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Recent progress on a precise dispersive data analysis of the $f_0(600)$ pole

J.R. Peláez[∗†]
Dpto. Física Teórica II, Universidad Complutense, Madrid, Spain
E-mail: `jrpelaez@fis.ucm.es`

R. García-Martín
Dpto. Física Teórica II, Universidad Complutense, Madrid, Spain and
Groupe de Physique Théorique, Institut de Physique Nucléaire (IN2P3-CNRS), Orsay, France.
E-mail: `garcia@ipno.in2p3.fr`

R. Kamiński
Institute of Nuclear Physics, Polish Academy of Science, Cracow, Poland
E-mail: `Robert.Kaminski@ifj.edu.pl`

We report on our recent progress describing experimental $\pi\pi$ data, including the newest precise data on kaon decays, together with forward dispersion relations (FDR) and Roy's equations in order to determine the $f_0(600)$ – or sigma – resonance pole position precisely. In particular, we present a new dispersive method and advance some preliminary results to obtain a precise description of the sigma pole from data. In particular, we propose, in addition to standard Roy equations, the use of a set of Roy-like equations but with one subtraction only, that, given the same experimental input, show a remarkable improvement in the precision in the 400 to 1100 MeV region that is relevant for the determination of the σ parameters.

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[∗]Speaker.

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1. Introduction

For long, it has been known that the correlated exchanged of two pions in the scalar-isoscalar channel, I=0, J=0, plays a key role in the nucleon-nucleon attractive interaction [1], usually modeled by the exchange of a scalar isoscalar meson, the so called "sigma" resonance. This channel is also of relevance since one expects the lightest glueball (a characteristic feature of the non-abelian nature of QCD) to show up with these quantum numbers. However, its identification can be complicated due to the presence of other states, not just the usual $\bar{q}q$ mesons but also possible exotic states, like tetraquarks, molecules, all of them possibly mixed. Actually, most of the interest in this channel comes from the fact that the identification of the scalar multiplets of mesons is still rather controversial (see the Particle Data Group – PDG from now on– "Note on Scalar Mesons"[2] for a brief account and references). Being the lightest hadronic resonance with the quantum numbers of the vacuum, this sigma state plays in many models a relevant role in the realization of the spontaneous chiral symmetry breaking of QCD. Actually it has many similarities – but also many differences– with the linear sigma model used to implement the Higgs mechanism in the Electroweak Symmetry Breaking Sector of the Standard Model. Finally, the interest of this state for Effective Field Theories, and in particular for Chiral Perturbation Theory, is to understand why, despite being so light and strongly coupled to pions, it plays such a small role, if any at all, in the saturation of the values of the low energy constants of Chiral Perturbation Theory. Moreover, the position of this pole could be setting the applicability limits of the chiral expansion.

The existence and properties of this "sigma" resonance have been very controversial since its proposal. Just for illustration, until 1974 the PDG listed a 0+ isoscalar state as "not well established", but it was removed from 1976 until 1994, coming back in 1996. The reason, of course, is that this state is extremely wide: its width is comparable to its mass, so that for many authors this was not even considered a particle or a resonance since it barely propagates and it is very hard to see experimentally. For this reason its mass and width are usually quoted from its pole position, namely $\sqrt{s_{\rm pole}} \sim M - i\Gamma/2$. Note that the pole position is well defined but its relation with the mass and width is just a notation abuse, since that identification is only correct for narrow Breit-Wigner resonances. The effect of the sigma, without the complications and subtleties of nucleon-nucleon scattering, has been traditionally seen as a broad enhancement in difficult $\pi\pi$ scattering experiments. Generically, these data are extracted from $\pi N \to \pi\pi N$ scattering using different models and are thus affected by large systematic uncertainties. This was one of the main reasons why, in its 1996 return to the PDG, the properties of the sigma, now called $f_0(600)$, were very conservatively estimated as: "Mass: 400 to 1200 MeV" and "Width: 600 to 1000 MeV". After 2000 the sigma pole has been observed in decays of heavier mesons, which have much better defined initial states and very different systematics from $\pi\pi$ scattering. Actually, the PDG, in 2002, declared the $f_0(600)$ a "well established state", although they kept the same conservative range for its mass and width. This huge uncertainty was due to the fact that old $\pi\pi$ scattering data have the huge systematic uncertainties and are often contradictory. Moreover, the choice of data sets varies among different works. To make things worse, a wide variety of extrapolations from the data on the real axis to the complex plane had been performed, and the sigma pole position was thus affected by strong model dependences.

In recent years, some light has been shed in this longstanding problem from the Effective Field

Theory approach combined with Dispersion Theory. The interest of these techniques is that they provide a *model independent approach* to analytically continue an amplitude away from the real axis provided we know its imaginary part for physical values of the energy, which is obtained from data. The Effective Field Theories provide constraints on the subtraction constants (subtractions are needed to make the dispersive integrals converge). For example, dispersion relations for partial waves of definite isospin and angular momentum, but using low energy constraints from ChPT, and some other approximations on the left cut, had already been successfully used for determining the position of the sigma pole

$$440 - i\,245 \text{ MeV} \qquad \text{Dobado, Pelaez (1997) [3]} \tag{1.1}$$

$$(470 \pm 50) - i\,(260 \pm 25) \text{ MeV} \quad \text{Zhou } et\ al. \text{ (2005) [4]} \tag{1.2}$$

showing a remarkably good agreement. Similar results were obtained in 2004 with unitarized coupled channel ChPT in [5]. Although there is no dispersive derivation for this coupled channel formalism, the single channel version, which is the most relevant for the sigma, is obtained from dispersion relations. As usual with this kind of dispersion relations, the biggest problems come from the evaluation or the approximations made on the left cut and the associated crossing symmetry. However, there exists a twice-subtracted dispersive representation that incorporates crossing exactly, written by Roy [6], and widely used in the 70's (see [7] and references therein). In recent years, these Roy equations have been used either to obtain predictions for low energy $\pi\pi$ scattering, sometimes using Chiral Perturbation Theory (ChPT) [8, 9], or to test ChPT [10], as well as to solve old data ambiguities [11]. Most recently, it has been shown that the sigma pole region lies within the applicability range of Roy Eqs. [12] which, combined with the ChPT *predictions* for the scalar scattering lengths, yields:

$$\sqrt{s_\sigma} = 441^{+16}_{-8} - i\,272^{+9}_{-19.5} \text{ MeV} \tag{1.3}$$

Note that they use no data below 800 MeV on S and P waves.

We want to remark that we are concentrated here on dispersive analysis of the sigma pole, but similar values have also been obtained with other models in the last years, like for instance in [13] and [5] (see also [14] for a general review and references).

Over the last few years, our group [15] has been analyzing the existing data using dispersive techniques with the aim of obtaining a precise description of $\pi\pi$ scattering. We have used a combination of a complete set of Forward Dispersion Relations, sum rules, and most recently also Roy Eqs. to constrain the fits to the data. Contrary to other works, we do not solve Roy equations, but we just include them as *constraints* in the fits, to be satisfied within experimental uncertainties. In addition, note that we do not include ChPT predictions on purpose, so that, on the one hand, we can test its predictions and on the other hand we can use our results to determine some of the ChPT parameters with precision. Furthermore, since *ours is a data analysis* our works include all available experimental data, in particular those from K_{l4} decays, since the results from the E865 collaboration at Brookhaven [16], and especially the recently published data from NA48/2 [17] provide us with very precise data on pion-pion scattering at very low energies.

2. Work in progress: once subtracted Roy-like Equations

As explained by R. García-Martín in his talk in this workshop, we have recently derived a new set of Roy-like equations but with just one subtraction – that we call GKPY equations. Actually, one subtraction alone still ensures convergence due to the u-s cut cancellations of the Pomeron contribution. A brief description of these equations can be found in the talk of R. García-Martín and in R. Kamiński's talk in the Meson08 Conference [18]. The relevant remark is that, given the same experimental input in the integrals, they yield much smaller uncertainties in the 400-1100 MeV region than standard Roy Eqs., which nevertheless are much more accurate below 400 MeV. Thus, the GKPY equations are very well suited for a precise analytic continuation of the $\pi\pi$ amplitude to the complex plane and the determination of the sigma pole from data.

Very briefly, our approach can be summarized as follows:

We use the simple data parametrizations already detailed in Ref. [15], except that we now impose a continuous matching of the derivative between the low-energy and intermediate energy S0 wave parametrizations (see R. García-Martín talk in this workshop). It is enough to say here that two different sets of parameters are considered:

- *Unconstrained Fits to Data* (UFD), in which each partial wave is fitted independently. This set satisfies both FDR and Roy's equations within the experimental errors in all waves except: the Roy equation for the S2 wave, for which the deviation is about 1.3σ, and the antisymmetric FDR above 930 MeV by a couple of standard deviations. In addition, the GKPY equations for the P wave and the S0 wave between 1 and 1.1 GeV are only satisfied within roughly 1.7 standard deviations.

- *Constrained Fits to Data* (CFD), We have obtained a new CFD set by constraining the fits to satisfy FDR, Roy and, improving what we did in KPY08, we now also impose GKPY equations. Note that, in this way, all waves are correlated. This new CFD set provides a remarkably precise and reliable description of the experimental data, and at the same time satisfies all equations remarkably well, with the only exception of the GKPY S0 wave between 1 and 1.1 GeV, which is satisfied within 2 sigmas. Since we impose Roy and GKPY equations as constraints up to 1.1 GeV, it was expected that close to that energy their behavior could be somewhat worse. A similar effect, but less marked, occurs with the fulfillment FDR equations up to where we use them as constraints, namely 1.42 GeV.

These two sets, and particularly the CFD, provide a very reliable and accurate parametrization of the partial waves up to 1 GeV, and a rather good one above, up to 1.42 GeV. These parametrizations provide the imaginary part of the amplitude used as input in the dispersive integrals of Roy or GKPY equations that we will use to obtain the sigma pole position.

In particular, an elastic resonance has an associated pole on the second Riemann sheet of the complex plane S-matrix, which, as it is well known, corresponds by unitarity to a zero on the first sheet. As usual then, we just look for zeroes of the S-matrix on the physical sheet, $S_0^0(s) = 1+2i\sigma(s)t_0^0(s)$, using the analytic extension provided by Roy's or GKPY equations. Their domain of validity has been shown to cover the region of the complex plane where the sigma lies [12].

3. Preliminary dispersive data analysis for the sigma pole

Despite Roy and GKPY equations are satisfied only approximately by the UFD set, we have also used it as input for Roy's equations, which yields $\sqrt{s_\sigma} = (426\pm25) - i(241\pm17)$ MeV. This can be compared with the pole $\sqrt{s_\sigma} = (484\pm17) - i(255\pm10)$ MeV directly obtained from the UFD parametrization [19], that fitted elastic scattering data only, without using Roy equations for the analytic continuation.

Of course, the pole position will be much more reliable if the input satisfies the dispersive representation better, as it is the case for the new CFD set, for which we find:

$$\sqrt{s_\sigma} = (455\pm29) - i(254\pm14)\ \text{MeV}, \quad \text{(preliminary CFD set with Roy equations.)} \tag{3.1}$$

which still has big uncertainties due to the strong dependence of Roy's equations on the scattering lengths, in particular of the a_0^2, which is known experimentally with less precision. Let us remark that these values are preliminary and will be subject to further improvement. As commented above, the most accurate results for the sigma pole are obtained when using the GKPY equations with the new CFD set:

$$\sqrt{s_\sigma} = (465\pm11) - i(251\pm12)\ \text{MeV}, \quad \text{(preliminary CFD set with GKPY equations.)} \tag{3.2}$$

which we consider our best result, although keeping in mind that it is just *preliminary*, since we are still improving our fits and parametrizations, and particularly the estimation of uncertainties. Note that both the mass and width resulting from our dispersive data analysis lie only slightly above one standard deviation of the ChPT+Roy equations prediction in [12], i.e., Eq.(1.3) above. A fairly good agreement.

In Fig.1 we show our results versus the PDG compilation of sigma poles. We have highlighted the other dispersive results commented above in order to show that all dispersive results are in remarkable good agreement, taking into account the large systematic uncertainties present in the experimental data. Actually, in that plot it can be easily seen that *even being extremely conservative*, all dispersive results from $\pi\pi$ scattering (and also the determinations from production processes) are comfortably included in a region which, at least for the mass is almost an order of magnitude smaller than the present estimate of the PDG and a factor of two smaller for the width. We consider that such an estimate should be reconsidered in view of the good agreement of the model independent dispersive techniques.

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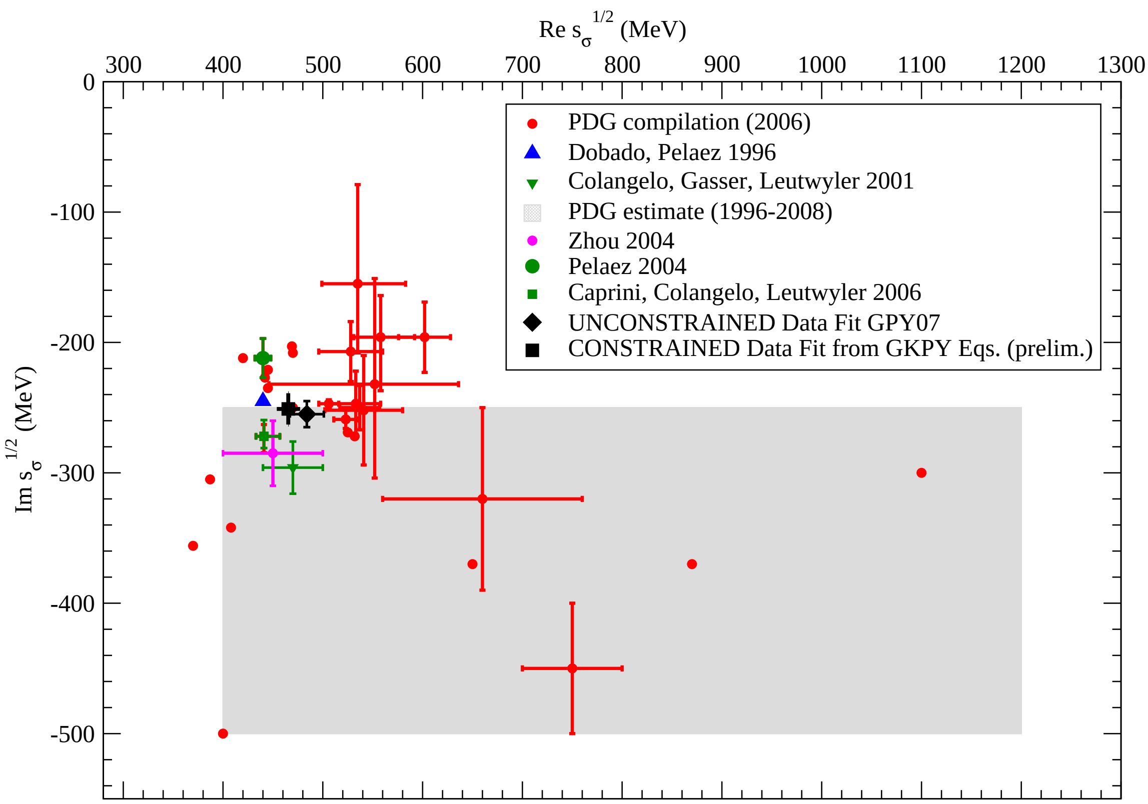

Figure 1: The sigma poles listed in the PDG(from the 2006 edition, the 2008 edition is almost identical when including the rest of the points in this plot). We have highlighted those poles obtained from dispersive approaches using some Chiral Perturbation Theory input in different forms, together with the preliminary results of the model independent dispersive data analysis reported in this talk. Note the good agreement of the dispersive results, all of them concentrated in a small region of the complex plane, versus the present estimate in the PDG (light grey rectangle).

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