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A new tool for constrained vertex fitting in ATLAS

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The precise reconstruction of trajectories of charged and neutral particles and their decay vertices is crucial for many physics analyses. Studying the tracking performance on well-known benchmark channels helps to understand the properties of the ATLAS detector during the initial phase of the LHC. In order to exploit the correlations between reconstructed parameters of final state tracks having the same mother particle, a new tool for vertex fitting with the possibility of simultaneous application of kinematic constraints has been developed. Using this tool with a mass constraint on a benchmark channel such as $J/\psi \rightarrow \mu^+\mu^-$ helps to correct shifts in the reconstructed curvature induced by systematic deformations of the detector.

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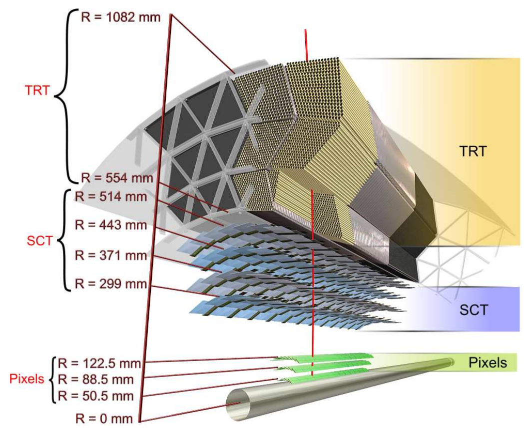

Figure 1: Drawing showing the sensors and structural elements traversed by a track in the central region of the ATLAS Inner Detector.

1. Track reconstruction in the ATLAS Inner Detector

The ATLAS experiment [1] is a particle detector in operation at the LHC at CERN. It consists of several sub-detectors. Closest to the interaction point in ATLAS is the Inner Detector, which is designed to reconstruct the trajectories of charged particles produced in the proton-proton collisions at the LHC. The Inner Detector is located in a superconducting solenoid magnet producing a magnetic field of 2 T. Presented in Figure 1 is a schematic layout of a transverse slice of the ATLAS Inner Detector. Starting from the innermost radii (closest to the interaction point) the detector consists of three different components:

- Pixel detector: provides three measurements per trajectory of a charged particle with average resolutions of 14 μm in $R\phi$ and 115 μm in z.
- Silicon-strip detector (SCT): provides four three-dimensional measurements per trajectory of a charged particle. The average resolutions are 17 μm in Rϕ and 580 μm in z.
- Transition Radiation Tracker (TRT): provides an average of 30 measurements per trajectory of a charged particle with a resolution of about 140 μm in $R\phi$.

The ATLAS Inner Detector provides on average 37 measurements per trajectory of a charged particle, thus allowing for the precise determination of its parameters. This highly precise reconstruction in particular allows for an efficient determination of interaction vertices.

2. Building a modular tool for constrained vertex fitting

Reconstruction of interaction vertices as the common intersection point of a set of particle trajectories allows for further understanding of the underlying physics processes. Several methods for vertex reconstruction are available within the ATLAS software framework Athena [2], [3]. Some of these methods can exploit kinematic constraints during the vertex fit [3], [4]. Described in this paper is a new tool for vertex fitting, which extends this framework by providing a flexible way of adding kinematic constraints to the vertex fitting process.

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2.1 Vertex fitting using Lagrange multipliers in the Cartesian frame

The new vertex fitting tool is based on the χ^2 minimisation with Lagrange multipliers [5]. The calculations are performed in the Cartesian frame. The chosen trajectory parametrisation consists of a three-dimensional position along the trajectory, the particle momentum at this position and the particle energy.

The standard track object in Athena describes the particle trajectory using the perigee parameters [6]. The Cartesian track parameters are obtained by converting the perigee parameters and assigning a mass to the final-state particle.

Each kinematic constraint is expressed in terms of a vector of equations $H(\vec{\alpha},\vec{x}) = 0$, where $\vec{\alpha}$ is a vector of Cartesian parameters of all tracks participating in the fit and $\vec{x}$ is the vertex position. The constraint equations are linearised by using the first order Taylor expansion around a convenient point $(\vec{\alpha}_A;\vec{x}_A)$:

$$\begin{aligned} 0 &= \left[\frac{\partial \mathbf{H}(\vec{\alpha}_A,\vec{x}_A)}{\partial \vec{\alpha}}\right](\vec{\alpha}-\vec{\alpha}_A) + \left[\frac{\partial \mathbf{H}(\vec{\alpha}_A,\vec{x}_A)}{\partial \vec{x}}\right](\vec{x}-\vec{x}_A) + \mathbf{H}(\vec{\alpha}_A,\vec{x}_A) \\ &\equiv \mathbf{D}(\vec{\alpha}-\vec{\alpha}_A) + \mathbf{E}(\vec{x}-\vec{x}_A) + \mathbf{d}, \end{aligned} \tag{2.1}$$

where $\mathbf{d}$ is the vector of values of the constraint equations at the expansion point $\mathbf{H}(\vec{\alpha}_A,\vec{x}_A)$, $\mathbf{D}$ is the matrix of partial derivatives of the constraint equations at the expansion point with respect to the track parameters and $\mathbf{E}$ is the matrix of the partial derivatives of the constraint equations at the expansion point with respect to the vertex coordinates.

The χ^2 function to be minimised can be written as:

$$\chi^2 = (\vec{\alpha}-\vec{\alpha}_0)^T V_{\alpha_0}^{-1}(\vec{\alpha}-\vec{\alpha}_0) + 2\lambda^T(\mathbf{D}(\vec{\alpha}-\vec{\alpha}_A) + \mathbf{E}(\vec{x}-\vec{x}_A) + \mathbf{d}), \tag{2.2}$$

where the first term represents the χ^2 contribution from tracks participating in the fit, the second and the third term represent contributions from the constraints and $\vec{\lambda}$ is a vector of Lagrange multipliers. The length of the vector $\vec{\lambda}$ is equal to the number of constraint equations. The values of track parameters $\vec{\alpha}$ and vertex position $\vec{x}$ that satisfy the given set of constraints can then be found by minimising the χ^2 equation with respect to $\vec{\alpha}$, $\vec{x}$ and $\vec{\lambda}$. The constraints currently implemented in the tool are the mass and the vertex constraints.

The vertex constraint consists of two equations per track, forcing the tracks to pass through a common point. For a solenoidal magnetic field parallel to the z-axis these equations are: [5]:

$$\begin{aligned} H_1 &= p_x\Delta y - p_y\Delta x - \frac{a}{2}(\Delta x^2 + \Delta y^2) = 0, \\ H_2 &= \Delta z - \frac{p_z}{a}\sin^{-1}\left(a(p_x\Delta x + p_y\Delta y)/p_T^2\right) = 0, \end{aligned} \tag{2.3}$$

where Δx, Δy, Δz are the differences between the position of the track and the vertex position, p_x, p_y, p_z are the momentum components of the track, a is a bending factor which, for momentum units in MeV and coordinate units in mm, is given by $a = -0.2997 \cdot Bq$, where q is the charge of the particle and B the magnetic field strength in Tesla.

The mass constraint consist of one equation, which forces a set of trajectories to have a common invariant mass m_{constr}:

$$\mathbf{H} = (\sum_{i=0}^{ntrack} E_i)^2 - (\sum_{i=0}^{ntrack} p_{xi})^2 - (\sum_{i=0}^{ntrack} p_{yi})^2 - (\sum_{i=0}^{ntrack} p_{zi})^2 - (m_{constr})^2 = 0. \tag{2.4}$$

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In the current approach the linearised constraints are implemented in a modular way separately from the fit. An extra constraint can thus be easily added within the same framework.

2.2 Modular tool for vertex fitting with kinematic constraints

The tracking and vertexing framework of ATLAS is organised as set of independent tools and algorithms [7]. The use of abstract interfaces allows the implementation of the new algorithm for constrained vertex fitting as an additional vertexing tool. The input to the constrained vertex fitter is a set of tracks with associated particle masses and a set of kinematic constraints. The output is the reconstructed vertex. This object contains the vertex position and its covariance matrix as well as the updated track parameters defined at the vertex position.

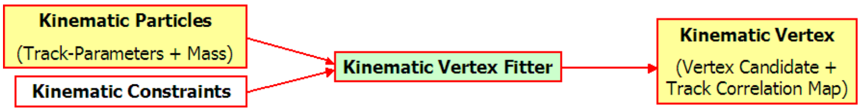

Figure 2: Schematic illustration of the input/output used by the Kinematic Vertex Fitter tool

The calculations of the derivatives and values of the constraint equations are contained in the implementations of the different kinematic constraints, while the main method only holds the constraint equations for the vertex constraint. When no additional constraints are present, the tool behaves as a conventional χ^2 vertex fitter.

3. Results

3.1 Vertex fit with mass constraint in $J/\psi \to \mu^+\mu^-$ events

The kinematic vertex fit with mass constraint was tested using simulated $J/\psi \to \mu^+\mu^-$ events. The result is compared with the conventional χ^2-based vertex fitting algorithm implemented in Athena. Shown in Figure 3 is the resolution on the inverse transverse momentum of the muon track refitted with the vertex constraint as a function of transverse momentum. It can be noted that the use of a mass constraint improves the resolution of q/p_T of the muon track by 20% over the whole momentum range. It was found that in the particular case of $J/\psi \to \mu^+\mu^-$ events, the resolution observed with the default vertex fitter is essentially the resolution coming from the track fit alone, as the vertex constraint does not significantly affect the reconstructed momentum. It was also checked that in this case the mass constraint does not change the result for the resolution of the fitted vertex position.

3.2 Weak-mode misalignments

Constrained vertex fitting can be used as a tool to help correct shifts in the reconstructed curvature induced by certain weak-mode misalignments. These misalignments represent systematic deformations of the detector, affecting the properties of reconstructed charged particles without increasing the hit residuals (i.e. the difference between the measured and predicted position of the hits). The alignment procedure [8], based on minimisation of the hit residuals, is not sensitive to weak-mode misalignments.

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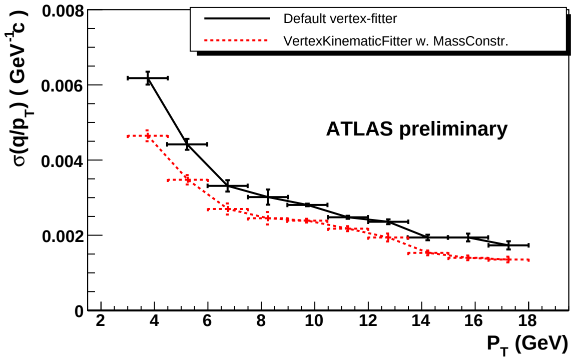

Figure 3: Resolution of the reconstructed value of q/p_T of the muon track parameters at the vertex as a function of the transverse momentum, using simulated $J/\psi \rightarrow \mu^+\mu^-$ events, obtained with the two vertex fitting methods.

An example of a possible weak-mode misalignment in the ATLAS detector is the so-called 'curl' (or 'clocking') misalignment, which rotates the detector elements by a an amount of azimuthal angle ϕ, increasing as a function of transverse radius R. The effect of this misalignment was studied by using simulated events reconstructed with a normal and 'curl' geometry. In this study, the 'curl' geometry is based on reasonable shifts of the detector modules, so that the position of an SCT module can be shifted by a maximum of 200 μm. The effect of the 'curl' geometry used in this study is such that negatively charged tracks are more bent ($1/p_T$ increases) while positively charged tracks are more stiff ($1/p_T$ decreases).

The $J/\psi \rightarrow \mu^+\mu^-$ events can be used as a benchmark channel to search for the possible presence of weak-mode misalignments in the data. Shown in Figure 4 is the reconstructed invariant di-muon mass as function of the transverse momentum difference between the positive and negatively charged muons in simulated $J/\psi \rightarrow \mu^+\mu^-$ events. It can be observed that the reconstructed invariant di-muon mass is no longer always equal to the J/ψ mass (3.097 GeV) when reconstructing simulated events with the 'curl' geometry.

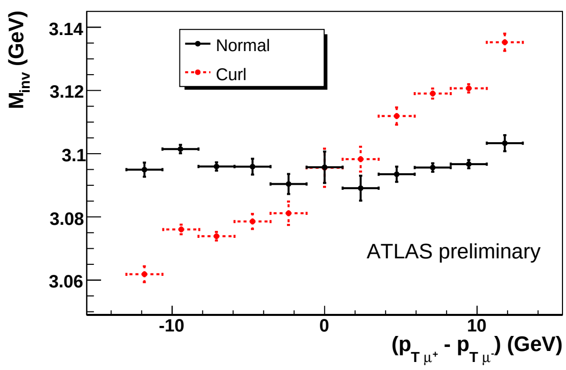

Figure 4: Reconstructed invariant di-muon mass for simulated $J/\psi \rightarrow \mu^+\mu^-$ events as a function of the p_T difference between the positive and negatively charged muon with and without the 'curl' misalignment.

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Shown in Figure 5 is the difference between the reconstructed and true value of q/p_T of the muon tracks, obtained with the conventional χ^2 vertex fit and the mass constrained vertex fit. It can be seen that the kinematic vertex fit with mass constraint can recover some of the q/p_T shift of the muon tracks in $J/\psi \rightarrow \mu^+\mu^-$ events reconstructed with a 'curl' misalignment. The amount of curvature that is recovered with the mass constrained vertex fit depends on the p_T distribution of the muon tracks. The result shown in Figure 5 was obtained by requiring a p_T difference of 9 GeV or more between the positively and the negatively charged muon track. Plotted are the differences between the reconstructed and the true value of q/p_T for muon tracks with $p_T > 9$ GeV. In this particular example, the systematic shift of q/p_T is halved when using the mass constrained vertex fit. The influence of the background processes on this correction requires a further study.

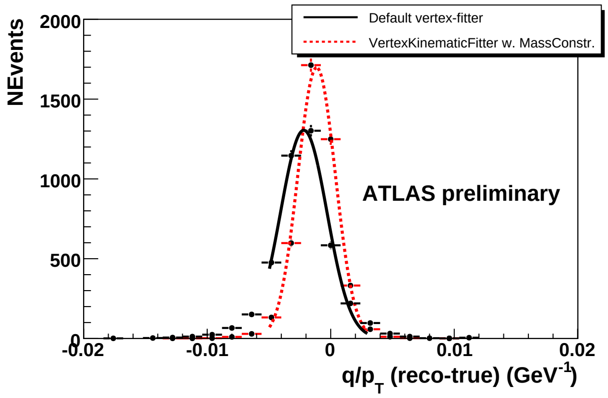

Figure 5: Difference between the reconstructed and true value of q/p_T of the muon tracks in $J/\psi \rightarrow \mu^+\mu^-$ events with a 'curl' misalignment in the detector, obtained with the two vertex fitting methods.

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4. Conclusions

A new tool for kinematic vertex fitting in ATLAS was developed. This tool allows the application of additional kinematic constraints during the vertex fit. These constraints are implemented as independent modules which makes the addition of other constraints simple and flexible. The use of a mass constraint in a benchmark channel $J/\psi \rightarrow \mu^+\mu^-$ improves significantly the experimental resolution on the momentum of the final-state muons. The use of this constraint allows also for the recovery of shifts in the particle momentum, which are present in possible weak-mode misalignments of the ATLAS detector.

Acknowledgments

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References

[1] The ATLAS Collaboration, *The ATLAS Experiment at the CERN Large Hadron Collider*, JINST 3 (2008)

[2] The ATLAS Collaboration, *ATLAS Computing Technical Design Report*, ATLAS TDR-017, CERN-LHCC-2005-022 (2005)

[3] E. Bouhova-Thacker et al., *Vertex Reconstruction in the ATLAS Experiment at the LHC*, in proceedings of *IEEE NSS Dresden*, CERN-ATL-INDET-PROC-2008-003 (2008)

[4] V. Kostyukhin, *VKalVrt - A package for vertex reconstruction in ATLAS*, ATL-PHYS-2003-031 (2003)

[5] P. Avery, *Fitting theory write-ups*, `http://www.phys.ufl.edu/~avery/fitting.html` (1998)

[6] F. Akesson et al., *The ATLAS Tracking Event Data Model*, CERN-ATL-SOFT-PUB-2006-004 (2006)

[7] A. Salzburger et al., *Concepts, Design and Implementation of the ATLAS New Tracking (NEWT)*, CERN-ATL-SOFT-PUB-2007-007 (2007)

[8] T. Golling, *Alignment strategy for the ATLAS tracker*, in proceedings of *the 16th International Workshop on Vertex Detectors*, `PoS(Vertex 2007)030` (2007)

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