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A double Myers-Perry black hole in five dimensions: an Inverse Scattering Construction

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Using the inverse scattering method we construct a six-parameter family of exact, stationary, asymptotically flat solutions of the 4+1 dimensional vacuum Einstein equations, with $U(1)^2$ rotational symmetry. It describes the superposition of two Myers-Perry black holes, each with a *single* angular momentum parameter, both in the same plane. The black holes live in a background geometry which is the Euclidean C-metric with an extra flat time direction. We discuss several aspects of the black holes geometry, including the conical singularities arising from force imbalance, and the torsion singularity arising from torque imbalance. The double Myers-Perry solution presented herein might be of interest in studying spin-spin interactions in five dimensional general relativity.

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1. Introduction

Over the last few years a great effort has been made to tackle the black hole classification problem in higher dimensions [1]. It is well known that the “phase space” of regular (i.e. free of curvature singularities on and outside an event horizon) and asymptotically flat black objects is rather richer than in four dimensions, containing exotic objects such as black rings [2, 3, 4, 5, 6, 7] and black saturns [8].

The new stationary solution presented herein describes the superposition of two Myers-Perry black holes in five dimensions, each with a single angular momentum parameter, both in the same plane. The new solution is built upon a non-trivial background geometry - the Euclidean C-metric with an extra flat time direction - having conical singularities, which are still present, generically, when the black holes are included. This solution also has a torsion singularity, which can be removed by an appropriate choice of parameters.

2. A Double Myers-Perry Solution in 5D

2.1 Generating the solution using the inverse scattering method

In five spacetime dimensions, the inverse scattering method (or Belinskii-Zakharov method) [9, 10] can be used to construct new Ricci flat metrics with three commuting Killing vector fields from known ones, by using purely algebraic manipulations. Such metrics can always be written in the form

$$ds^2 = G_{ab}(\rho,z)dx^a dx^b + e^{2\nu(\rho,z)}(d\rho^2+dz^2),$$

where $a,b=1,\dots,3$, and are fully characterized by their rod structure [11]. Indeed, all the physical information is given by the sizes of the finite rods and the corresponding eigenvectors to each rod.

To use the BZ method, we need a seed solution, which we take to be the double Schwarzschild-Tangherlini solution built in [12]. It is defined by

$$G_0 = \mathrm{diag}\left\{-\frac{\mu_1\mu_4}{\mu_2\mu_5}, \frac{\rho^2\mu_3}{\mu_1\mu_4}, \frac{\mu_2\mu_5}{\mu_3}\right\}\ , e^{2\nu_0} = \frac{k\mu_2\mu_5\mu_3^{-1}\prod_{i<j}(\rho^2+\mu_i\mu_j)}{(\rho^2+\mu_1\mu_4)^3(\rho^2+\mu_2\mu_5)^3\prod_{i=1}^{5}(\rho^2+\mu_i^2)}\ ,$$

where k is an integration constant and $\mu_k \equiv \sqrt{\rho^2+(z-a_k)^2}-(z-a_k)$. The a_i's determine the edges of the rod intervals and we choose the ordering $a_1<a_2<a_3<a_4<a_5$.

Using the method suggested by Pomeransky [13], we remove two anti-solitons, at $z=a_1,a_4$, and two solitons, at $z=a_2,a_5$, all with trivial BZ vectors $(1,0,0)$. Thus we divide $(G_0)_{tt}$ by $(\mu_1^2\mu_4^2)/(\mu_2^2\mu_5^2)$ and we actually used this new metric as our seed.

After computing the corresponding generating matrix that solves the Lax pair constructed in the BZ method, the double Myers-Perry solution is now obtained adding two anti-solitons, at $z=a_1$ with BZ vector $(1,b,0)$ and at $z=a_4$ with a BZ vector $(1,c,0)$, and re-adding the two solitons at $z=a_2,a_5$ with BZ vector $(1,0,0)$.

The resulting metric is explicitly given in [14] and its rod structure is represented in figure 1. The metric gives a six parameter family of solutions that can be taken to be the four finite rod sizes, together with the two BZ parameters (b and c, which are responsible for the rotational features).

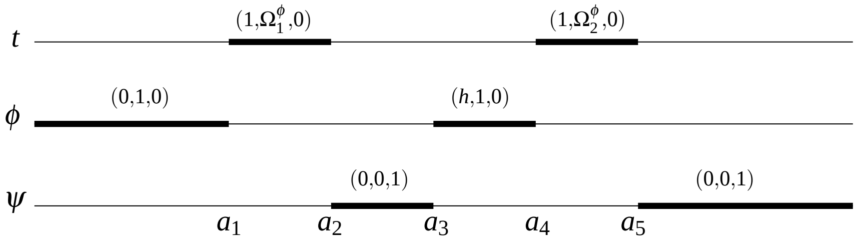

Figure 1: Rod structure for the double Myers-Perry spacetime. Next to each rod the corresponding eigenvector [11] is displayed.

The eigenvector of the two timelike rods shows a spatial component along the ϕ direction. These components (Ω_1^ϕ and Ω_2^ϕ) are the angular velocities of the individual black hole horizons and they take the form:

$$\Omega_1^\phi = \frac{a_{41}b}{2a_{21}a_{51}+a_{31}b^2}\ , \qquad \Omega_2^\phi = \frac{a_{41}(a_{54}a_{31}b+a_{51}a_{43}c)}{2a_{41}^2a_{51}a_{54}+(a_{31}b+a_{43}c)(a_{54}a_{31}b+a_{51}a_{43}c)}.$$

These angular velocities reduce to the horizon angular velocities of single Myers-Perry black holes in the limits $a_3 = a_4 = a_5$ and $a_1 = a_2 = a_3$, respectively. If $b = 0 = c$, both angular velocities are zero.

The finite spacelike rod's eigenvector between a_3 and a_4 also shows a timelike component h. If $h \neq 0$, $\rho = 0$ and $a_3 < z < a_4$ is not an axis for $\partial/\partial\phi$ and we have a torsion singularity. Thus, we demand $h = 0$, which is sometimes called the *axis condition* [15] and yields the constraint

$$\Delta_{axis} = 0\ , \qquad \Delta_{axis} \equiv (a_{31}b+a_{43}c)(2a_{42}a_{51}-a_{31}bc)-2a_{41}^2a_{51}c\ . \tag{2.1}$$

In particular, this equation is obeyed if $b = 0 = c$, as expected.

2.2 Background Geometry

In the absence of the two black holes, i.e. considering $a_2 = a_1$, $a_4 = a_5$ and $b = 0 = c$, we end up with a metric whose rod structure has four spacelike rods, all with trivial eigenvectors. This is our background geometry which is exactly the Euclidean C-metric with an added time direction. In principle one could have a double Myers-Perry solution in five dimensions that would reduce to flat space when the two black holes are removed. However, such solution could not have the three commuting killing vectors fields necessary to apply the inverse scattering method.

The Euclidean C-metric as conical singularities due to the three fixed points a_1,a_3 and a_5, that yields the desired $U(1)^2$ spatial isometry. Choosing k (the integration constant on the conformal factor $e^{2\nu(\rho,z)}$) to ensure asymptotically flatness, in the remaining two finite rods there are conical *excesses* given by $\delta_\psi = 2\pi(a_{53}/a_{31})$, for $a_1 < z < a_3$ and $\delta_\phi = 2\pi(a_{31}/a_{53})$, for $a_3 < z < a_5$.

2.3 Conical Singularities and Komar Integrals

The conical *excesses* for the generic solution are

$$\delta_\psi = 2\pi\left(\frac{a_{41}a_{52}}{\sqrt{a_{51}a_{31}a_{32}a_{42}}}\cdot\left|\frac{2a_{42}a_{51}}{2a_{42}a_{51}+a_{43}bc}\right|-1\right)\ ,\ \ a_2 < z < a_3\ , \tag{2.2}$$

$$\delta_\phi = 2\pi\left(\frac{a_{41}a_{52}}{\sqrt{a_{51}a_{43}a_{53}a_{42}}}\cdot\left|\frac{2a_{42}a_{51}}{2a_{42}a_{51}-a_{31}bc}\right|-1\right)\quad a_3 < z < a_4. \tag{2.3}$$

It is clear that the introduction of rotation could eliminate either of these conical singularities, but not *both* simultaneously. However, one must note that the requirement for either of these conical singularities to vanish is *incompatible* with the axis condition. Note also that the second condition should only be considered if one imposes the axis condition. The incompatibility of the axis and regularity conditions is reminiscent of the result obtained in [15] for $D = 4$ using a post-post Newtonian analysis.

From the next to leading order term in G_{tt} and leading order in $G_{t\phi}$ we can read off the ADM mass and angular momentum to be

$$M_{ADM} = \frac{3\pi}{8}\left[2a_{21} + 2a_{54} + \frac{(a_{31}b + a_{43}c)^2}{a_{41}^2}\right] ,$$

$$J^{\phi}_{ADM} = \frac{\pi}{4}\left[\frac{2[a_{31}b(a_{21} + a_{54} + a_{34}) + a_{43}c(a_{21} + a_{54} + a_{31})]}{a_{41}} + \frac{(a_{31}b + a_{43}c)^3}{a_{41}^3}\right] .$$

We can also compute the individual mass and the intrinsic spin, of each black hole, as a Komar integral at the horizon of each black hole, i.e. at $\rho = 0$, $a_1 < z < a_2$ and $\rho = 0$, $a_4 < z < a_5$. However, unlike the static case and, for instance, the black saturn solution [8], for our solution the Komar masses and angular momenta do not add up to the ADM quantities. In general, there is a non-trivial Komar integral coming from the surface $\rho = 0$ and $a_3 < z < a_4$. This contribution is proportional to the axis condition and it accounts for the extra piece in the relations

$$M_{ADM} = M_1^{Komar} + M_2^{Komar} + M_{extra}^{Komar} , \quad J_{ADM} = J_1^{Komar} + J_2^{Komar} + J_{extra}^{Komar} .$$

Imposing the axis condition, the Komar masses and angular momenta do add up to the ADM mass and angular momentum; in other words the strut is massless and spinless.

3. Conclusions

We have used the inverse scattering technique to generate a new asymptotically flat, vacuum solution of five dimensional general relativity describing two Myers-Perry black holes, each with a singular angular momentum parameter, both in the same plane. The addition of rotation, to the black holes, brings a torsion singularity and changes both conical singularities, already present on the background geometry. When the axis condition is imposed the torsion singularity disappears, but none of the conical singularities can be removed. It remains to be seen if, by including the second angular momentum parameter or other fields, like the electromagnetic field, such singularities can be removed.

For the study of spin-spin interactions in higher-dimensions, it would be important to have a physical interpretation of these axis and regularity conditions in terms of the different forces and torques that play a role in this geometry.

One somewhat unexpected feature that we found was a contribution to the ADM mass and angular momentum of one part of the geometry exterior to the black hole horizons, if the axis condition is not obeyed. This suggests that, in the post-post Newtonian analysis of this type of problems, along the lines of [15], one should indeed include one further parameter describing the rotating rod, as suggested in [16].

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