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Unitarity constraints on chiral perturbative amplitudes

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Low lying scalar resonances emerge as a necessary part to adjust chiral perturbation theory to experimental data once unitarity constraint is taken into consideration. I review recent progress made in this direction in a model independent approach. Also I briefly review studies on the odd physical properties of these low lying scalar resonances, including in the $\gamma\gamma \to \pi^+\pi^-, \pi^0\pi^0$ processes.

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Low lying scalar resonances emerge as a necessary part to adjust chiral perturbation theory to experimental data when the constraint of unitarity is taken into consideration. This is most clearly seen if one writes down a dispersion relation for $\sin(2\delta_\pi)$ where δ_π is the $\pi\pi$ scattering phase shift in the scalar–iso-scalar channel. The data exhibits a convex curvature below 1GeV whereas chiral estimation to the nearby cut contribution is negative and concave – the huge gap between the two can only be made up by including a pole contribution, according to the standard S–matrix theory principal. [1] Unitarization of the chiral perturbative amplitude also predicts the existence of a light and broad pole structure in the IJ=00 channel $\pi\pi$ scattering,nevertheless it was not very clear to what extent one should trust the output of unitarized chiral perturbative amplitude.

In section 1 we briefly introduce a novel dispersion representation for partial wave amplitudes developed in recent few years, [2, 3] and physical results read out from it, including a better understanding on the Padé unitarization approximation. In section 2 we discuss how one can get a better understanding on chiral perturbation theory and resonance chiral perturbation theory (RχPT) parameters based on the use of dispersion techniques. In section 3 we investigate studies on the dynamical properties of the low lying scalar resonances. Finally in section 4 we introduce a recent work on the $\gamma\gamma \to \pi\pi$ process. Based on which we find that the $\sigma \to \gamma\gamma$ coupling is significantly smaller than that of a naive $\bar{q}q$ assignment.

1. The PKU representation – a unitarized dispersion representation for elastic scattering amplitudes

The S-matrix element of partial wave elastic scattering amplitude satisfies the following dispersive representation: [2, 3]

$$S^{phy.} = \prod_i S^{R_i} \cdot S^{cut} \ , \tag{1.1}$$

where S^{R_i} denotes the i-th *second* sheet pole contribution and S^{cut} denotes the contribution from cuts except the elastic one. The information from higher sheet poles is hidden in the right hand integral which consists of one part of the total background contribution. We have,

$$\begin{aligned} S^{cut} &= e^{2i\rho f(s)} \ , \\ f(s) &= \frac{s}{\pi}\int_L \frac{\mathrm{Im}_L f(s')}{s'(s'-s)} + \frac{s}{\pi}\int_R \frac{\mathrm{Im}_R f(s')}{s'(s'-s)} \ , \end{aligned} \tag{1.2}$$

where the 'left hand' cut $L = (-\infty, 0]$ for equal mass scatterings and may contain a rather complicated structure for unequal mass scatterings. The right hand cut R starts from first *inelastic* threshold to positive infinity. It can be demonstrated that the dispersive representation for f is free from the subtraction constant. [2] The PKU representation, Eq. (1.1), is sensitive to S matrix poles not too far away from physical threshold, hence providing a useful tool to explore the light and broad resonance σ and κ. In the data fit it is found that crossing symmetry plays an important role in fixing the σ pole location. Taking this fact into account [3] it gives the σ pole location at $M_\sigma = 470 \pm 50\text{MeV}$, $\Gamma_\sigma = 570 \pm 50\text{MeV}$, in good agreement with the determination using more sophisticated Roy equation analysis. [4, 5] The application of Eq. (1.1) to LASS data [6] also unambiguously establish the existence of the κ meson with the pole location: [2]

PoS(EFT09)048

$M_\kappa = 694 \pm 53$MeV , $\Gamma_\kappa = 606 \pm 59$MeV , which are also in agreement with the later determination on κ pole parameters using Roy–Steiner equations. [7]

The dispersion representation Eq. (1) safely embeds chiral perturbative amplitudes into a unitarized scheme. This property is not always trivial in the practice of unitarization. For example, contrary to the input chiral perturbative amplitudes, Padé approximants lead to completely different singularity structure in the vicinity of $s = 0$ – a region where the former ought to be trustworthy. The reliability of χPT predictions in the small $|s|$ region can be vividly seen in the I=2 s wave amplitude. One may use χPT result to estimate the contribution of the left hand cut to the scattering length a_0^2. The estimate is rough but gives a value qualitatively much larger in magnitude than a_0^2 extracted from experiment. This difficulty is resolved, recalling that χPT also predicts a virtual pole near $s = 0$, which contribution cancels a large amount of the cut contribution and leads to the correct prediction of a_0^2. This example further illustrate that the singularity structure as predict by χPT in the vicinity of $s = 0$ is indeed reliable and self-consistent.

The PKU representation, Eq. (1.1) affords another interesting opportunity to study the relation between resonance parameters and low energy constants of the chiral lagrangian. On the $l.h.s.$ of Eq. (1.1) one may replace $S^{phy.}$ by χPT result at low energies, on the $r.h.s.$ one does not know how many resonances are there, nevertheless one may formally make a threshold expansion to match the $l.h.s.$. In this situation the cut integrals in Eq. (1.2) are however difficult to calculate. So we firstly neglect completely the cut integrals and assuming there is only one pole on the $r.h.s.$ of Eq. (1.1). Under this approximation we can get a prediction on the resonance parameters expressed in terms of the low energy constants. It is shown in Ref. [8] that the pole location is exactly the same as the prediction of [1,1] Padé amplitude in the large N_c and chiral limit. In my knowledge there had never been serious examination on what does the predictions of Padé approximation mean in the literature. Hence Ref. [8] provides a first understanding on this question: in the large N_c and chiral limit the pole location as predicted by [1,1] Padé approximant is equivalent to the approximation that: 1) neglecting crossed channel cut completely, 2) assuming single pole dominance in the s channel. However, in Ref. [9] it is shown that crossed channel cut contributions are *not* negligible. A direct consequence of neglecting the cut contribution is the violation of crossing symmetry as will be discussed in the next section.

2. Matching between two expansions

Threshold expansion on both sides of Eq. (1.1) (the $l.h.s.$ replaced by $S^{\chi PT}$) could provide useful relations between resonance parameters and low energy constants, if one can reliably estimate cut integrals in some way. Fortunately, this can indeed be done in the large N_c limit. In such a limit the Eq. (1.1) leads to the same result as the partial wave dispersion relation. Hence Eq. (1.1) can actually be understood as a simple combination of single channel unitarity and partial wave dispersion relation. [9]

The matching results in a set of relations at different chiral order.

PoS(EFT09)048

	$T(0)$	t_0^{tR}	t_0^{sR}	$t_0^{\chi PT} = m_\pi a_J^I$
$IJ = 11$	$-\frac{m_\pi^2}{24\pi f^2}$	$\frac{4\Gamma_S}{9M_S^3} + \frac{2\Gamma_V}{M_V^3}$	$\frac{4\Gamma_V}{M_V^3}$	0
$IJ = 00$	$-\frac{m_\pi^2}{32\pi f^2}$	$-\frac{4\Gamma_S}{3M_S^3} + \frac{36\Gamma_V}{M_V^3}$	$\frac{4\Gamma_S}{M_S^3}$	$\frac{7m_\pi^2}{32\pi f^2}$
$IJ = 20$	$\frac{m_\pi^2}{16\pi f^2}$	$-\frac{4\Gamma_S}{3M_S^3} - \frac{18\Gamma_V}{M_V^3}$	0	$-\frac{m_\pi^2}{16\pi f^2}$

Table 1: Summary of the different contributions $T(0)$, cross channel resonance exchange contribution t_0^{tR}, and s-channel resonance contribution t_0^{SR} to the scattering lengths at leading order in the m_π^2 expansion. The generalized KSRF-relation derives from the matching of the sum of the first three columns to the χPT prediction, $t_0^{\chi PT}$. In the last line, $T(0)$ contains the sum of $-|T(0)|$ and the $IJ = 20$ virtual pole contribution.

At $O(p^2)$:

$$\frac{1}{16\pi f^2} = \frac{9\Gamma_V^{(0)}}{M_{\mathrm{V}}^{(0)\,3}} + \frac{2\Gamma_S^{(0)}}{3M_{\mathrm{S}}^{(0)\,3}}, \tag{2.1}$$

It is remarkable to notice that three different channels produce the same results. The conclusion is that *partial wave amplitudes remember crossing symmetry.* It is interesting to compare Eq. (2.1) with the old version of the so called KSRF relation:

$$\frac{1}{16\pi f^2} = \frac{6\Gamma_V^{(0)}}{M_{\mathrm{V}}^{(0)\,3}} \,. \tag{2.2}$$

In the IJ=11 channel, one may obtain Eq. (2.2) if neglecting crossed channel vector and scalar exchanges, see table 1 for illustration. [9]

At $O(p^4)$:

$$L_2 = 12\pi f^4 \frac{\Gamma_{\mathrm{V}}^{(0)}}{M_{\mathrm{V}}^{(0)\,5}} \,, \quad L_3 = 4\pi f^4 \left(\frac{2\Gamma_{\mathrm{S}}^{(0)}}{3M_{\mathrm{S}}^{(0)\,5}} - \frac{9\Gamma_{\mathrm{V}}^{(0)}}{M_{\mathrm{V}}^{(0)\,5}} \right) . \tag{2.3}$$

It is remarkable to notice that the Eq. (2.3) rewrites the old results of Ref. [10] without even knowing how to write down an effective resonance chiral lagrangian!

At $O(m_\pi^2 p^2)$:

Matching at this order led to a novel relation any lagrangian model has to obey, which is a consequence of high energy constraint combined with chiral symmetry:

$$0 = \frac{2}{3}\,\frac{\Gamma_{\mathrm{S}}^{(0)}}{M_{\mathrm{S}}^{(0)\,5}}\,[\alpha_S + 6] + \frac{9\,\Gamma_{\mathrm{V}}^{(0)}}{M_{\mathrm{V}}^{(0)\,5}}\,[\alpha_V + 6]\,. \tag{2.4}$$

PoS(EFT09)048

The physical widths and masses, $\Gamma_{\rm R}$ and $M_{\rm R}$, carry an implicit dependence on m_π^2, which can be expressed in the form

$$\frac{\Gamma_{\rm R}}{M_{\rm R}^3} = \frac{\Gamma_{\rm R}^{(0)}}{M_{\rm R}^{(0)\,3}} \left[1 + \alpha_R \frac{m_\pi^2}{M_{\rm R}^{(0)\,2}} \; + \; \mathscr{O}\left(m_\pi^4\right) \right] . \tag{2.5}$$

The matching project has been further extended to $O(p^6)$ and interesting results are obtained, [11] I refer to the talk given by Sanz–Cillero in this conference for details. Based on these new formulas, Guo and Sanz–Cillero made a systematic re-estimation to the coupling constants in $O(p^6)$ chiral lagrangian in a model independent way. [12]

3. On the nature of the f0(600) pole

It has long been argued that the $f_0(600)$ pole is the σ meson of linear σ (-like) model. [13] Nevertheless due to its strong interaction nature, it is very difficult to solve this problem at fundamental level. On the other side, it is argued using the inverse amplitude method or chiral unitarization approach, that the $f_0(600)$ pole is a 'dynamically generated' resonance. [14] Yet the wording 'dynamically generated' itself needs clarification, [15] it can be understood from the discussion in section 1 and Refs. [16, 8] that in the approach of unitarization of chiral perturbative amplitudes the $f_0(600)$ pole does fall back to the real axis in the large N_c limit. Hence one has to put it explicitly in the lagrangian in the very beginning, therefore being 'fundamental'. The odd pole trajectory of $f_0(600)$ with respect to the variation of N_c was used to argue its dynamical nature. Nevertheless it is shown, using a solvable $O(N)$ linear σ model, that the 'fundamental' σ pole trajectory looks indeed being odd. [17] One may expect that the study of other light scalar resonances like $\kappa(700)$, $f_0(980)$ and $a_0(980)$ could shed some light on the understanding of $f_0(600)$. However, the inclusion of these resonances does not seem to be very helpful up to now, if not merely making the situation more complicated.

4. The $\gamma\gamma \to \pi\pi$ process in a partially couple channel approach

There remains the hope in understanding better the property of $f_0(600)$ through the study on the $\gamma\gamma \to \pi^+\pi^-, \pi^0\pi^0$ process, as emphasized by Pennington, [18] since the di-photon coupling of a resonance may be used as probe to investigate hadron internal structure at quark level. Again unitarity plays a crucial role in such investigations. However the σ pole locates quite far away from the physical region, the di-photon coupling extracted as such is found not very stable. This problem is reinvestigated recently, [19] where the fit to data at first step is up to 1.4GeV, aiming at fixing the d-wave background. Then a refined analysis is made by fitting data up to 0.8GeV, using the $\pi\pi$ scattering T matrix obtained in Ref. [3]. The fit quality can seen in fig. 1 borrowed from Ref. [19]. In this way the two photon decay width is obtained to be $\Gamma(\sigma \to \gamma\gamma) \simeq 2.1$KeV, a result significantly smaller than that expected from a naive $\bar{q}q$ model calculation. This further stresses the unconventional nature of the $f_0(600)$ nature except the large width it has. The value of $\sigma\pi\pi$ coupling is also given in Ref. [19], $g^2_{\sigma\pi\pi} = (-0.20 - 0.13i)\mathrm{GeV}^2$, which is compatible with the value given in Ref. [22]: $g^2_{\sigma\pi\pi} = -0.25 - 0.06i\mathrm{GeV}^2$ which was used by the authors of Ref. [23] to argue in favor of the gluonium nature of $f_0(600)$. It is interesting to notice that, for a narrow

PoS(EFT09)048

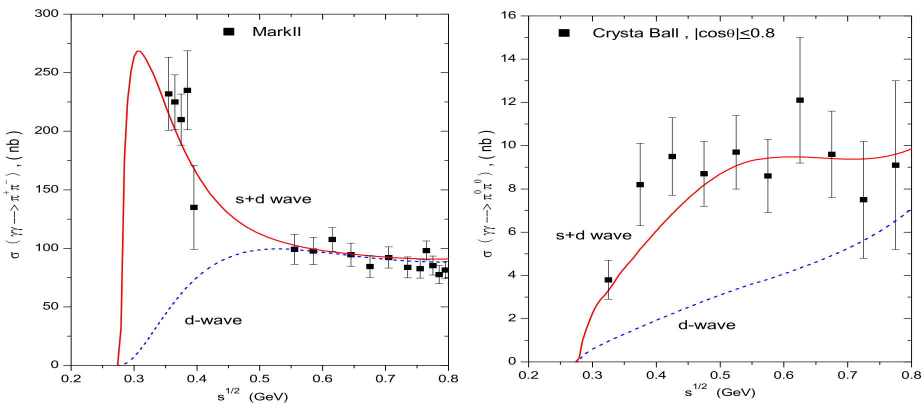

Figure 1: A fit up to 0.8GeV using single channel s-wave T matrices of Ref. [3], with only one fit parameter. The $\pi^+\pi^-$ and $\pi^0\pi^0$ data are from Refs. [20] and [21], respectively. Dashed curve represents d-wave background, solid curve represents the total contributions, including the I=0 s–wave to be fitted.

resonance Re$[g^2_{\sigma\pi\pi}]$ should be positive, otherwise it would be a ghost rather than a particle and violates probability conservation. Nevertheless for a broad resonance this constraint does not need to hold anymore. The negative Re$[g^2_{\sigma\pi\pi}]$ indicates another peculiarity of $f_0(600)$.

To conclude, the correct use of unitarity, when combined with chiral symmetry, plays a powerful role in studying resonance physics, especially the property of the light and broad $f_0(600)$. However, there still remains many interesting and mysterious characters of $f_0(600)$ waiting to be resolved in future.

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PoS(EFT09)048

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PoS(EFT09)048