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On distinguishing the direct and spontaneous CP violation in 2HDM

Dorota Sokolowska[*†]
University of Warsaw
E-mail: dsok@fuw.edu.pl

Konstantin Kanishev
Sobolev Institute of Mathematics and Novosibirsk State University
E-mail: kanishev@ngs.ru

Maria Krawczyk
University of Warsaw
E-mail: krawczyk@fuw.edu.pl

The most general Two Higgs Doublet Model (2HDM) allows both for the explicit and the spontaneous CP violation in the scalar sector. Here we discuss CP violation in terms of basis-independent quantities and show how using two different sets of known CP-odd weak-basis invariants one can distinguish between CP conservation, explicit CP violation and spontaneous CP violation. The special case of CP violation without CP mixing, in which the neutral Higgs gauge boson interaction respects CP while Higgs self-interaction violates CP symmetry, is also presented.

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[*]Speaker.

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1. Introduction

CP symmetry is one of the most crucial symmetries in particle physics. In the Standard Model CP symmetry is violated through a complex phase in the CKM matrix. However, this violation is not big enough to lead to the observed baryon asymmetry of the Universe [1]. Determining all possible sources of CP violation is a fundamental challenge for high energy physics.

In a CP conserving model both the Lagrangian and the vacuum state conserve CP. There are two possible sources of CP violation - either by parameters of the Lagrangian (direct or explicit violation) or by the vacuum state (spontaneous violation). The 2HDM is one of the simplest extension of the Standard Model that allows both for spontaneous and explicit CP violation [2].

In 2HDM we deal with two doublets of scalar fields with identical quantum numbers, so there is a freedom of choosing the basis in the space of fields [3, 4, 5, 6]. Because of this freedom it is convenient to study CP violation in the 2HDM not in terms of parameters of the potential that depend on the choice of basis but using the CP-odd weak-basis invariant quantities.

We are interested in distinguishing the direct and spontaneous CP violation in 2HDM and for simplicity we focus here on scalar sector only. We will use the two different sets of weak-basis invariants: J-invariants introduced in [7] and I-invariants [5]. They are the analogs of the Jarlskog invariant [8] allowing to distinguish between CP conserving and CP violating models. We show that by combining these invariants one can determine the way CP is violated - explicitly (directly) or spontaneously. We discuss separately a special case of CP violation without CP mixing, where CP violation appears only in the interaction [7].

2. The general 2HDM

Let us consider the most general 2HDM (without the Yukawa interaction) [2, 3, 4, 5]:

$$\mathscr{L} = \mathscr{L}_{gauge} + \left(D_\mu \Phi_1\right)^\dagger \left(D^\mu \Phi_1\right) + \left(D_\mu \Phi_2\right)^\dagger \left(D^\mu \Phi_2\right) - V, \tag{2.1}$$

$$\begin{aligned} V = {} & \frac{\lambda_1}{2}\left(\Phi_1^\dagger \Phi_1\right)^2 + \frac{\lambda_2}{2}\left(\Phi_2^\dagger \Phi_2\right)^2 + \lambda_3 \left(\Phi_1^\dagger \Phi_1\right)\left(\Phi_2^\dagger \Phi_2\right) + \lambda_4 \left(\Phi_1^\dagger \Phi_2\right)\left(\Phi_2^\dagger \Phi_1\right) \\ & + \left[\frac{1}{2}\lambda_5 \left(\Phi_1^\dagger \Phi_2\right)\left(\Phi_1^\dagger \Phi_2\right) + \lambda_6 \left(\Phi_1^\dagger \Phi_1\right)\left(\Phi_1^\dagger \Phi_2\right) + \lambda_7 \left(\Phi_2^\dagger \Phi_2\right)\left(\Phi_1^\dagger \Phi_2\right) + H.c.\right] \\ & - \frac{1}{2} m_{11}^2 \left(\Phi_1^\dagger \Phi_1\right) - \frac{1}{2} m_{22}^2 \left(\Phi_2^\dagger \Phi_2\right) - \frac{1}{2} m_{12}^2 \left(\Phi_1^\dagger \Phi_2\right) - \frac{1}{2} (m_{12}^2)^* \left(\Phi_2^\dagger \Phi_1\right), \end{aligned} \tag{2.2}$$

where $\Phi_{1,2}$ are SU(2) doublet with weak hypercharge Y=+1, $\lambda_{1-4}, m_{11}^2, m_{22}^2 \in \mathbb{R}$ and $\lambda_{5-7}, m_{12}^2 \in \mathbb{C}$.

In the Higgs (Georgi) basis, in which only one doublet acquires the non-zero vacuum expectation value, scalar doublets can be decomposed in the following way:

$$\Phi_1 = \begin{pmatrix} G^+ \\ \frac{v+\eta_1+iG^0}{\sqrt{2}} \end{pmatrix}, \qquad \Phi_2 = \begin{pmatrix} H^+ \\ \frac{\eta_2+iA}{\sqrt{2}} \end{pmatrix}. \tag{2.3}$$

In this basis we use special symbols for parameters of the potential, namely Λ_i, μ_{ij}^2 instead of λ_i, m_{ij}^2.

There are various CP-transformations possible [4, 9], which does not change the kinetic term of (2.1). For our purpose it is enough to apply the simplest CP transformation (see also [4]):

$$\Phi_1(\vec{x},t) \to \Phi_1^\dagger(-\vec{x},t), \qquad \Phi_2(\vec{x},t) \to \Phi_2^\dagger(-\vec{x},t). \tag{2.4}$$

Under this transformation the neutral fields from the decomposition (2.3) transform as follows:

$$\eta_{1,2} \to \eta_{1,2}, \qquad A \to -A, \tag{2.5}$$

so we can identify η_1 and η_2 as the CP-even fields and A as the CP-odd field (see also [10]).

The squared-mass matrix for the neutral fields (η_1, η_2, A) in the Higgs basis is given by:

$$\mathscr{M}^2 = \begin{pmatrix} M_{11} & M_{12} & M_{13} \\ M_{12} & M_{22} & M_{23} \\ M_{13} & M_{23} & M_{33} \end{pmatrix} = \begin{pmatrix} v^2\Lambda_1 & v^2 \mathrm{Re}\,\Lambda_6 & -v^2 \mathrm{Im}\,\Lambda_6 \\ v^2 \mathrm{Re}\,\Lambda_6 & \left[v^2\Lambda_{345} - \mu_{22}^2\right]/2 & -v^2 \mathrm{Im}\,\Lambda_5/2 \\ -v^2 \mathrm{Im}\,\Lambda_6 & -v^2 \mathrm{Im}\,\Lambda_5/2 & \left[v^2\tilde{\Lambda}_{345} - \mu_{22}^2\right]/2 \end{pmatrix}, \tag{2.6}$$

where $\Lambda_{345} = \Lambda_3 + \Lambda_4 + \mathrm{Re}\Lambda_5$, $\tilde{\Lambda}_{234} = \Lambda_3 + \Lambda_4 - \mathrm{Re}\Lambda_5$. There are two distinct cases here:

- *CP-mixing.* This is a case with non-zero off-diagonal element M_{13} or M_{23} leading to a mixing between states of different CP properties. Model allows for the CP violation since physical states h_i, being combinations of η_1, η_2, A, have indefinite CP quantum numbers. This is considered to be a standard way CP is violated in the 2HDM (and in other models).

- *no CP-mixing.* When $M_{13} = M_{23} = 0$ then there is no CP-mixing and physical (mass-)states have defined CP properties. Neutral Higgs bosons are: CP-odd A and CP-even combinations of η_1 and η_2 denoted as h, H.

Note that parameter Λ_7 (in general complex) does not appear in the mass matrix (2.6) in the Higgs basis, and therefore it can be related only to the interaction (ie. couplings).

3. CP-odd weak-basis invariants

The CP-odd weak-basis invariants are quantities that are invariant under the weak-basis transformation, but change their sign under the CP transformation. In the Standard Model such invariant was introduced by C. Jarlskog for the quark sector [8]:

$$J \propto (m_t^2 - m_c^2)(m_t^2 - m_u^2)(m_c^2 - m_u^2)(m_b^2 - m_s^2)(m_b^2 - m_d^2)(m_s^2 - m_d^2)\mathrm{Im}(V_{ud}V_{cs}V_{us}^*V_{cd}^*), \tag{3.1}$$

where m_i is a mass of i-quark, V_{ij} are elements of the CKM matrix. In the Standard Model J is a only CP-odd invariant and if $J = 0$ then CP is conserved. Note, that $J = 0$ if two masses of quarks are equal. We know that in the SM $J \neq 0$ and the CP is violated through the complex phase in the CKM matrix. In the 2HDM the situation is more complicated, however also here there is a close analog of the Jarlskog invariant, this time for scalars [7]:

$$J_1 \propto (m_1^2 - m_2^2)(m_1^2 - m_3^2)(m_2^2 - m_3^2)T_{11}T_{21}T_{31}, \tag{3.2}$$

where $m_i, i = 1,2,3$ are masses of neutral scalars and T_{ij} – elements of rotation matrix between the mass-states and (η_1, η_2, A) fields. Also here if two masses of neutral Higgs bosons are equal $J_1 = 0$. It is important to point out that in the 2HDM $J_1 = 0$ does not imply CP conservation in the scalar sector, in contrast to the Standard Model case. Vanishing of the J_1 is now a necessary but not sufficient condition for CP conservation [7].

3.1 *J*-invariants

The J_1 invariant (3.2) is the only CP-odd invariant in 2HDM which can be constructed from the squared-mass matrix [7]. It can be written also in two other forms by using elements of squared-mass matrix (2.6) before diagonalization and by parameters of the potential V (2.2), respectively:

$$J_1 = M_{12}M_{13}(M_{22}-M_{33}) + M_{23}\left(M_{13}^2 - M_{12}^2\right) = -8v^6 \mathrm{Im}\left(\Lambda_5^* \Lambda_6^2\right). \tag{3.3}$$

From the above form of J_1 it is easy to see that if any two of three off-diagonal elements of the matrix $\mathscr{M}^2$ vanish then $J_1 = 0$. However, even when $J_1 = 0$ CP violation is possible [7], signalizing by two additional CP-odd invariants containing Λ_7, the only complex parameter which is absent in $\mathscr{M}^2$. So, the set of relevant CP-odd weak-basis invariant is [1]

$$J_1 = -8v^6 \mathrm{Im}\left(\Lambda_5^* \Lambda_6^2\right), \qquad J_2 = -4v^4 \mathrm{Im}\left(\Lambda_5^* \Lambda_7^2\right), \qquad J_3 = 2\sqrt{2}v^3 \mathrm{Im}\left(\Lambda_7^* \Lambda_6\right). \tag{3.4}$$

To have CP conserving model all J-invariants must vanish [7]. If any of those three invariants does not vanish, then there is CP violation in the model. As we already discussed there are two cases with and without CP-mixing, which in terms of $J_{1,2,3}$ can be described as follows:

- If $J_1 \neq 0$ $(M_{13}, M_{23} \neq 0)$ then there is mixing between states of different CP properties and physical states h_1, h_2, h_3 have no defined CP quantum numbers.
- If $J_1 = 0$ and $J_{2,3} \neq 0$ we have CP violation from interactions even if there is no mixing between states. This case will be discussed further in section 4.

It is worth noticing that all that these findings, in particular the fact of possible CP violation without CP-mixing, are valid not only in the Higgs basis [7]. Note, however that CP violation without CP mixing can not be realized in the 2HDM with soft violation of Z_2 symmetry.

It is important to realize that J-invariants do not distinguish between explicit and spontaneous CP violation. So, in order to pin down kind of CP violation other types of invariants are needed.

3.2 *I*-invariants

The potential (2.2) can be written in the following form [4, 6]:

$$V = Y_{a\bar{b}} \Phi_{\bar{a}}^\dagger \Phi_b + \frac{1}{2} Z_{a\bar{b}c\bar{d}} (\Phi_{\bar{a}}^\dagger \Phi_b)(\Phi_{\bar{c}}^\dagger \Phi_d). \tag{3.5}$$

Four CP-odd weak-basis invariants can be build from parameters $Y_{a\bar{b}}, Z_{a\bar{b}c\bar{d}}$ of V (3.5) [5]:

$$\begin{aligned} I_1 &= \mathrm{Im}(Z^{(1)}_{a\bar{c}} Z^{(1)}_{e\bar{b}} Z_{b\bar{e}c\bar{d}} Y_{d\bar{a}}), & I_2 &= \mathrm{Im}(Y_{a\bar{b}} Y_{c\bar{d}} Z_{b\bar{a}d\bar{f}} Z^{(1)}_{f\bar{c}}), \\ I_3 &= \mathrm{Im}(Z_{a\bar{c}b\bar{d}} Z^{(1)}_{b\bar{f}} Z^{(1)}_{d\bar{h}} Z_{f\bar{a}j\bar{k}} Z_{k\bar{j}m\bar{n}} Z_{n\bar{m}h\bar{c}}), & I_4 &= \mathrm{Im}(Z_{a\bar{c}b\bar{d}} Z_{c\bar{e}d\bar{g}} Z_{e\bar{h}f\bar{q}} Y_{g\bar{a}} Y_{h\bar{b}} Y_{q\bar{f}}). \end{aligned} \tag{3.6}$$

(Here $Z^{(1)}_{a\bar{b}}$ denotes a combination of the parameters $Z_{a\bar{b}c\bar{d}}$.) Note that in I_i both quartic and quadratic parameters enter in contrast to J_i, derived for physical states, which can be expressed by quartic parameters (Λ_i) only.

[1] Only two of three J-invariants are independent in the general 2HDM without fermions [7].

The potential is explicitly CP conserving if and only if all I_i vanish [5]. This means that there exists „a real basis" of fields Φ_1, Φ_2 in which all λ_i, m_{ij}^2 are real. In such case still CP can be violated spontaneously, since I-invariants are not sensitive to vacuum expectation value of fields. So, if $\forall I_i = 0$ then CP symmetry in the model can be either conserved (both explicitly and spontaneously) or violated spontaneously by the vacuum state.

3.3 Distinguishing between various kinds of CP violation

As we already discussed J- and I-invariants are sensitive to the different aspects of CP violation in the scalar sector of 2HDM. In particular we see that they have different sensitivity for the spontaneous CP violation (I_i have none). Combining the information provided by J- and I-invariants allows us to distinguish between conservation and violation of CP symmetry and if CP is violated to establish the pattern of this violation. A comparison of J- and I-invariants is shown in the table 1.

Table 1: A comparison of J- and I-invariants in the 2HDM

CP properties of 2HDM	J-invariants	I-invariants
CP explicitly violated	$\exists J_i \neq 0$	$\exists I_i \neq 0$
CP spontaneously violated	$\exists J_i \neq 0$	$\forall I_i = 0$
CP conserved	$\forall J_i = 0$	$\forall I_i = 0$

4. CP violation without CP mixing

Let us now consider a special case, which we mentioned earlier, when physical states with defined CP properties have CP-violating interaction. In such model (in the Higgs basis) let us take

$$\mathrm{Im}\,\Lambda_5 = \mathrm{Im}\,\Lambda_6 = 0.$$

Due to the chosen values of Λ_5 and Λ_6 there is no mixing between states of different CP and the J_1 invariant (3.3) is equal to zero. After standard diagonalization we get states h, H and A with defined CP properties.

The interaction of these particles h, H, A with gauge bosons is described by

$$\mathscr{L}_{s-g} \supset \frac{g^2 v}{2}\chi_h^V h W^+W^- + \frac{g^2 v}{2}\chi_H^V H W^+W^- + \frac{g^2 v}{2}\chi_A^V A W^+W^-, \tag{4.1}$$

where

$$\chi_h^V = \cos\alpha, \qquad \chi_H^V = \sin\alpha, \qquad \chi_A^V = 0. \tag{4.2}$$

Here α is a mixing angle between η_1 and η_2 fields. The χ_i are the relative couplings with respect to the Standard Model coupling between the SM Higgs boson and W/Z, with a sum rule $(\chi_h^V)^2 + (\chi_H^V)^2 + (\chi_A^V)^2 = 1$. All these couplings are like in the CP conserving case, in contrast to the case of CP mixing, where all couplings χ_i^V for h_i are in principle nonvanishing, in particular $\chi_3^V \neq 0$.

Although the mass-squared matrix and interactions with gauge bosons point to the CP conservation, the remaining nonvanishing for $\mathrm{Im}\,\Lambda_7 \neq 0$ J-invariants

$$\mathrm{Im}\,\Lambda_7 \neq 0 \Rightarrow J_2, J_3 \neq 0, \tag{4.3}$$

ensure us that there is the CP violation in the considered (no CP-mixing) case. This CP violation shows up for example in the trilinear self-interaction of physical scalars $\mathscr{L}_{self}$:

$$\begin{aligned}\mathscr{L}_{self} \supset & -\frac{1}{2}\mathrm{Im}\Lambda_7 v AAA - \frac{1}{2}\mathrm{Im}\Lambda_7 v \sin^2\alpha\, Ahh - \frac{1}{2}\mathrm{Im}\Lambda_7 v \cos^2\alpha\, AHH, \\ & + \mathrm{Im}\Lambda_7 v \cos\alpha \sin\alpha\, AhH - \mathrm{Im}\Lambda_7 v AH^+H^-. \end{aligned} \tag{4.4}$$

We see that if $\mathrm{Im}\Lambda_7 \neq 0$ there are possible couplings with odd number of the CP-odd field A, which cannot occur in the CP conserving 2HDM. For example, new decay channels appear for A: $A \to hh$, HH, H^+H^-. J-invariants tell us about CP violation in the model, however if we want to know what kind of CP violation occurs here we need to use I-invariants.

5. Summary

We studied the CP violation in the 2HDM with the aim to distinguish between the explicit and spontaneous form of this violation. We used two sets of well known CP-odd weak-basis invariants and we found that both of them are needed to pin down the nature of CP violation.

We discuss a special case of CP violation without CP mixing, which is in contradiction with the usual treatment, where CP violation in the 2HDM is considered as being equivalent to the mixing between states with different CP properties.

We found a case in which there is no CP mixing and the interaction of neutral Higgs bosons with gauge bosons preserves CP, however the Higgs self-interaction violates CP symmetry. This results in non-zero vertices with odd number of A (eg. $A \to H^+H^-$).

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