

Chiral corrections to transverse vector meson couplings

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We study chiral corrections to the ratio between the longitudinal and transverse decay couplings f_V^\perp/f_V for the lightest (1^{--}) vector mesons. This ratio is of relevance in the determination of CKM matrix elements from exclusive B decays and has recently attracted considerable interest inside the lattice community. With the present accuracy, knowledge of the chiral corrections to f_V^\perp/f_V is an essential ingredient in order to extrapolate the results from quark masses in lattice simulations to the physical masses. We compute the leading order chiral corrections using Heavy Meson Effective Theory. We find that kaon logarithms are absent, while pion logarithms contribute to f_ρ^\perp/f_ρ , but not to the K^* and ϕ ratios. NLO chiral corrections are $\mathcal{O}(m_q^{3/2})$ and purely analytic. As an illustration, we apply our results to recent (unquenched) lattice data.

International Workshop on Effective Field Theories: from the pion to the upsilon

February 2-6 2009

Valencia, Spain

*Speaker.

†Work supported by the EU under contract MTRN-CT-2006-035482 FLAVIANet.

1. Introduction

Due to their quantum numbers, (1^{--}) vector mesons can be interpolated both by the vector $(\bar{q}\gamma_\mu q)$ and tensor $(\bar{q}\sigma_{\mu\nu}q)$ currents. The resulting couplings are denoted by f_V and f_V^\perp respectively, and are important quantities in the determination of CKM matrix elements from Light-Cone Sum Rules (LCSR) applied to semileptonic and radiative B meson decays.

However, while the longitudinal coupling f_V can be determined experimentally, for instance from hadronic τ decays or e^+e^- annihilation, the transverse coupling is experimentally inaccessible and has to be computed using non-perturbative techniques, predominantly LCSR (see [1] for a review of the latest results) and lattice QCD [2, 3, 4, 5, 6].¹

As it was noted in Ref. [2], the extraction of $|V_{ub}|$ from the semileptonic B meson decay $B \rightarrow \rho \ell \nu$ is particularly sensitive to f_ρ^\perp , which in particular means that an effort in accuracy is needed. In this respect, the main advantage of lattice QCD is that it is the ratio f_ρ^\perp/f_ρ which enters the sum rule. This means that in principle one can get a very clean determination of this quantity, since systematic errors cancel in the ratio.

However, an essential ingredient in all lattice analyses is the extrapolation of the results from the masses used in the simulations to the physical masses. Presently, a χ PT-based formula to do the extrapolation is lacking and the different lattice groups use *ad hoc* extrapolation formulae. For instance, in Ref. [2, 6] the chiral extrapolation was made with linear and quadratic fits in the quark masses. With the advent of the first unquenched lattice data on f_V^\perp/f_V [6], we think that a better, theoretically motivated formula should be employed instead.

2. Theoretical framework

In order to study the influence of the (π, K, η) octet on the ratio between vector meson decay couplings, we need an effective field theory that couples the (1^{--}) states with pseudo-Goldstone bosons in a chirally-symmetric way. However, a generic problem one encounters is that, since vector meson masses are not protected by chiral symmetry, derivatives on vector fields spoil the chiral power counting of the theory. A possible solution is to factor out the heavy component of the momentum and work with fields which only depend on the residual (soft) momentum. This approach, inspired on HQET and HB χ PT, was first introduced in Ref. [10] and is usually referred to as Heavy Meson Effective Theory (HMET).

In Ref. [11], HMET was generalized to include generic external vector currents and used to compute the chiral corrections to the longitudinal vector meson decay couplings. In this work, we will have to extend the theory to account for external tensor currents.

The chiral properties of tensor external sources were worked out in Ref. [12], and we refer there for further details. By adding the source term $\bar{q}\sigma_{\mu\nu}t^{\mu\nu}q$ to the QCD Lagrangian, the effective field theory acquires the new fields $t_{LR}^{\mu\nu}$ and $t_{RL}^{\mu\nu}$, where the subscripts indicate their transformation properties under $SU(n_f)_L \times SU(n_f)_R$:

$$\{t_{LR}^{\mu\nu}, t_{RL}^{\mu\nu}\} \longmapsto \{V_L t_{LR}^{\mu\nu} V_R^\dagger, V_R t_{RL}^{\mu\nu} V_L^\dagger\}, \quad (2.1)$$

¹See [7, 8, 9] for different determinations.

with $V_{L,R} \in SU(n_f)_{L,R}$. Its relation to the QCD tensor source is

$$t^{\mu\nu} = P_+^{\mu\nu\lambda\rho} t_{\lambda\rho}^{LR} + P_-^{\mu\nu\lambda\rho} t_{\lambda\rho}^{RL}, \quad (2.2)$$

where

$$P_{\pm}^{\mu\nu\lambda\rho} = \frac{1}{4}(g^{\mu\lambda}g^{\nu\rho} - g^{\mu\rho}g^{\nu\lambda} \pm i\epsilon^{\mu\nu\lambda\rho}) \quad (2.3)$$

are chiral projectors [12].

The effective field theory is then build out of the Goldstone fields, collected as

$$u = \exp\left(i\frac{\Pi}{\sqrt{2}F_0}\right); \quad \Pi = \begin{bmatrix} \frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} & \pi^+ & K^+ \\ \pi^- & \frac{-\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} & K^0 \\ K^- & \bar{K}^0 & -\frac{2\eta}{\sqrt{6}} \end{bmatrix}, \quad (2.4)$$

together with the external fields and the vector meson states. It is convenient to work in the following basis

$$\begin{aligned} u_\mu &= iu^\dagger D_\mu U u^\dagger = i[u^\dagger(\partial_\mu - ir_\mu)u - u(\partial_\mu - il_\mu)u^\dagger], \\ h_{\mu\nu} &= \nabla_\mu u_\nu + \nabla_\nu u_\mu, \\ \chi_\pm &= u^\dagger \chi u^\dagger \pm u \chi^\dagger u, \\ \hat{Q}_{\pm\nu} &= u \hat{l}_\nu u^\dagger \pm u^\dagger \hat{r}_\nu u, \\ \hat{T}_{\pm\mu\nu} &= u^\dagger \hat{t}_{\mu\nu}^{LR} u^\dagger \pm u \hat{t}_{\mu\nu}^{RL} u, \end{aligned} \quad (2.5)$$

such that all elements transform in the adjoint representation of the unbroken $SU(n_f)_{L+R}$. As for the vector meson fields, in this work we will assume ideal mixing between octet and singlet states, such that the ϕ field is a pure $s\bar{s}$ state, while ρ^0 and ω are orthogonal combinations of u and d quarks. Within this approximation, the vector mesons can be collected in the $U(3)$ matrix

$$S_\mu = \begin{bmatrix} \frac{\omega+\rho^0}{\sqrt{2}} & \rho^+ & K^{*+} \\ \rho^- & \frac{\omega-\rho^0}{\sqrt{2}} & K^{*0} \\ K^{*-} & \bar{K}^{*0} & \phi \end{bmatrix}_\mu. \quad (2.6)$$

As mentioned above, in the framework of HMET it is convenient to factor out the heavy components of the momentum and work instead with labelled fields $S_\mu^{(v)}$, which only depend on the soft residual momentum:

$$S_\mu = \frac{1}{\sqrt{2m}} \left[e^{-imv \cdot x} S_\mu^{(v)} + e^{+imv \cdot x} S_\mu^{(v)\dagger} \right] + S_\mu^\parallel. \quad (2.7)$$

The parallel component S_μ^\parallel can be integrated out and therefore will play no role in our computation. In the following we will drop the velocity label from the effective fields and write them simply as \hat{S}_μ . Finally, since we want the vector meson fields to couple to external sources, an analogous decomposition has to be performed on them. The last two lines of Eq. (2.5) are already written

	\mathcal{P}	\mathcal{C}	\mathcal{T}	h.c.
S_μ	S^μ	$-S_\mu^T$	S^μ	S_μ^\dagger
v_μ	v^μ	v_μ	v^μ	v_μ
∇_μ	∇^μ	∇_μ	$-\nabla^\mu$	∇_μ
u	u^\dagger	u^T	u	u^\dagger
u_μ	$-u^\mu$	u_μ^T	u^μ	u_μ
\hat{l}_μ	\hat{r}^μ	$-\hat{r}_\mu^T$	\hat{l}^μ	\hat{l}_μ^\dagger
\hat{r}_μ	\hat{l}^μ	$-\hat{l}_\mu^T$	\hat{r}^μ	\hat{r}_μ^\dagger
χ_\pm	$\pm\chi_\pm$	χ_\pm^T	$\pm\chi_\pm$	$\pm\chi_\pm^\dagger$
$\hat{Q}_{\pm\nu}$	$\pm\hat{Q}_{\pm\nu}^v$	$\mp\hat{Q}_{\pm\nu}^T$	$\hat{Q}_{\pm\nu}^{v\dagger}$	$\hat{Q}_{\pm\nu}^\dagger$
$\hat{T}_{\pm\mu\nu}$	$\pm\hat{T}_{\pm\nu}^{\mu v}$	$-\hat{T}_{\pm\mu\nu}^T$	$-\hat{T}_{\pm\nu}^{\mu v\dagger}$	$\pm\hat{T}_{\pm\mu\nu}^\dagger$

Table 1: Transformation properties of the various fields entering HMET under discrete symmetries.

down in the heavy meson limit.² Finally, we will assume that both \hat{Q}_μ^\pm and $\hat{T}_{\mu\nu}^\pm$ are $\mathcal{O}(1)$ in the chiral power-counting. This choice is completely arbitrary and it only entails that the leading operators in the effective Lagrangian will be $\mathcal{O}(p^0)$.

One can then proceed to build the most general Lagrangian compatible with chiral symmetry and invariant under \mathcal{P} , \mathcal{C} and \mathcal{T} .³ Using Table 1, one can show that the relevant operators at $\mathcal{O}(p^0)$, $\mathcal{O}(p)$ and $\mathcal{O}(p^2)$ read

$$\mathcal{L}^{(0)} = -i\hat{S}_\mu^\dagger(v \cdot \partial)\hat{S}^\mu + \lambda_1\langle\hat{S}_\mu\hat{Q}_+^{\mu\dagger}\rangle + i\lambda_2\langle\hat{S}_\mu\hat{T}_{+\mu\nu}^\dagger\rangle v^\nu \quad (2.8)$$

$$+ \lambda_3\langle\hat{S}_\mu\rangle\langle\hat{Q}_+^{\mu\dagger}\rangle + i\lambda_4\langle\hat{S}_\mu\rangle\langle\hat{T}_{+\mu\nu}^\dagger\rangle v^\nu + \text{h.c.} ,$$

$$\mathcal{L}^{(1)} = \varepsilon^{\mu\nu\rho\lambda} \left[i\mu_1\langle u_\mu \{ \hat{S}_\rho, \hat{S}_\lambda^\dagger \} \rangle v_\nu + i\mu_2\langle u_\lambda \{ \hat{S}_\mu, \hat{Q}_{+\rho}^\dagger \} \rangle v_\nu + \mu_3\langle u_\mu \{ \hat{S}_\rho, \hat{T}_{+\nu\lambda}^\dagger \} \rangle \quad (2.9)$$

$$+ i\mu_4 \left\{ \langle u_\mu \hat{S}_\rho \rangle \langle \hat{S}_\lambda^\dagger \rangle + \langle u_\mu \hat{S}_\lambda^\dagger \rangle \langle \hat{S}_\rho \rangle \right\} v_\nu + i\mu_5\langle u_\lambda \hat{S}_\mu \rangle \langle \hat{Q}_{+\rho}^\dagger \rangle v_\nu + i\mu_6\langle u_\lambda \hat{Q}_{+\rho}^\dagger \rangle \langle \hat{S}_\mu \rangle v_\nu$$

$$+ \mu_7\langle u_\mu \hat{S}_\rho \rangle \langle \hat{T}_{+\nu\lambda}^\dagger \rangle + \mu_8\langle u_\mu \hat{T}_{+\nu\lambda}^\dagger \rangle \langle \hat{S}_\rho \rangle \Big] + \text{h.c.} ,$$

$$\mathcal{L}^{(2)} = \lambda_5\langle \{ \hat{S}^\mu, \chi_+ \} \hat{Q}_{+\mu}^\dagger \rangle + i\lambda_6\langle \{ \hat{S}^\mu, \chi_+ \} \hat{T}_{+\mu\nu}^\dagger \rangle v^\nu \quad (2.10)$$

$$+ \lambda_7\langle \hat{S}^\mu \chi_+ \rangle \langle \hat{Q}_{+\mu}^\dagger \rangle + i\lambda_8\langle \hat{S}^\mu \chi_+ \rangle \langle \hat{T}_{+\mu\nu}^\dagger \rangle v^\nu$$

$$+ \lambda_9\langle \hat{S}^\mu \rangle \langle \chi_+ \hat{Q}_{+\mu}^\dagger \rangle + i\lambda_{10}\langle \hat{S}^\mu \rangle \langle \chi_+ \hat{T}_{+\mu\nu}^\dagger \rangle v^\nu$$

$$+ \lambda_{11}\langle \chi_+ \rangle \langle \hat{S}^\mu \hat{Q}_{+\mu}^\dagger \rangle + i\lambda_{12}\langle \chi_+ \rangle \langle \hat{S}^\mu \hat{T}_{+\mu\nu}^\dagger \rangle v^\nu$$

$$+ \lambda_{13}\langle \hat{S}^\mu \rangle \langle \chi_+ \rangle \langle \hat{Q}_{+\mu}^\dagger \rangle + i\lambda_{14}\langle \hat{S}^\mu \rangle \langle \chi_+ \rangle \langle \hat{T}_{+\mu\nu}^\dagger \rangle v^\nu + \text{h.c.}$$

²In this work the χ_+ field will only play the role of a mass insertion operator. Therefore, for our purposes, $\chi_+ = 2\chi = 4B_0\text{diag}(m_u, m_d, m_s)$.

³Since the field decomposition of Eq. (2.7) manifestly breaks Lorentz invariance, the *CPT* theorem does not apply and one has to ensure that each discrete symmetry is separately conserved.

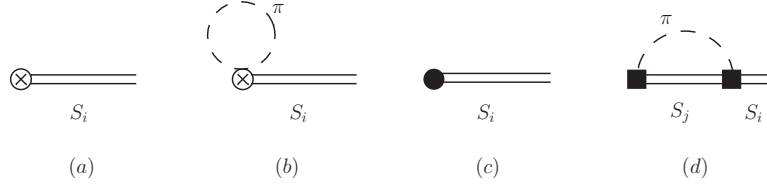


Figure 1: Diagrams contributing to the ratio f_V^\perp/f_V up to $m_q^{3/2}$ in HMET. From left to right, LO [(a)], NLO [(b) and (c)] and NNLO [(d)] contributions. Circle-cross vertices depict operators of $\mathcal{O}(1)$, while box vertices and dotted vertices represent $\mathcal{O}(p)$ and $\mathcal{O}(p^2)$ operators, respectively.

The quantities we want to evaluate are defined as follows:

$$\begin{aligned} \langle 0 | \bar{u} \gamma_\mu d(0) | \rho^+(p, \lambda) \rangle &\doteq f_\rho m_\rho \varepsilon_\mu^{(\lambda)}, \\ \langle 0 | \bar{u} \sigma_{\mu\nu} d(0) | \rho^+(p, \lambda) \rangle &\doteq i f_\rho^\perp m_\rho (\varepsilon_\nu^{(\lambda)} v_\rho - \varepsilon_\rho^{(\lambda)} v_\nu), \end{aligned} \quad (2.11)$$

and similar expressions for the rest of the (1^{--}) nonet, where it should be noted that we define the couplings f_V and f_V^\perp to be the ones associated to isospin currents.

In order to identify the number of diagrams that contribute at each given order in the chiral expansion, it is useful to use the power counting scheme of HMET [13]:

$$d = 2 + 2N_L + N_R + \sum_n (n-2)N_n \quad (2.12)$$

where N_L is the number of loops, N_R the number of internal resonance lines and N_n the number of operators with chiral counting n . Using the previous formula it is easy to show that the only diagrams contributing up to $\mathcal{O}(m_q^{3/2})$ are the ones depicted in Figure 1.⁴

3. Results and conclusions

We will present our results for two and three dynamical flavours. For $SU(2)$, there are a number of simplifications that can be made. In the first place, some of the operators in Eq. (2.8) turn out to be linearly dependent. Using the so-called Cayley-Hamilton relations among traces, it is easy to show that the following subsets of couplings are related: $\{\mu_1, \mu_4\}$, $\{\mu_3, \mu_7, \mu_8\}$, $\{\mu_2, \mu_5, \mu_6\}$, $\{\lambda_5, \lambda_7, \lambda_9, \lambda_{11}, \lambda_{13}\}$ and $\{\lambda_6, \lambda_8, \lambda_{10}, \lambda_{12}, \lambda_{14}\}$ and therefore one coupling in each subset can be eliminated. Additionally, if one is working in the isospin limit the mass insertion matrix χ_+ is proportional to the identity, which results in further simplifications: only one representative of the subsets $\{\lambda_7, \lambda_9, \lambda_{13}\}$ and $\{\lambda_8, \lambda_{10}, \lambda_{14}\}$ is independent. In the following we will take the isospin limit and set $m_u = m_d = m$.

Taking all these considerations into account, our final result reads

$$\begin{aligned} \frac{f_\rho^\perp}{f_\rho} &= \frac{f_V^\perp}{f_V} \left[1 + \frac{m}{32\pi^2 F_0^2} \log \left(\frac{m^2}{\hat{m}_\rho^2} \right) \right], \\ \frac{f_\omega^\perp}{f_\omega} &= \frac{f_V^\perp}{f_V} \left[1 + 2\Lambda - \frac{3m}{32\pi^2 F_0^2} \log \left(\frac{m^2}{\hat{m}_\omega^2} \right) \right], \end{aligned} \quad (3.1)$$

⁴Wave function renormalization and mass corrections are not included since they cancel identically in the ratio.

Table 2: Results for f_V^T/f_V from the various fits (data taken from Ref. [6]).

	Linear	Quadratic	Chiral
f_ρ^T/f_ρ	0.619(15)	0.600(38)	0.585(64)
χ^2/dof	0.17	0.09	0.06
f_K^T/f_K	0.6498(62)	0.644(17)	0.638(31)
χ^2/dof	0.11	0.09	0.07
f_ϕ^T/f_ϕ	0.6838(33)	0.6815(95)	0.680(17)
χ^2/dof	0.10	0.12	0.15

where $f_V^\perp/f_V = \lambda_2/\lambda_1$ and $\hat{m}_\rho, \hat{m}_\omega$ are combinations of couplings from $\mathcal{L}^{(2)}$. Λ is a Zweig-suppressed term coming from the second line of $\mathcal{L}^{(0)}$ and is expected to be extremely small.

For the $SU(3)$ case the results are

$$\begin{aligned}
\frac{f_\rho^\perp}{f_\rho} &= \frac{f_V^\perp}{f_V} \left[1 + 2m\bar{\Lambda} + m_s\tilde{\Lambda} + \frac{m_\pi^2}{32\pi^2 F_0^2} \log\left(\frac{m_\pi^2}{\mu^2}\right) - \frac{m_\eta^2}{96\pi^2 F_0^2} \log\left(\frac{m_\eta^2}{\mu^2}\right) \right], \\
\frac{f_K^\perp}{f_K} &= \frac{f_V^\perp}{f_V} \left[1 + m(\bar{\Lambda} + \tilde{\Lambda}) + m_s\bar{\Lambda} + \frac{m_\eta^2}{48\pi^2 F_0^2} \log\left(\frac{m_\eta^2}{\mu^2}\right) \right], \\
\frac{f_\phi^\perp}{f_\phi} &= \frac{f_V^\perp}{f_V} \left[1 + m\Lambda_1 + m_s\Lambda_2 + \Lambda - \frac{m_\eta^2}{24\pi^2 F_0^2} \log\left(\frac{m_\eta^2}{\mu^2}\right) \right].
\end{aligned} \tag{3.2}$$

Two comments are in order at this point: (i) kaon loops are non-vanishing for each decay coupling but cancel in the ratios, whereas pion loops are absent in the K^* and ϕ ratios. Therefore, in $SU(2) \times U(1)_S$ χ PT, which is the framework used by many lattice groups, only f_ρ^\perp/f_ρ is sensitive to pion loops; (ii) in the preceding equations we have omitted the $\mathcal{O}(m_q^{3/2})$ terms. Its actual computation involves a sizeable number of new operators (see Ref. [11] for the full set) but its contribution gives a purely analytic piece. We refer to Ref. [13] for further details.

As a simple application of our results, one can consider the recent unquenched RBC data of Ref. [6] and check if they can be qualitatively described by Eq. (3.2). In particular, we would like to test the absence of light-quark logarithms in the K^* and ϕ channels. A possible strategy is to fit the data points for the different channels with a chiral logarithm and compare the results with the linear and quadratic fits of [6]. In Table 2 we summarize the values for the resulting chiral extrapolations, where the errors quoted are only statistical. As an illustration, the results for the different fits for f_ρ^\perp/f_ρ are shown in Figure 2.

We want to emphasize that the results shown in Table 2 and Figure 2 (and the conclusions thereof) should be taken only as indicative: the data reported in Ref. [6] are obtained at a finite lattice spacing, i.e., they are not in the continuum limit. Moreover, data are still scarce to be statistically significant and the values of the quark masses used in the simulations are still too high

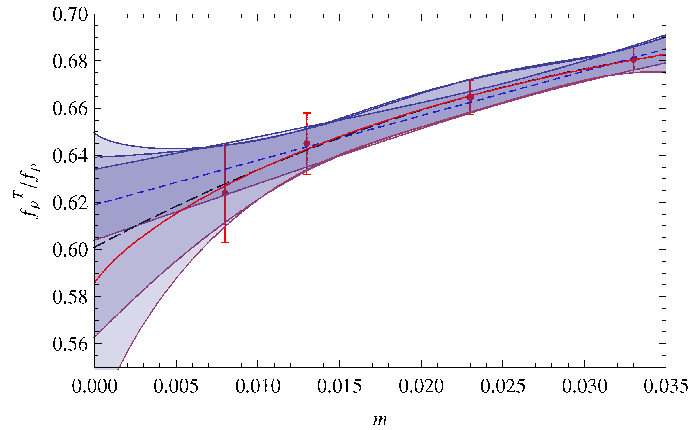


Figure 2: Chiral extrapolations for linear (blue dotted line), quadratic (dashed line) and logarithmic (solid red line) fits. We have absorbed the lattice spacing into m . Data taken from Ref. [6].

to unambiguously show the presence or absence of logarithmic behaviour. Therefore, the use of our chiral extrapolation formulae should be seen only as a plausible exercise at this point. With these caveats in mind, data seem to be in fair agreement with Eq. (3.2): for the ρ meson the best fit is the logarithmic one, while for the ϕ the linear fit gives the best results. For the K^* the situation is less clear and at present no conclusions can be drawn.

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