

Microscopic entropy of black holes and AdS₂ quantum gravity

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Quantum gravity (QG) on two-dimensional anti-de Sitter spacetime (AdS₂) takes always the form of a chiral conformal field theory (CFT). However, the actual content of the CFT, and in particular its central charge, depends on the background values of the dilaton and Maxwell field. We review the main features of AdS₂ QG with linear dilaton and of AdS₂ QG with constant dilaton and Maxwell field. We also show that the 3D charged Bañados-Teitelboim-Zanelli black hole interpolates between these two versions of AdS₂ QG. Applications to the computation of the microscopic entropy of black holes are also discussed.

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1. Introduction

The problem of the microscopic origin of the Bekenstein-Hawking (BH) entropy of black holes is one of the most intriguing challenges for modern theoretical physics. Its solution is not only important for delivering a microscopic basis for black hole thermodynamics. It also represents one crucial test, perhaps the most relevant one, that any quantum theory of gravity has to pass. It has been tackled using many different frameworks and approaches: String theory, AdS/CFT correspondence, asymptotic symmetries, D-branes, induced gravity and entanglement entropy, loop quantum gravity.

Many of these approaches reproduce correctly the BH black hole entropy (some exactly others up to some numerical constant), in such a good way that this success is considered by some physicists almost as a problem [1]. It is likely that this universality, rather than a problem, is a consequence of some fundamental underlying feature of semiclassical quantum gravity that has to be shared by the different approaches. A strong hint that this may be indeed the case is represented by the wide, successfully, use of an asymptotical level formula for two-dimensional (2D) conformal field theory (CFT), the so called Cardy formula, to count black hole microstates [2],

$$S = 2\pi \left(\sqrt{\frac{cl_0}{6}} + \sqrt{\frac{c\bar{l}_0}{6}} \right), \quad (1.1)$$

where l_0 and \bar{l}_0 are the eigenvalues of the L_0 and \bar{L}_0 operators and c is the central charge in the conformal algebra.

Obviously, Cardy's formula has a chance to reproduce BH black hole entropy only if there is an underlying (at least approximate) 2D conformal symmetry. This is for instance the case of black holes in anti de Sitter (AdS) spacetime. The AdS/CFT correspondence should allow us to describe black holes as thermal states of the dual CFT. An other approach is to use the built-in conformal symmetry of event horizons and 2D diffeomorphisms and the related algebra of constraints, to model the black hole as a microstate gas of a CFT (see e.g. Ref. [3]).

Counting microstates using the AdS/CFT correspondence works well only when the black hole geometry factorizes as $\text{AdS}_3 \times M$ (M compact manifold) or $\text{AdS}_2 \times M$. For instance, the famous Strominger-Vafa [4] calculation of the entropy of the 5D Reissner-Nordström (RN) SUSY black hole has been made possible because of the AdS_3 factor in the near-horizon geometry of the 5D black hole solution. Precise computation of the statistical entropy of generic AdS_d black hole (e.g. the Schwarzschild-AdS black hole in $d=4$) is out of our reach, because we do not know how to compute in strongly-coupled gauge theories.

Thus, AdS_3 and AdS_2 QG together with the three and two-dimensional black holes in AdS spacetime play a very special role for the computation of the statistical entropy of black holes. In the large N limit AdS_3 QG can be identified as a 2D CFT with central charge $c = 3G/2l$ (G and l are respectively the 3D Newton constant and the AdS length) describing Brown-Henneaux-like boundary excitations, i.e. deformations of the asymptotic boundary of AdS_3 [5]. The CFT reproduces correctly the entropy of the Bañados-Teitelboim-Zanelli (BTZ) black hole [6] and of a wide class of higher-dimensional black holes. The related thermodynamic system describes a 2D field theory with extensive entropy $S \sim T$ with a ground state of zero entropy at zero temperature.

On the other hand, it is still not completely clear whether AdS₂ QG has to be considered as the chiral half of 2D CFT or a conformal quantum mechanics living on the asymptotic one-dimensional boundary of AdS₂ [7, 8, 9, 10, 11, 12, 13, 14, 15, 16]. Nevertheless, it has been used with success to compute the statistical entropy of AdS₂ black holes and related higher dimensional solutions.

One important application of AdS₂ QG is its use in the description of the near-horizon regime of charged extremal (BPS) black holes, in which the near-horizon geometry factorizes as AdS₂ × M. In this case the dynamical system has peculiar features, such as the attractor mechanism [17, 18, 19], whereas from the thermodynamical point of view the system is characterized by a ground state of nonvanishing entropy at zero temperature. Interesting examples of this kind of behavior are the near-horizon geometries of asymptotically flat, extremal, black *p*-branes in *d* space-time dimensions,

$$AdS_{p+2} \times S^{d-p-2} = \frac{SO(p+1, 2)}{SO(p+1, 1)} \times \frac{SO(d-p-1)}{SO(d-p-2)}. \quad (1.2)$$

For *p* = 0 we have charged, BPS, black holes in *d* = 4, 5. For *p* = 1 and *d* = 5, 6 we have the black string. It is interesting to notice that although Eq. (1.2) holds also for *p* = 0 and *d* = 3, this geometry cannot be obtained as the near-horizon geometry of a 3D charged black hole. This is because there are no asymptotically flat 3D black holes.

The first, oldest, version of AdS₂ QG has been constructed following closely the Brown-Henneaux formulation of AdS₃ QG [7]. It is based on AdS₂ endowed with a linear dilatonic background. Recently, there has been renewed interest for the AdS/CFT correspondence in two-spacetime dimensions [13, 14, 15, 20]. In particular, a second formulation for AdS₂ QG, based on AdS₂ endowed with constant dilaton and Maxwell field has been proposed in Ref. [13]. In this paper we will argue that the two different formulations of AdS₂ QG and their relationship with AdS₃ could be the clue for understanding the complicate pattern of near-horizon geometries of higher-dimensional charged black holes and their entropies. A key role in this context is played by 3D charged BTZ black hole. This black hole interpolates between an asymptotic AdS₃ and a near-horizon AdS₂ × S¹ geometry. Circular symmetric dimensional reduction allows us to describe AdS₃ as AdS₂ with a linear dilaton. Thus, the charged BTZ black hole interpolates between the two different versions of AdS₂ QG.

The plan of this paper is as follows. In Sect 2 we give a short review of the Brown-Henneaux formulation of AdS₃ quantum gravity. In Sect. 3 we briefly review AdS₂ QG with a linear dilaton. In Sect. 4 we consider AdS₂ QG with constant dilaton and *U*(1) field. Some basic features of the charged BTZ black hole are discussed in Sect. 5. In Sect. 6 we will show that charged BTZ black hole interpolates between the two formulations of AdS₂ QG. In Sect. 7 we discuss the application to the calculation of the microscopic black hole entropy. Finally in Sect. 8 we state our conclusions.

2. A short review of AdS₃ quantum gravity

Classical AdS₃ gravity is described by the action

$$I = \frac{1}{16\pi G} \int d^3x \sqrt{-g} (R + 2\Lambda), \quad (2.1)$$

where *G* is the 3D Newton constant and $\Lambda = \frac{1}{l^2} > 0$ is the cosmological constant. We are using units where *G* and *l* have both the dimension of a length. Black hole solutions in AdS₃, called BTZ

after their discoverers Bañados, Teitelboim and Zanelli [21, 22], are characterized by mass M and angular momentum J . The corresponding line element in Schwarzschild coordinates is

$$ds^2 = -f(r)dt^2 + f^{-1}dr^2 + r^2 \left(d\theta - \frac{4GJ}{r^2} dt \right)^2, \quad (2.2)$$

with metric function:

$$f(r) = -8GM + \frac{r^2}{l^2} + \frac{16G^2J^2}{r^2}. \quad (2.3)$$

The outer and inner horizons, r_+ , r_- are given by

$$r_{\pm}^2 = 4Gl^2 \left(M \pm \sqrt{M^2 - \frac{J^2}{l^2}} \right). \quad (2.4)$$

AdS₃ quantum gravity was discovered by Brown and Henneaux [5] ten years before Maldacena conjecture about the correspondence between gravity on AdS and conformal field theories [23, 24]. They realized that the asymptotic symmetry group (ASG) of AdS₃, i.e. the group that leaves invariant the asymptotic form of the metric, is the conformal group in two spacetime dimensions.

In order to determine the ASG one has first to fix boundary conditions for the fields at $r = \infty$ then to find the Killing vectors leaving these boundary conditions invariant. The boundary conditions must be relaxed enough to allow for the action of the conformal group and for the right boundary deformations, but tight enough to keep finite the charges associated with the ASG generators, which are given by boundary terms of the action (2.1). These charges can be calculated using a canonical realization of the ASG [5, 25]. Alternatively, one can use a lagrangian formalism and work out the stress-energy tensor for the boundary CFT [26]. For the BTZ black hole suitable boundary conditions for the metric are [5]

$$\begin{aligned} g_{tt} &= -\frac{r^2}{l^2} + O(1), & g_{t\theta} &= O(1), & g_{tr} &= g_{r\theta} = O\left(\frac{1}{r^3}\right), \\ g_{rr} &= \frac{l^2}{r^2} + O\left(\frac{1}{r^4}\right), & g_{\theta\theta} &= r^2 + O(1), \end{aligned} \quad (2.5)$$

whereas the vector fields preserving them are

$$\begin{aligned} \chi^t &= l(\varepsilon^+(x^+) + \varepsilon^-(x^-)) + \frac{l^3}{2r^2}(\partial_+^2 \varepsilon^+ + \partial_-^2 \varepsilon^-) + O\left(\frac{1}{r^4}\right), \\ \chi^\theta &= \varepsilon^+(x^+) - \varepsilon^-(x^-) - \frac{l^2}{2r^2}(\partial_+^2 \varepsilon^+ - \partial_-^2 \varepsilon^-) + O\left(\frac{1}{r^4}\right), \\ \chi^r &= -r(\partial_+ \varepsilon^+ + \partial_- \varepsilon^-) + O\left(\frac{1}{r}\right), \end{aligned} \quad (2.6)$$

where $\varepsilon^+(x^+)$ and $\varepsilon^-(x^-)$ are arbitrary functions of the light-cone coordinates $x^\pm = (t/l) \pm \theta$ and $\partial_\pm = \partial/\partial x^\pm$. The generators L_n (\bar{L}_n) of the diffeomorphisms with $\varepsilon^+ \neq 0$ ($\varepsilon^- \neq 0$) obey the Virasoro algebra,

$$\begin{aligned}
[L_m, L_n] &= (m-n)L_{m+n} + \frac{c}{12}(m^3 - m)\delta_{m+n,0}, \\
[\bar{L}_m, \bar{L}_n] &= (m-n)\bar{L}_{m+n} + \frac{c}{12}(m^3 - m)\delta_{m+n,0}, \\
[L_m, \bar{L}_n] &= 0,
\end{aligned} \tag{2.7}$$

where c is the central charge. In the semiclassical regime $c \gg 1$, the central charge can be calculated using a canonical realization of the ASG algebra. Explicit computation of c gives [5]

$$c = \frac{3l}{2G}. \tag{2.8}$$

In a further development, Strominger reproduced the entropy of the rotating, BTZ black hole counting states of the Hilbert space of the CFT, i.e. using the Cardy formula (1.1) and identifying the eigenvalues of the L_0 and \bar{L}_0 operators in terms of the mass and angular momentum of the hole [6],

$$lM = l_0 + \bar{l}_0, \quad J = l_0 - \bar{l}_0. \tag{2.9}$$

Strominger calculation holds for $c \gg 1$ and for large mass, large angular momentum black holes. What AdS₃ QG for $c \sim 1$ really is, it is still not clear (see recent developments about this topic in Ref. [27, 28])

3. AdS₂ quantum gravity with linear dilaton

The simplest theory of classical AdS₂ gravity contains a scalar field (the dilaton η), parametrizing (the inverse of) 2D Newton constant,

$$A = \frac{1}{2} \int d^2x \eta \left(R + \frac{2}{l^2} \right). \tag{3.1}$$

The ensuing field equations do not allow for a constant dilaton but require a linear dilaton background. Black hole solutions of mass M are given by [29]

$$ds^2 = - \left(\frac{r^2}{l^2} - \frac{2Ml}{\eta_0} \right) dt^2 + \left(\frac{r^2}{l^2} - \frac{2Ml}{\eta_0} \right)^{-1} dr^2, \quad \eta = \eta_0 \frac{r}{l}. \tag{3.2}$$

The BH entropy of the 2D black hole is [29]

$$S = 2\pi\eta_h = 2\pi\sqrt{2\eta_0 M l}, \tag{3.3}$$

where η_h is the value of the dilaton at the black hole horizon.

Linear dilaton AdS₂ quantum gravity has been formulated following closely the Brown-Henneaux derivation of AdS₃ QG [7]. Suitable boundary condition for the metric and Killing vectors at the timelike boundary of AdS₂ are

$$g_{tt} = -\frac{r^2}{l^2} + O(1), \quad g_{tr} = O\left(\frac{1}{r^3}\right), \quad g_{rr} = \frac{l^2}{r^2} + O\left(\frac{1}{r^4}\right), \tag{3.4}$$

$$\eta = O(r), \quad \chi^t = \varepsilon(t) + O\left(\frac{1}{r^2}\right), \quad \chi^r = -r\dot{\varepsilon}(t) + O\left(\frac{1}{r}\right). \tag{3.5}$$

The ASG of AdS₂ is generated by one single copy of the Virasoro algebra spanned by the L_0 generators in Eq. (2.7). Thus AdS₂ quantum gravity can be seen as the chiral half of a 2D CFT. The main difference between the AdS₂ and the AdS₃ case is the origin of the central charge c in the Virasoro algebra (2.7). In the 2D case the origin of the central charge can be traced back to the breaking of the $SL(2, R)$ isometry of AdS₂ owing to the linear dilaton background given by Eq. (3.2) [30].

The central charge c can be calculated using a canonical realization of the ASG algebra [7]¹,

$$c = 12\eta_0. \quad (3.6)$$

Using Eq. (3.6), identifying l_0 in terms of the black hole mass, $l_0 = MI$, the Cardy formula (1.1) reproduces exactly the entropy of the AdS₂ black hole given by Eq. (3.3).

4. AdS₂ quantum gravity with constant dilaton and U(1) field

Recently Hartmann and Strominger have found an independent formulation of AdS₂ QG, which works for a background with constant dilaton and differs in the mechanism generating the central charge [13]. The classical theory considered in Ref. [13] is 2D Maxwell-Dilaton gravity,

$$I = \frac{1}{2} \int d^2x \sqrt{-g} \left[\eta \left(R + \frac{8}{l^2} \right) - \frac{l^2}{4} F^2 \right], \quad (4.1)$$

where $F_{\mu\nu}$ is the Maxwell tensor. The ensuing equations of motion admit solutions describing AdS₂ endowed with a constant dilaton and $U(1)$ field parametrized by a constant E . In the conformal gauge the solutions are given by

$$ds^2 = -\frac{l^2}{4\sigma^2} dx^+ dx^-, \quad F_{+-} = 2E\epsilon_{+-}, \quad A_{\pm} = \frac{El^2}{4\sigma}, \quad \eta = \frac{l^4 E^2}{4}, \quad \sigma = \frac{1}{2}(x^+ - x^-). \quad (4.2)$$

We fix the diffeomorphisms and $U(1)$ gauge freedom using a conformal, respectively, Lorentz gauge,

$$ds^2 = -e^{2\rho} dx^+ dx^-, \quad \partial_{\mu} A^{\mu} = 0. \quad (4.3)$$

After gauge fixing, conformal diffeomorphisms are described by two arbitrary functions $\epsilon^+(x^+)$, $\epsilon^-(x^-)$. The stress-energy tensor and the $U(1)$ current are the constraints enforcing gauge fixing and generate, respectively, residual diffeomorphisms and gauge transformations

$$T_{\pm\pm} = \frac{2}{\sqrt{-g}} \frac{\delta I}{\delta g^{\pm\pm}} = -2\partial_{\pm} \eta \partial_{\pm} \rho + \partial_{\pm} \partial_{\pm} \eta + 2\partial_{\pm} h \partial_{\pm} a = 0, \quad (4.4)$$

$$J_{\pm} = 2 \frac{\delta I}{\delta A^{\pm}} = \pm 2\partial_{\pm} h = 0, \quad (4.5)$$

where h is an auxiliary field used to linearize the quadratic term for the $U(1)$ field and we have dualized the vector potential A_{μ} in terms of a scalar a . If one now requires that the asymptotic

¹The outcome of early calculations was two times the actual value of the central charge, $c = 24\eta_0$ [7]. The origin of the mismatch has been later clarified in several independent ways [31, 32, 12].

boundary of AdS₂ remains at $\sigma = 0$ under the action of conformal diffeomorphisms (this is equivalent to fix boundary conditions for the metric) ε^- is determined in terms of ε^+ . We are left with only a chiral half of the 2D CFT. Analogously to the previous realization of AdS₂ QG the symmetry algebra is one single copy of the Virasoro algebra.

Being the dilaton constant, one is led to conclude that we are dealing with pure 2D QG, which has vanishing central charge [33]. This is not the case because of the presence of the $U(1)$ field. We need boundary conditions for the vector potential at $\sigma = 0$. Absence of charged current flow out of the boundary of AdS₂ requires $A_\mu(\sigma = 0) = 0$. The problem is that this boundary condition is not invariant under the action of conformal diffeomorphisms,

$$\delta_\varepsilon A_\mu|_{\sigma=0} = \frac{l^2 E}{2} \partial_+^2 \varepsilon^+|_{\sigma=0}. \quad (4.6)$$

This term can be cancelled by a gauge transformation $A \rightarrow A + \partial\lambda$ with

$$\lambda(x^+) = -\frac{l^2 E}{2} \partial_+ \varepsilon^+. \quad (4.7)$$

Hence, the conformal symmetry group is a chiral half of conformal diffeomorphisms supplemented by the gauge transformation (4.7). We have a twisted CFT. Conformal transformations are generated by Virasoro generators given in terms of an improved stress energy tensor,

$$\tilde{L} = \frac{1}{2} \int dx^+ \tilde{T}_{++} \varepsilon^+, \quad \tilde{T}_{++} = T_{++} + \frac{El^2}{4} \partial_+ J_+. \quad (4.8)$$

The central charge in the Virasoro algebra can be calculated expanding in Laurent modes and using the transformation law of the improved stress-energy tensor

$$\delta_\varepsilon \tilde{T}_{--} = \varepsilon^+ \partial_+ \tilde{T}_{++} + 2\partial_+ \varepsilon^+ \tilde{T}_{++} + \frac{c}{12} \partial_+^3 \varepsilon^+. \quad (4.9)$$

The transformation law of the original T_{++} is anomaly-free, but that of the current J_+ may have an anomalous term proportional to its level k [13],

$$\delta_\lambda J_+ = k \partial_+ \lambda^+. \quad (4.10)$$

This produces a central charge c in the Virasoro algebra given by,

$$c = 3kE^2 l^4 = \frac{3}{4} k \sqrt{\frac{\pi}{G}} l Q. \quad (4.11)$$

Let us close this sections by summarizing the main results of the last two sections. We have two different formulations of AdS₂ QG; both are described by the chiral half of a 2D CFT but the origin of the central charge is drastically different. In the first case, AdS₂ with a linear dilaton, the central charge is originated by the breaking of the conformal symmetry caused by a nonconstant dilaton and is determined by 2D inverse Newton constant η_0 . In the second case, AdS₂ with a constant dilaton and $U(1)$ field, the central charge is produced by a Schwinger effect and by a twisting of the CFT and is determined by the electric field E . To find a bridge between the two formulations we have to go up to three dimensions and to bring into the play the charged BTZ black hole.

5. The charged BTZ black hole

AdS gravity in three spacetime dimensions admits also charged black hole solutions, which are the charged version of the BTZ black hole [34]. They are solution of the action

$$I = \frac{1}{16\pi G} \int d^3x \sqrt{-g} \left(R + \frac{2}{l^2} - 4\pi G F_{\mu\nu} F^{\mu\nu} \right), \quad (5.1)$$

where $F_{\mu\nu}$ is the electromagnetic (EM) field strength. Considering for simplicity solutions with zero angular momentum, we have the two-parameter (M, Q) family of electric charged black hole solutions [34]

$$ds^2 = -f(r)dt^2 + f^{-1}dr^2 + r^2d\theta^2, \quad f(r) = -8GM + \frac{r^2}{l^2} - 8\pi GQ^2 \ln\left(\frac{r}{w}\right), \quad F_{tr} = \frac{Q}{r}, \quad (5.2)$$

where M, w are constants and $-\infty < t < +\infty$, $0 \leq r < +\infty$, $0 \leq \theta \leq 2\pi$. Notice that the parameter w can be reabsorbed in the definition of M . The striking differences with the BTZ black hole is represented by the presence of a power-law curvature singularity at $r = 0$. The charged BTZ black hole has an inner, $r = r_-$, and outer, $r = r_+$, event horizon. It also has well-defined temperature and entropy,

$$T_H = \frac{r_+}{2\pi l^2} - \frac{2GQ^2}{r_+}, \quad S = \frac{\pi r_+}{2G} = \frac{\pi l}{G} \sqrt{2GM + 2\pi GQ^2 \ln\left(\frac{r_+}{w}\right)}. \quad (5.3)$$

The charged BTZ black hole has been considered as the Cinderella in the family of 3D AdS black hole celebrities. The reason is that it has some unpleasant features. By varying the action one gets logarithmic divergent boundary terms. This makes the mass of the solution a poorly defined concept. Moreover, in order to avoid naked singularities one must impose a BPS-like bound involving M and Q ,

$$\Delta = 8GM - 4\pi GQ^2 \left[1 - 2 \ln\left(\frac{2Ql}{w} \sqrt{\pi G}\right) \right] \geq 0. \quad (5.4)$$

Unfortunately, this bound can be satisfied for arbitrary negative values of M , making the definition of thermodynamic ensembles problematic.

These problems can be handled using renormalization group ideas and the IR/UV relation for the AdS/CFT correspondence [20, 35]. The system is enclosed in a circle of radius r_0 (the UV cutoff for the CFT), one takes $r, r_0 \rightarrow \infty$, keeping the ratio $r/r_0 = 1$, and writes,

$$f(r) = -8GM_0(r, w) + \frac{r^2}{l^2} - 8\pi GQ^2 \ln\left(\frac{r}{r_0}\right), \quad M_0(r_0, w) = M + \pi Q^2 \ln\left(\frac{r_0}{w}\right). \quad (5.5)$$

The parameter w is interpreted as a running scale and $M(r_0, w)$ is the regularized black hole mass, the total energy (gravitational plus electromagnetic) inside a circle of radius r_0 . Basically, one has now two options:

1. M is kept fixed and the metric (hence the position of the horizon) is scale-dependent. In this case M is seen as the black hole mass [34].
2. The metric (hence the horizon position) is w -independent and M runs with w [20, 35].

Because we want to keep the horizon (the IR scale for the CFT) fixed, we use prescription 2. M runs with w : $w \rightarrow \lambda w$, $M \rightarrow M + \pi Q^2 \ln \lambda$, but M_0 is w -independent. We fix now $w = l$ and $r_0 = r_+$. The invariant black hole mass, to be identified with the conserved charge associated with time-translations, becomes

$$M_0(r_+, l) = M + \pi Q^2 \ln\left(\frac{r_+}{l}\right). \quad (5.6)$$

This solves the problem of divergent boundary terms in the variation of the action (5.1). Moreover, the use of the mass M_0 of Eq. (5.6) instead of M as the energy of the system allows for a consistent formulation of the thermodynamics of the charged BTZ black hole [36].

5.1 The near-horizon limit

We expect the generic near-horizon, extremal behavior of black branes given by Eq. (1.2) to hold also for $p = 0$ and $d = 3$ not for asymptotically flat but for asymptotically AdS black holes. Thus, we expect an AdS₂ \times S¹ near horizon geometry for our extremal charged BTZ black hole.

In the extremal limit the charged BTZ black hole saturates the bound (5.4), i.e. we have $\Delta = 0$, $r_- = r_+ = \gamma = 2\sqrt{\pi G} Q l$. Expanding near the horizon, $r = \gamma + x$ one finds that the 3D geometry factorize as AdS₂ \times S¹, whereas the EM field becomes constant,

$$ds^2 = - \left(\frac{2}{l^2} x^2 - 8G\Delta M \right) dt^2 + \left(\frac{2}{l^2} x^2 - 8G\Delta M \right)^{-1} dr^2 + \gamma^2 d\theta^2, \quad F_{tx} = \frac{1}{2\sqrt{\pi G} l}, \quad (5.7)$$

where $\Delta M = M - M(\gamma) = M - \pi Q^2 (\frac{1}{2} - \ln(2Q\sqrt{\pi G}))$ is the mass above extremality. This black hole solution shares with its higher-dimensional, asymptotically flat, cousins the thermodynamical behavior. The extremal charged BTZ black hole is a state of zero temperature and constant entropy $S_{ext} = \pi\gamma/2G$. Thus, the charged BTZ black hole interpolates between an asymptotic, $r \rightarrow \infty$, AdS₃ geometry and a near horizon AdS₂ \times S¹ geometry.

6. Interpolating the two versions of AdS₂ quantum gravity

The two limiting regimes, the asymptotic and near-horizon one, of the BTZ black hole can be both described by an effective 2D Maxwell-Dilaton theory of gravity. The 2D effective theory can be obtained performing a circular symmetric dimensional reduction 3D \rightarrow 2D, with the dilaton parametrizing the radius of the transverse circle and with an electric ansatz for the Maxwell field, $F_{t\theta} = F_{r\theta} = 0$,

$$ds_{(3)}^2 = ds_{(2)}^2 + l^2 \eta^2 d\theta^2. \quad (6.1)$$

The 2D Maxwell-Dilaton gravity theory turns out to be,

$$I = \frac{1}{2} \int d^2x \sqrt{-g} \eta \left(R + \frac{2}{l^2} - 4\pi G F^2 \right). \quad (6.2)$$

The corresponding 2D field equations admit two classes of solutions whose metric part is always a 2D AdS spacetime:

- AdS₂ with linear dilaton and Maxwell field $F_{tr} = Q/r$. This corresponds to the asymptotic $r \rightarrow \infty$ regime of the charged BTZ black hole.
- AdS₂ with constant dilaton and electric field. This corresponds to the near horizon regime.

6.1 AdS₂ with linear dilaton

These solutions are nothing but the 3D solution written in a 2D form. They are given by the 2D sections of the 3D solutions (5.2) and with $\eta = \bar{\eta}_0(r/l)$. Owing to a scale symmetry, $\eta \rightarrow \lambda\eta$, of the 2D field equations, the constant $\bar{\eta}_0$ is determined by the dimensional reduction:

$$\bar{\eta}_0 = \frac{l}{4G}. \quad (6.3)$$

Mass, temperature and entropy of the 2D black hole are the same as those of the 3D black hole.

The dual CFT can be constructed following the same procedure used in Sect. 3 for 2D dilaton gravity without Maxwell field. There is, however, a non trivial detail. Not only the charge associated with the L_0 Virasoro operator (the mass) diverges, but also the other charges associated with the other Virasoro operators L_m . The renormalization procedure used in the previous section for the mass allows also to define finite charges for the other Virasoro operators (see for details Ref. [35]). It turns out that the central charge of the Virasoro algebra is also finite and matches exactly that of pure AdS₂ with linear dilaton,

$$c = 12\bar{\eta}_0 = \frac{3l}{G}. \quad (6.4)$$

The EM field does not contribute to the central charge but only enters in the renormalization of the eigenvalue of L_0 , which is given in terms the mass M_0 of Eq. (5.6),

$$L_0 = lM_0(r_+, l). \quad (6.5)$$

6.2 AdS₂ with constant dilaton and electric field

The 2D field equations stemming from the action (6.2) admit also solutions describing AdS₂ with constant dilaton and electric field. They are given by the 2D sections of the near-horizon 3D solution (5.7). A Weyl transformation of the metric together with a rescaling by a constant of the $U(1)$ field strength brings the 2D action into the form [20],

$$I = \frac{1}{2} \int d^2x \sqrt{-g} \left[\eta \left(R + \frac{(\partial\eta)^2}{\eta} + \frac{2\eta}{l^2\eta_0} \right) - \frac{l^2}{2} F^2 \right]. \quad (6.6)$$

The classical solutions are

$$\begin{aligned} ds^2 &= -\left(\frac{2}{l^2}x^2 - a^2\right)dt^2 + \left(\frac{2}{l^2}x^2 - a^2\right)^{-1}dx^2, \quad F_{\mu\nu} = 2E\varepsilon_{\mu\nu}, \\ \eta &= 2l^4E^2, \quad E^2 = \frac{1}{4l^3} \sqrt{\frac{\pi}{G}} Q. \end{aligned} \quad (6.7)$$

Apart from a trivial redefinition of the AdS length, this 2D model differs from the Hartmann-Strominger model just for the presence of a kinetic term for the dilaton and a dilaton potential $V(\eta)$. In a constant dilaton background these terms do not contribute to the central charge. It is a simple exercise to construct the dual twisted CFT describing AdS₂ QG using the Hartmann-Strominger procedure described in Sect. 4 (see for details Ref. [20]). The central charge of the twisted CFT turns out to be

$$c = 3kE^2l^4 = \frac{3}{4}k\sqrt{\frac{\pi}{G}}lQ. \quad (6.8)$$

7. Microscopic black hole entropy

We can easily reproduce the Bekenstein-Hawking entropy of the 2D AdS black hole and hence the entropy of the charged BTZ black hole calculating the asymptotic density of states for the linear dilaton CFT. Using Eqs. (6.4), (6.5) and (5.6) into the Cardy formula (1.1) we find exactly the BH entropy (5.3).

In principle, one should also be able to reproduce the entropy of the extremal (and near-extremal) charged BTZ black hole by calculating the asymptotic density of states for the twisted CFT. However this requires using in the Cardy formula the eigenvalues of the twisted operator \tilde{L}_0 instead of that for the untwisted one. Careful analysis of the CFT spectrum and detailed knowledge of the effect of twisting on the CFT Hilbert space is needed.

8. Conclusions

The two different realizations of AdS_2 QG investigated in this paper describe different states: AdS_2 QG with linear dilaton describes Brown-Henneaux-like boundary excitations, which are relevant for explaining the entropy of the BTZ black hole whereas AdS_2 QG with constant dilaton and maxwell field describes D-brane-like excitations, which should account correctly for the entropy of extremal BPS black holes. Both realizations have a dual gravitational description in terms of an asymptotic and near-horizon geometry. Similarly to what happens for higher-dimensional charged RN solutions, there is an interpolating gravitational solution, the charged BTZ black hole bridging the two descriptions. These features make AdS_2 QG a powerful tool for investigating microscopic black hole physics and to shed light on several features of the AdS/CFT correspondence.

There is a long list of open questions and possible further developments. One should be able to reproduce the entropy of extremal and near-extremal (BPS) black holes using the near-horizon CFT. From the gravitational side this requires the use of the entropy function formalism [14, 37], whereas from the CFT side requires careful investigation of the Hilbert space of the twisted CFT.

An other key issue is the understanding, at the pure 2D level, of the relationship between the two sectors of 2D Maxwell-Dilaton gravity, the one with constant dilaton and the other with linear varying dilaton. In Ref. [38] it has been shown that the constant dilaton sector requires a negative 2D Newton constant. A true unified description of both constant and linear dilaton sector would shed light on these issues.

Also from the CFT point of view the relationship between the asymptotic CFT and the near-horizon CFT is far from being understood. The relevant question here is whether or not these two realizations correspond to two different conformal points. In the case of 3D AdS gravity minimally coupled with a scalar field it has been shown that the two dual CFTs are related by renormalization group flow and that the Zamolodchikov c -theorem holds [39]. Presently it is not clear if the same holds for Maxwell-Dilaton AdS_3 gravity.

Finally, one would like to extend our arguments to $d > 3$ spacetime dimensions. Here the main question is whether or not the interpolating feature of the charged BTZ black hole is a peculiarity of $d = 3$ and whether we can extend it to a wide class of charged and/or rotating black holes in $d > 3$.

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