a potential [3] (POT), see Fig. 2 (right).

A further hint for Serber symmetry comes from the late 50's Skyrme proposal [7] to introduce a pseudopotential representing the NN effective interaction in nuclei in the form

$$V_{\text{effective}}(\mathbf{p}', \mathbf{p}) = t_0(1 + x_0 P_\sigma) + t_1(1 + x_1 P_\sigma)(\mathbf{p}'^2 + \mathbf{p}^2) + t_2(1 + x_2 P_\sigma)\mathbf{p}' \cdot \mathbf{p} + \ldots \qquad (3.3)$$

with $P_\sigma = (1 + \sigma_1 \cdot \sigma_2)/2$ the spin exchange operator. $P_\sigma = -1$ for spin singlet $S = 0$ and $P_\sigma = 1$ for spin triplet $S = 1$ states. Serber symmetry corresponds to take $x_2 = -1$ in the P-wave term, $\mathbf{p}' \cdot \mathbf{p}$. Mean field theory calculations fitting single nucleon states yield $x_2 = -0.99$ [8].

4. Renormalization and Long Distance Symmetry

In the meson exchange picture [9] the NN interaction can be decomposed as the sum

$$V(x) = V_{\text{short}}(x) + V_{\text{long}}(x) \tag{4.1}$$

where the short range and scheme dependent piece is given by distributional contact terms

$$V_{\text{short}}(r) = C_0 \delta(\vec{x}) + C_2 \{\nabla^2, \delta(\vec{x})\} + \dots, \tag{4.2}$$

whereas the long distance piece $V_{\text{long}}(x)$ is scheme independent and usually produces power divergences $\sim 1/r^n$ at short distances. We introduce a short distance cut-off, r_c, which will be removed in the end [2]. LDS means that even if $V_{1S_0}(r) = V_{3S_1}(r)$ for any $r > r_c$ one has $C_{0,{}^1S_0} \neq C_{0,{}^3S_1}$. We analyze the implications by looking at finite energy $S-$wave scattering states

$$u_p(r) = u_{p,c}(r) + p \cot \delta_0(p) u_{p,s}(r) \rightarrow \cos(pr) + \cot \delta_0(p) \sin(pr), \tag{4.3}$$

where $p = \sqrt{2\mu_{pn}E}$ is CM momentum. For $p \rightarrow 0$ then $\delta_0(p) \rightarrow -\alpha_0 p$ and zero energy states are

$$u_0(r) = u_{0,c}(r) - u_{0,s}(r)/\alpha_0 \rightarrow 1 - r/\alpha_0, \tag{4.4}$$

Here $u_{p,c}(r)$, $u_{p,s}(r)$, $u_{0,c}(r)$ and $u_{0,s}(r)$ depend on $V(r)$ *only*. Orthogonality in $r_c \leq r < \infty$ requires

$$0 = \int_{r_c}^{\infty} dr \left[u_{0,c}(r) - \frac{1}{\alpha_0} u_{0,s}(r) \right] \left[u_{p,c}(r) + p \cot \delta_0(p) u_{p,s}(r) \right]. \tag{4.5}$$

Note that the potential $V(r)$ and the scattering length α_0 are *independent variables*. Thus we assume Wigner symmetry for the potential $V_{1S_0}(r) = V_{3S_1}(r)$ but experimentally different scattering lengths $\alpha_{1S_0} = -23.74$fm and $\alpha_{3S_1} = 5.42$fm, yielding from Eq. (4.5) the structure for $r_c \rightarrow 0$,

$$p \cot \delta_{1S_0}(p) = \frac{\alpha_{1S_0} \mathscr{A}(p) + \mathscr{B}(p)}{\alpha_{1S_0} \mathscr{C}(p) + \mathscr{D}(p)}, \qquad p \cot \delta_{3S_1}(p) = \frac{\alpha_{3S_1} \mathscr{A}(p) + \mathscr{B}(p)}{\alpha_{3S_1} \mathscr{C}(p) + \mathscr{D}(p)}, \tag{4.6}$$

showing that a symmetry of the potential for any $r > r_c$, $r_c \rightarrow 0$, *is not* necessarily a symmetry of the S-matrix. The result for $\pi + \sigma$ exchange, while not exact, works rather well (see Fig. 1).

5. Sum rules

Based on the LDS idea we have recently derived the sum rules for phase shifts [3, 4]

$$\underbrace{\delta_{3L}(p) = \delta_{1L}(p) \qquad \text{all } L}_{\text{Wigner}} \quad , \quad \underbrace{\delta_{3L}(p), \delta_{1L}(p) = 0 \qquad \text{odd } L}_{\text{Serber}} \quad , \tag{5.1}$$

where we have defined the multiplet center $\delta_L^{ST} = 1/(3(2L+1)) \sum_{J=L-1}^{L+1} (2J+1) \delta_{LJ}^{ST}$. From data Fig. 3 shows that one has Wigner for *even* L and Serber for triplet *odd* L. The LDS character accommodates the symmetry for increasing p and L; what matters is the impact parameter, $b \sim L/p$.

The previous sum rules have a parallel long distance potential analog , and are also well verified for $r > 1.5$fm [4]. This suggests that a coarse graining of the interaction using e.g. the V_{lowk} potentials [11] works and justifies *per se* the symmetry obtained phenomenologically by fitting single particle states [8] for the Skyrme effective force, Eq. (3.3), [4]. We find that $V_{3L,\text{lowk}}(p,p) \ll V_{1L,\text{lowk}}(p,p)$ for $(-1)^L = -1$ and $V_{3L,\text{lowk}}(p,p) \sim V_{1L,\text{lowk}}(p,p)$ for $(-1)^L = 1$.

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[2]The constants C_0, C_2 etc. are scale dependent. The equivalence with momentum space renormalization is shown in Ref. [10] where the limit $r_c \rightarrow 0$ implies the irrelevance of C_2 in the presence of a singular chiral potential.
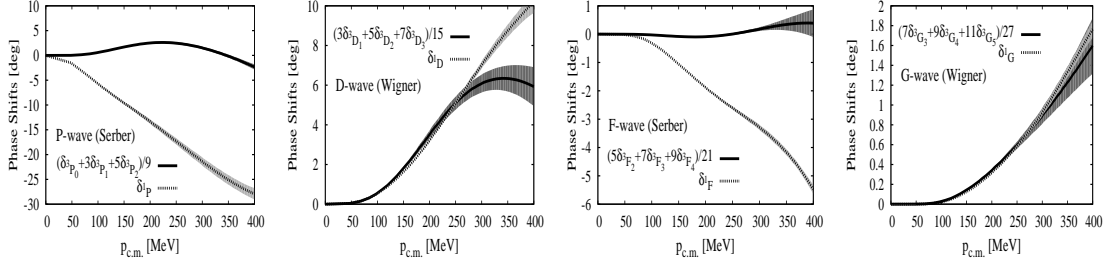
Figure 3: Phase shifts sum rules based on spin-orbit and tensor force breaking to first order. Wigner symmetry is verified only by even-L states while Serber symmetry is verified only by spin triplet odd-L waves.

6. Large N_c nucleon-nucleon potentials

As it is well known, in the large N_c limit with $\alpha_s N_c$ fixed, nucleons are heavy, $M_N \sim N_c$ [12], and the NN potential $\sim N_c$ becomes meaningful. The amazing aspect is that the symmetry pattern of the sum rules for the old nuclear Wigner and Serber symmetries largely complies to the large N_c and QCD based contracted $SU(4)_C$ symmetry [13, 14] where the tensorial spin-flavour structure is

$$V(r) = V_C(r) + \tau_1 \cdot \tau_2[\sigma_1 \cdot \sigma_2 W_S(r) + S_{12}W_T(r)] \sim N_c \qquad (6.1)$$

Other operators are $\mathcal{O}(N_c^{-1})$ and hence suppressed by a relative $1/N_c^2$ factor. One has the sum rules

$$V_{1_L}(r) = V_{3_L}(r) = V_C(r) - 3W_S(r) + \mathcal{O}(N_c^{-1}), \quad (-1)^L = +1 \qquad (6.2)$$
$$V_{1_L}(r) = V_C(r) + 9W_S(r) + \mathcal{O}(N_c^{-1}), \quad (-1)^L = -1 \qquad (6.3)$$
$$V_{3_L}(r) = V_C(r) + W_S(r) + \mathcal{O}(N_c^{-1}), \quad (-1)^L = -1 \qquad (6.4)$$

Thus, large N_c *implies* Wigner symmetry *only* in even-L channels, exactly as observed in Fig. 3. Serber symmetry is possible but less evident (see [4]). This suggests to use large N_c itself and its contracted spin-flavour group $SU(4)_C$ as a long distance symmetry. Actually, the energy independent potential may be obtained in a multi-meson exchange picture consistently with large N_c counting rules [15] [3]. Retaining one boson exchange (OBE) with π, σ, ρ and ω mesons one has

$$V_C(r) = -\frac{g_{\sigma NN}^2}{4\pi}\frac{e^{-m_\sigma r}}{r} + \frac{g_{\omega NN}^2}{4\pi}\frac{e^{-m_\omega r}}{r}, \qquad (6.5)$$

$$W_S(r) = \frac{g_{\pi NN}^2}{48\pi}\frac{m_\pi^2}{\Lambda_N^2}\frac{e^{-m_\pi r}}{r} + \frac{f_{\rho NN}^2}{24\pi}\frac{m_\rho^2}{\Lambda_N^2}\frac{e^{-m_\rho r}}{r}, \qquad (6.6)$$

$$W_T(r) = \frac{g_{\pi NN}^2}{48\pi}\frac{m_\pi^2}{\Lambda_N^2}\frac{e^{-m_\pi r}}{r}\left[1 + \frac{3}{m_\pi r} + \frac{3}{(m_\pi r)^2}\right] - \frac{f_{\rho NN}^2}{48\pi}\frac{m_\rho^2}{\Lambda_N^2}\frac{e^{-m_\rho r}}{r}\left[1 + \frac{3}{m_\rho r} + \frac{3}{(m_\rho r)^2}\right], (6.7)$$

where $\Lambda_N = 3M_p/N_c$ and $g_{\sigma NN}, g_{\pi NN}, f_{\rho NN}, g_{\omega NN} \sim \sqrt{N_c}$ and $m_\pi, m_\sigma, m_\rho, m_\omega \sim N_c^0$. To leading and subleading order in N_c one may neglect spin orbit, meson widths and relativity. The tensor force W_T is singular at short distances $\sim 1/r^3$ and requires renormalization (see [17] for the π case).

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[3]The LDS character implies relaxing the contact interaction piece *not* to be of the same form as the long distance potentials, i.e. $V_{\text{short}}(\vec{x}) \neq (C_C + \tau_1 \cdot \tau_2[\sigma_1 \cdot \sigma_2 C_S + S_{12}C_T])\delta(\vec{x})$ avoiding the extra symmetry, $\tau_a \sigma_i \to -\tau_a \sigma_i$ [16].

Table 1: Deuteron properties for renormalized large N_c OBE potentials. We use $\gamma = \sqrt{2\mu_{np}B_d}$ with $B_d = 2.224575(9)$ and take $g_{\pi NN} = 13.1083$, $m_\pi = 138.03\,\text{MeV}$, $m_\rho = 770\,\text{MeV}$, $m_\omega = 782\,\text{MeV}$. A fit to the $^1S_0$ phase shift gives $m_\sigma = 501\,\text{MeV}$ and $g_{\sigma NN} = 9.1$ [18]. $\pi\sigma\rho\omega$ uses $f_{\rho NN} = 15.5$ and $g_{\omega NN} = 9.857$ while $\pi\sigma\rho\omega^*$ uses $f_{\rho NN} = 17.0$ and $g_{\omega NN} = 10.147$. Experimental or recommended values from Ref. [19].

	$\gamma(\text{fm}^{-1})$	η	$A_S(\text{fm}^{-1/2})$	$r_m(\text{fm})$	$Q_d(\text{fm}^2)$	P_D	$\langle r^{-1}\rangle$
$\pi([17])$	Input	0.02633	0.8681	1.9351	0.2762	7.88%	0.476
$\pi\sigma$	Input	0.02599	0.9054	2.0098	0.2910	6.23%	0.432
$\pi\sigma\rho\omega$	Input	0.02597	0.8902	1.9773	0.2819	7.22%	0.491
$\pi\sigma\rho\omega^*$	Input	0.02625	0.8846	1.9659	0.2821	9.09%	0.497
NijmII([5])	Input	0.02521	0.8845(8)	1.9675	0.2707	5.635%	0.4502
Reid93([5])	Input	0.02514	0.8845(8)	1.9686	0.2703	5.699%	0.4515
Exp. ([19])	0.231605	0.0256(4)	0.8846(9)	1.9754(9)	0.2859(3)	5.67(4)	

Deuteron properties are shown in Table 1 for parameters always reproducing the $^1S_0$ phase shift, Fig. 1 (middle). Space-like electromagnetic form factors in the impulse approximation [20] for $G_E^p(-\mathbf{q}^2) = 1/(1 + \mathbf{q}^2/m_\rho^2)^2$ and without MEC are plotted in Fig. 4 (see [21] for the π case). Overall, the agreement is good for *realistic* couplings [4]. The inclusion of shorter range mesons induces moderate changes, due to the expected short distance insensitivity embodied by renormalization, *despite* the short distance singularity and *without* introducing strong meson-nucleon-nucleon vertex functions. In practice convergence is achieved for $r_c \sim 0.3\,\text{fm}$. Our calculation includes only the OBE part of the leading N_c potential but multiple meson exchanges could also be added [15].

For large N_c, the central potential is leading, Eq. (6.5). Energy independent potentials using power counting within Chiral Perturbation Theory (ChPT) [22] yield a central force V_C^{ChPT} only to $\mathcal{O}(1/f_\pi^4 M_N)$ i.e. N^{2}LO and ChPT potentials do not scale properly with N_c since $g_A \sim N_c$, $f_\pi \sim \sqrt{N_c}$ and there are terms scaling as $V_{2\pi}^{\text{ChPT}} \sim g_A^4/f_\pi^4 \sim N_c^2$ and not as $\sim N_c$, even after inclusion of Δ [23]. Moreover, Wigner and Serber symmetries are violated at long distances since

$$V_{2\pi}^{\text{ChPT}}(r) = (1 + 2\tau_1 \cdot \tau_2)\frac{e^{-2m_\pi r}}{r}\frac{3g_A^4 m_\pi^5}{1024 f_\pi^4 M_N \pi^2} + \ldots \tag{6.8}$$

These features might perhaps explain why renormalizing ChPT potentials in different schemes a mismatch of 10^0 at $p = 400\,\text{MeV}$ for the $^1S_0$ phase shift is persistently obtained [24, 25, 10, 26].

7. Conclusions

Wigner and Serber symmetries in the NN system are realized as long distance ones and are largely compatible with the large N_c picture. When large N_c NN-potentials are saturated by π,σ,ρ and ω exchange and subsequently renormalized, we obtain satisfactory results for the deuteron and central partial waves. This suggests that large N_c potentials might eventually provide a workable scheme, less directly related to ChPT but closer in spirit to the common wisdom of Nuclear Physics.

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[4]The Goldberger-Treiman relation gives $g_{\pi NN} = g_A M_N/f_\pi = 12.8$ for pions and $g_{\sigma NN} = M_N/f_\pi = 10.1$ for scalars for $f_\pi = 92.3\,\text{MeV}$ and $g_A = 1.26$. Sakurai's universality and KSFR yield $g_{\rho NN} = g_{\rho\pi\pi}/2 = m_\rho/f_\pi/\sqrt{8} = 2.9$. From $SU(3)$ we have $g_{\omega NN} = 3g_{\rho NN} - g_{\phi NN} = 8.7$ using OZI rule, $g_{\phi NN} = 0$. ρ−meson dominance yields $f_{\rho NN} = \kappa_\rho g_{\rho NN}$ with $\kappa_\rho = \mu_p - \mu_n - 1 = 3.7$ with $\mu_p = 2.79$ and $\mu_n = -1.91$. Adding ρ',ρ'' states yields $\kappa_\rho = 6.1$ and thus $f_{\rho NN} = 18$.
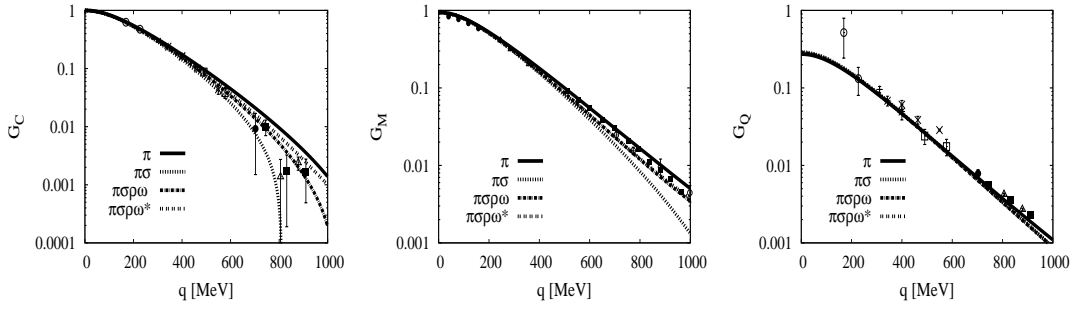
Figure 4: Deuteron charge (left), magnetic (middle) and quadrupole (right) form factors. See also Table 1.

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