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Chiral Perturbation Theory in the Nuclear Medium including both short- and long-range Interactions

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We review on a novel chiral power counting scheme for in-medium chiral perturbation theory with nucleons and pions as degrees of freedom [1]. It allows for a systematic expansion taking into account local as well as pion-mediated inter-nucleon interactions. Based on this power counting, one can identify classes of non-perturbative diagrams that require a resummation. We then calculate the pion self-energy in asymmetric nuclear matter up-to-and-including next-to-leading order (NLO). It is shown that the corrections involving in-medium nucleon-nucleon interactions cancel between each other at NLO.

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1. Introduction

An interesting achievement in nuclear physics would be the calculation of atomic nuclei and nuclear matter properties from microscopic inter-nucleon forces in a systematic and controlled way. This is a non-perturbative problem involving the strong interactions. In the last decades, Effective Field Theory (EFT) has proven to be an indispensable tool to accomplish such an ambitious goal. In this work we employ Chiral Perturbation Theory (CHPT) to nuclear systems [2, 3, 4], with nucleons and pions as the pertinent degrees of freedom. For the lightest nuclear systems with two, three and four nucleons, it has been successfully applied [5]. For heavier nuclei one common procedure is to employ the chiral nucleon-nucleon potential derived in CHPT combined with standard many-body methods, sometimes supplied with renormalization group techniques [6]. In ref.[1] we have recently derived a chiral power counting in nuclear matter that takes into account local multi-nucleon interactions simultaneously to the pion-nucleon interactions. Many present applications of CHPT to nuclei and nuclear matter only consider meson-baryon chiral Lagrangians (see e.g. [5] for a summary), without constraints from free nucleon-nucleon scattering. Our novel power counting is applied in ref.[1] to the problem of calculating the pion self-energy in asymmetric nuclear matter at next-to-leading order (NLO). This problem is tightly connected with that of pionic atoms [7, 8] due to the relation between the pion self-energy and the pion-nucleus optical potential. For recent calculations of the former see [9, 10, 11, 12, 13].

2. Chiral Power Counting

Ref.[14] establishes the concept of a "in-medium generalized vertex" (IGV). Such type of vertices result because one can connect several bilinear vacuum vertices through the exchange of baryon propagators with the flow through the loop of one unit of baryon number, contributed by the nucleon Fermi seas. At least one is needed because otherwise we would have a vacuum closed nucleon loop that in a low energy effective field theory is buried in the chiral higher order counterterms. It was also stressed in ref.[9] that within a nuclear environment a nucleon propagator could have a "standard" or "non-standard" chiral counting. To see this note that a soft momentum $Q \sim p$, related to pions or external sources can be associated to any of the vertices. Denoting by k the on-shell four-momenta associated with one Fermi sea insertion in the IGV, the four-momentum running through the j^{th} nucleon propagator can be written as $p_j = k + Q_j$. If $Q_j^0 = \mathcal{O}(m_\pi) = \mathcal{O}(p)$ one has the standard counting so that the baryon propagator scales as $\mathcal{O}(p^{-1})$. However, if Q_j^0 is of the order of a kinetic nucleon energy in the nuclear medium then the nucleon propagator should be counted as $\mathcal{O}(p^{-2})$. This is referred as the "non-standard" case in ref.[9]. In order to treat chiral Lagrangians with an arbitrary number of baryon fields (bilinear, quartic, etc) ref.[1] considered firstly bilinear vertices like in refs.[14, 9], but now the additional exchanges of heavy meson fields of any type are allowed. The latter should be considered as merely auxiliary fields that allow one to find a tractable representation of the multi-nucleon interactions that result when the masses of the heavy mesons tend to infinity. These heavy meson fields are denoted in the following by H, and a heavy meson propagator is counted as $\mathcal{O}(p^0)$ due to their large masses. On the other hand, ref.[1] takes the non-standard counting case from the start and any nucleon propagator is considered as $\mathcal{O}(p^{-2})$. In this way, no diagram whose chiral order is actually lower than expected if the nucleon propagators were counted assuming the standard rules is lost. In the following $m_\pi \sim k_F \sim \mathscr{O}(p)$ are taken of the same chiral order, and are considered much smaller than a hadronic scale Λ_χ of several hundreds of MeV that results by integrating out all other particle types, including nucleons with larger three-momentum, heavy mesons and nucleon isobars [4]. The final formula obtained in ref.[1] for the chiral order ν of a given diagram is

$$\nu = 4 - E + \sum_{i=1}^{V_\pi}(n_i + \ell_i - 4) + \sum_{i=1}^{V}(d_i + \omega_i - 1) + \sum_{i=1}^{m}(v_i - 1) + \sum_{i=1}^{V_\rho} v_i \;. \tag{2.1}$$

where E is the number of external pion lines, n_i is the number of pion lines attached to a vertex without baryons, ℓ_i is the chiral order of the latter with V_π its total number. In addition, d_i is the chiral order of the i^{th} vertex bilinear in the baryonic fields, v_i is the number of mesonic lines attached to it, ω_i that of only the heavy lines, V is the total number of bilinear vertices, V_ρ is the number of IGVs and m is the total number of baryon propagators minus V_ρ, $V = V_\rho + m$. It is important to stress that ν given in eq.(2.1) is bounded from below as explicitly shown in ref.[1]. Because of the last term in eq.(2.1) adding a new IGV to a connected diagram increases the counting at least by one unit because $v_i \geq 1$. The number ν given in eq.(2.1) represents a lower bound for the actual chiral power of a diagram, μ, so that $\mu \geq \nu$. The real chiral order of a diagram might be different from ν because the nucleon propagators are counted always as $\mathscr{O}(p^{-2})$ in eq.(2.1), while for some diagrams there could be propagators that follow the standard counting. Eq.(2.1) implies the following conditions for augmenting the number of lines in a diagram without increasing the chiral power by adding i) pionic lines attached to mesonic vertices, $\ell_i = n_i = 2$, ii) pionic lines attached to meson-baryon vertices, $d_i = v_i = 1$ and iii) heavy mesonic lines attached to bilinear vertices, $d_i = 0$, $\omega_i = 1$.

3. Meson-baryon contributions to the pion self-energy

Here, we apply the chiral counting given in eq.(2.1) to calculate the pion self-energy in the nuclear medium up to NLO or $\mathscr{O}(p^5)$, with the different contributions shown in fig.1. The nucleon propagator contains both the free and the in-medium piece [15],

$$\frac{\theta(\xi_{i_3} - |\mathbf{k}|)}{k^0 - E(\mathbf{k}) - i\varepsilon} + \frac{\theta(|\mathbf{k}| - \xi_{i_3})}{k^0 - E(\mathbf{k}) + i\varepsilon} = \frac{1}{k^0 - E(\mathbf{k}) + i\varepsilon} + i(2\pi)\theta(\xi_{i_3} - |\mathbf{k}|)\delta(k^0 - E(\mathbf{k})) \;. \tag{3.1}$$

In this equation the subscript i_3 refers to the third component of isospin of the nucleon, so that, $i_3 = +1/2$ corresponds to the proton and $-1/2$ to the neutron, and the symbol ξ_{i_3} is the Fermi momentum of the Fermi sea for the corresponding nucleon. Our convention for the pion self-energy, Σ, is such that the dressed pion propagators reads $i\Delta_\pi(q) = i/(q^2 - m_\pi^2 + \Sigma)$. The leading contribution to the pion self-energy corresponds to the diagrams 1 (Σ_1) and $2+3$, Σ_2, with [1]

$$\begin{aligned}
\Sigma_1 &= \frac{-iq^0}{2f^2}\varepsilon_{ij3}(\rho_p - \rho_n) \;, \\
\Sigma_2 &= \frac{ig_A^2\,\mathbf{q}^2}{2f^2 q^0}\varepsilon_{ij3}(\rho_p - \rho_n) - \frac{g_A^2}{4f^2}\frac{(\mathbf{q}^2)^2}{mq_0^2}\delta_{ij}(\rho_p + \rho_n) \;,
\end{aligned} \tag{3.2}$$

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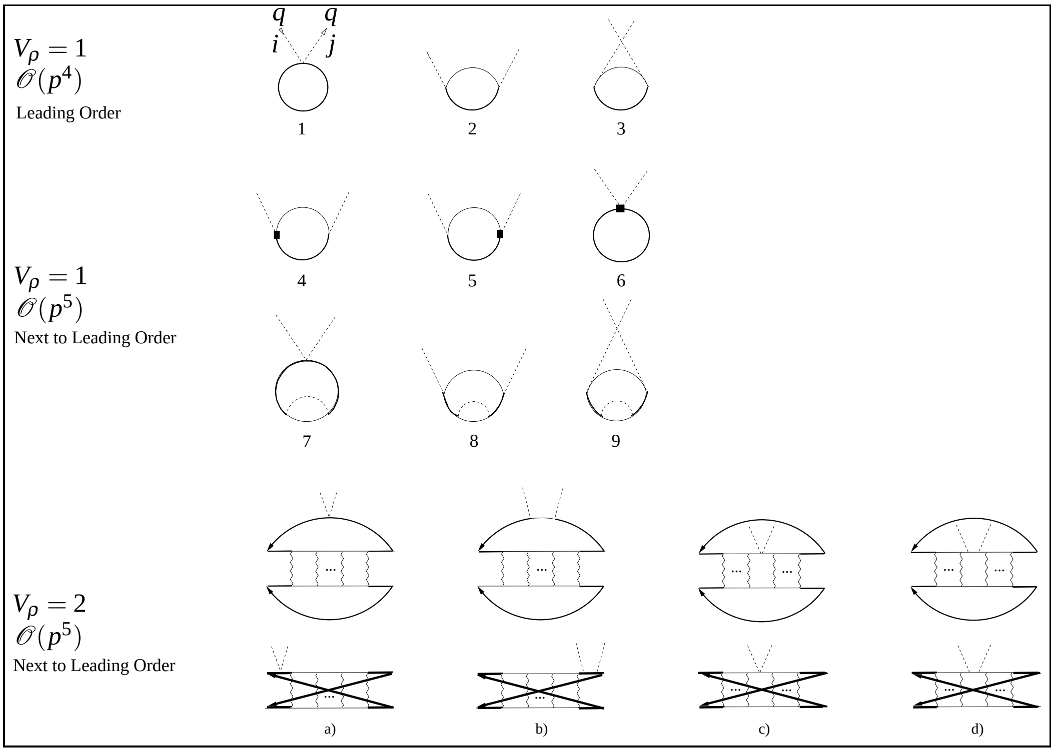

Figure 1: Contributions to the in-medium pion self-energy up to NLO or $\mathscr{O}(p^5)$. The pions are indicated by the dashed lines and the squares correspond to NLO pion-nucleon vertices. A wiggly line is the nucleon-nucleon interaction kernel, that it is iterated as meant by the ellipsis.

where $f = 92.4\,\mathrm{MeV}$ is the pion decay constant and the proton(neutron) density is given by $\rho_{p(n)} = \xi_{p(n)}^3/3\pi^2$. Now, we move to the NLO contributions. The sum of the diagrams 4 and 5 gives the result [1]

$$\Sigma_3 = \frac{g_A^2 \mathbf{q}^2}{2mf^2}(\rho_p + \rho_n)\delta_{ij} \ . \tag{3.3}$$

The diagram 6 of fig.1 is given by [1]

$$\Sigma_4 = \frac{-2\delta_{ij}}{f^2}\left(2c_1\, m_\pi^2 - q_0^2(c_2 + c_3 - \frac{g_A^2}{8m}) + c_3\, \mathbf{q}^2\right)(\rho_p + \rho_n) \ , \tag{3.4}$$

where the c_i are low-energy constants of the pion-nucleon Lagrangian $\mathscr{L}_{\pi N}^{(2)}$ [16]. Next, let us consider the contributions to the pion self-energy due to the nucleon self-energy from a one-pion loop as depicted in the diagrams 7–9 of fig.1. The one-pion loop nucleon self-energy can be written as,

$$\Sigma^\pi = \frac{1+\tau_3}{2}\Sigma_p^\pi + \frac{1-\tau_3}{2}\Sigma_n^\pi \ , \tag{3.5}$$

with Σ_p^π and Σ_n^π the proton and nucleon self-energies due to the in-medium pion-nucleon loop. The contributions from the diagrams 7 (Σ_5) and $8+9$ (Σ_6) are [1]

$$\begin{aligned}
\Sigma_5 &= \frac{iq^0}{f^2}\varepsilon_{ij3}\int\frac{d^3k}{(2\pi)^3}\left(\frac{\partial\Sigma_p^\pi}{\partial k^0}\theta_p^- - \frac{\partial\Sigma_n^\pi}{\partial k^0}\theta_n^-\right)_{k^0=E(\mathbf{k})}~, \\
\Sigma_6 &= \frac{-ig_A^2}{f^2}\frac{\mathbf{q}^2}{q^0}\varepsilon_{ij3}\int\frac{d^3k}{(2\pi)^3}\left(\frac{\partial\Sigma_p^\pi}{\partial k^0}\theta_p^- - \frac{\partial\Sigma_n^\pi}{\partial k^0}\theta_n^-\right)_{k^0=E(\mathbf{k})} \\
&\quad -\frac{g_A^2}{f^2}\frac{\mathbf{q}^2}{q_0^2}\delta_{ij}\int\frac{d^3k}{(2\pi)^3}\left(\Sigma_p^\pi\theta_p^- + \Sigma_n^\pi\theta_n^-\right)~. \qquad (3.6)
\end{aligned}$$

The last term in Σ_6 is a NNLO or $\mathcal{O}(p^6)$ contribution because the pion-loop nucleon self-energy is $\mathcal{O}(p^3)$ and we neglect it. The free pion-loop nucleon self-energy is calculated in heavy baryon CHPT [16]. Its derivative is $\mathcal{O}(p^2)$ [1] so that when inserted in Σ_5 and Σ_6 it gives rise to an $\mathcal{O}(p^6)$ contribution that we neglect in the present work. As shown in ref.[1] the in-medium contribution to the pion-loop nucleon self-energy is even further suppressed, being a contribution of $\mathcal{O}(p^7)$ to the pion self-energy. As a result, Σ_5 and Σ_6 are at least $\mathcal{O}(p^6)$.

4. In-medium nucleon-nucleon scattering contributions

We now consider those NLO contributions to the pion self-energy in the nuclear medium that involve the nucleon-nucleon interactions. They are depicted in the diagrams of the last two rows of fig.1, where the ellipsis indicate the iteration of the two-nucleon reducible loops. For the diagrams b) and d) of fig.1 the pion lines can leave or enter the diagrams. It is remarkable that these NLO contributions cancel between each other. On the other hand, since $V_\rho = 2$ in these contributions one needs only the nucleon-nucleon scattering amplitude at $\mathcal{O}(p^0)$ to match with our required precision at NLO. This amplitude is obtained by iterating in an infinite ladder of two nucleon reducible loops, with full in-medium nucleon propagators, the tree level amplitudes obtained from the $\mathcal{O}(p^0)$ Lagrangian with four nucleons [4] and from the one-pion exchange with the lowest order pion-nucleon coupling. The sum of the latter is represented diagrammatically in fig.1 by the exchange of a wiggly line. This procedure would correspond in vacuum to the leading nucleon-nucleon scattering amplitude according to refs.[3, 4].

The diagrams a) and c) of fig.1 involve the Weinberg-Tomozawa vertex (WT) while b) and d) contain the pole terms of pion-nucleon scattering. At leading order in the chiral counting the sum of the latter two has the same structure as the WT term, with the resulting vertex given by

$$-\frac{iq^0}{2f^2}\left(1-g_A^2\frac{\mathbf{q}^2}{q_0^2}\right)\varepsilon_{ijk}\tau^k~. \qquad (4.1)$$

We can then discuss simultaneously all the diagrams in the last two rows of fig.1 employing the latter vertex. Let us denote by $\Sigma_{p(n),NN}$ the proton (neutron) self-energy in the nuclear medium due to the nucleon-nucleon interactions,

$$\Sigma_{i_3,NN} = \sum_{\alpha_2,\sigma_2}\int\frac{d^3k_2}{(2\pi)^3}\theta(\xi_{\alpha_2}-|\mathbf{k}_2|)_A\langle\mathbf{k}_1\sigma_1 i_3,\mathbf{k}_2\sigma_2\alpha_2|T_{NN}|\mathbf{k}_1\sigma_1 i_3,\mathbf{k}_2\sigma_2\alpha_2\rangle_A~. \qquad (4.2)$$

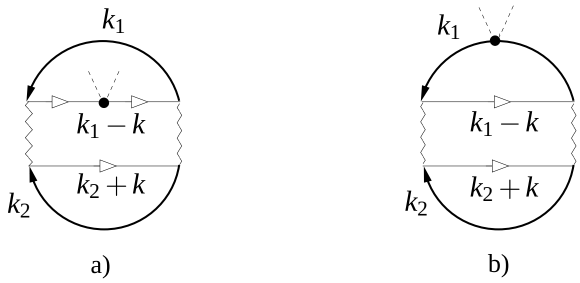

Figure 2: Contribution to the pion self-energy with a two-nucleon reducible loop. The pion scatters inside/outside the loop for the diagram a)/b).

The sum of the diagrams a) and b) of fig.1 can then be written as [1]

$$\begin{aligned}\Sigma_7 = &\frac{iq^0}{2f^2}\left(1-g_A^2\frac{\mathbf{q}^2}{q_0^2}\right)\varepsilon_{ij3}\sum_{\sigma_1,\sigma_2}\int\frac{d^3k_1}{(2\pi)^3}\frac{d^3k_2}{(2\pi)^3}\\ &\times\frac{\partial}{\partial k_1^0}\Big(\theta(\xi_p-|\mathbf{k}_1|)\theta(\xi_p-|\mathbf{k}_2|)_A\langle\mathbf{k}_1\sigma_1 p,\mathbf{k}_2\sigma_2 p|T_{NN}|\mathbf{k}_1\sigma_1 p,\mathbf{k}_2\sigma_2 p\rangle_A\\ &-\theta(\xi_n-|\mathbf{k}_1|)\theta(\xi_n-|\mathbf{k}_2|)_A\langle\mathbf{k}_1\sigma_1 n,\mathbf{k}_2\sigma_2 n|T_{NN}|\mathbf{k}_1\sigma_1 n,\mathbf{k}_2\sigma_2 n\rangle_A\Big)_{k_1^0=E(\mathbf{k}_1)}~. \end{aligned} \tag{4.3}$$

Let us now consider the diagrams c) and d) of fig.1 whose contribution is denoted by Σ_8. These diagrams consist of the pion-nucleon scattering in a two-nucleon reducible loop which is corrected by initial and final state interactions. The iterations are indicated by the ellipsis on both sides of the diagrams. In order to see that these diagrams cancel with eq.(4.3) let us take first the diagram of fig.2a with a twice iterated wiggly line vertex. It is given by

$$\begin{aligned}\Sigma_8^L = &\frac{iq_0}{2f^2}\left(1-g_A^2\frac{\mathbf{q}^2}{q_0^2}\right)\varepsilon_{ij3}\int\frac{d^3k_1}{(2\pi)^3}\frac{d^3k_2}{(2\pi)^3}\\ &\times\frac{\partial}{\partial k_1^0}(\theta(\xi_p-|\mathbf{k}_1|)\theta(\xi_p-|\mathbf{k}_2|)\Pi_p-\theta(\xi_n-|\mathbf{k}_1|)\theta(\xi_n-|\mathbf{k}_2|)\Pi_n)_{k_1^0=E(\mathbf{k}_1)}~, \end{aligned} \tag{4.4}$$

where

$$\Pi_{i_3} = i\int\frac{d^4k}{(2\pi)^4}VPro(i_3,k_1-k)Pro(i_3,k_2+k)V~, \tag{4.5}$$

Here, $Pro(i_3,k)$ is an in-medium nucleon propagator, eq.(3.1), and V is a shortcut notation to indicate a wiggly line nucleon-nucleon vertex. There is also the corresponding crossed contribution with the final nucleons exchanged. The isovector nature of the modified WT vertex of eq.(4.1) implies that only the difference between the proton-proton and neutron-neutron contributions arises. The derivative with respect to k_1^0 arises in eq.(4.4) because the nucleon propagator to which the two pions are attached appears squared [1]. We have in addition the diagram of fig.2b corresponding to the once iterated wiggly line exchange contribution to T_{NN} in eq.(4.3). The latter is given by $-\Pi_{i_3}$ and then, when inserted in eq.(4.3), it cancels with Σ_8^L. Notice as well that the contribution to T_{NN} given by the exchange of only one wiggly line vanishes when inserted in eq.(4.3) because it is independent of k_1^0. This process of mutual cancellation between Σ_7 and Σ_8 can be easily generalized to any number of two-nucleon reducible loops in figs.1a), b) and 1c) and d), respectively. An $n+1$ iterated wiggly line exchange in these figures implies n two-nucleon reducible loops. The two pions can be attached for Σ_8 to any of them, while for Σ_7 the derivative with respect to k_1^0 can also act on any of the loops. The iterative loops are the same for both cases but a relative minus sign results from the loop on which the two pions are attached with respect to the one on which the derivative is acting, as just discussed. Hence,

$$\Sigma_7 + \Sigma_8 = 0 \; . \tag{4.6}$$

The basic simple reason for such cancellation is that while for Σ_7 the presence of a nucleon propagator squared gives rise to $+i\partial/\partial k_1^0$, for Σ_8 it yields $-i\partial/\partial k_1^0$.

5. Conclusions and outlook

We have reviewed on the development in ref.[1] of a power counting in the nuclear medium that combines both short-range and pion-mediated inter-nucleon interactions. The power counting requires typically the resummation of infinite strings of two–nucleon reducible diagrams with the leading $\mathscr{O}(p^0)$ two-nucleon CHPT amplitudes. As a result, the power counting accounts for non-perturbative effects to be resummed. The pion self-energy in asymmetric nuclear matter has been calculated up-to-and-including $\mathscr{O}(p^5)$. As a novelty, it is shown that the leading corrections to the linear density approximation vanish. In particular, it is derived that the leading corrections from nucleon-nucleon scattering mutually cancel.

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