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Kerr black hole and rotating black string by intersecting M-branes

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We find eleven dimensional non-BPS black M-brane solutions from Kerr solution in M-theory by using U-duality. Under the transformation four dimensional Kerr metric change to non-BPS rotating intersecting M-brane solutions. To easy application for AdS/CFT correspondence we must need a supersymmetric black brane solution, which limits special configuration of M-branes, e.g., M2-M2-M2 branes in five dimensional black brane and M5-M5-M5 brane with pp-wave in four dimensional one. In this case we may easily apply the AdS/CFT correspondence for non-BPS black hole solutions.

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1. Introduction

AdS/CFT correspondence [1] is the most successful theory to describe strong coupling limit for boundary conformal field theory. There is a holographic duality between bulk quantum gravity and boundary quantum field theory. Four dimensional CFT is studied in all aspects because of CFT interests, and its dual theory is based on five dimensional supergravity black hole solutions. In recent years, Kerr/CFT correspondence are found only for the extremal limit case [2]. The paper shows the reason of the finite entropy of the extremal Kerr black hole, even the Hawking temperature is zero. AdS/CFT correspondence are very useful to show that reason. However non-extremal case we have no information about AdS/CFT correspondence, thus for non-extremal Kerr/CFT correspondence we will give the M-brane configuration about non-extremal Kerr solution.

Non BPS black ring solutions [3] using the same method we use below, but these solutions related to intersecting $M2 \perp M2 \perp M2$-brane solution in M-theory, or intersecting $F1 \perp D2 \perp D2$-brane solution in type IIA string theory with compactification on one-dimensional torus of M-Theory. These configuration in string theory is impossible to analysis for AdS/CFT correspondence [1], because we must choose typical coordinates for the compact space for near horizon limit, which is the kk-wave direction, but this solution has not any typical direction for compact spaces. Thus we show the $NS5 \perp D4 \perp D4$-brane solutions from general axisymmetric four-dimensional vacuum solution with two Killing vector, e.g., Kerr metric. We also get the general rotating black string solutions, which is rotating four-dimensional black hole with extra one dimension, by the $M2 \perp M2 \perp M2$-brane solutions from the Kerr metric.

In our previous work [4], black hole solutions with flat extra dimensions in M-theory are only exist the specific configuration of intersecting M-branes. However the earlier study for the micro state of Kerr black hole [5] using the D0⊥D6 branes, which is impossible to extension to the supersymmetric black brane solution in four dimension. The supersymmetric black brane means which have the regular event horizon with finite surface area.

Adding the new charge for the solution, we apply the boost for the ordinary metric along the extra dimension. Applying T-duality for the boost solution, we find D3-brane given by four-form field D_4, which is the self-dual field strength $dD_4 = *(d\tilde{D}_4)$. There are some technical difficulties for the Hodge dual for the stationary spacetime, since the stationary black hole remains the non-diagonal component after the boost. Such a component makes problem for integration in order to get the explicit description of the dual field $\tilde{D}_4$.

We will construct the intersecting M-brane solution consistent to the intersecting rule, and avoiding the difficulties of integration we apply the final boost after the lifting up to the eleven dimension. Thus we try to find the sequence to get the M5⊥M5⊥M5-brane soltuion, which is related to the D4⊥D4⊥NS5 brane solutions in type IIA superstring theory.

2. From Kerr metric to Intersecting D-branes

Before charging up the vacuum solution, we introduce the Kerr metric, which is a stationary solution of four-dimensional vacuum Einstein equation. The Kerr metric in spherical coordinate is written in

$$ds^2 = -f\,(dt+\Omega)^2 + \Sigma^2\left(\frac{dr^2}{\Delta} + d\theta^2\right) + \frac{\Delta}{f}\sin^2\theta\, d\phi^2$$

where the metric functions are defined by

$$\Sigma^2 = r^2 + a^2\cos^2\theta\,,\ \Delta = r^2 - 2mr + a^2$$
$$\Omega = \frac{2mr}{\Sigma^2 - 2mr}a\sin^2\theta\, d\phi\,,\ f = 1 - \frac{2mr}{\Sigma^2}\,.$$

The mass of the black hole is m and a is the specific angular momentum witch bounded for $m \geq a$. In the Kerr/CFT correspondence, we take the extremal limit $m = a$ and near horizon limit $r \to r_+$, where the event horizon r_+ is determined by $\Delta = 0$. For simplicity we denote $ds^2 = -f(dt+\Omega)^2 + ds^2_{\text{base}}$ in below. The base metric $ds^2_{\text{base}} = \gamma_{ij}dx^i dx^j$ are the orthogonal three-dimensional metric written by the variables r, θ, ϕ. The metric has the two Killing vector ξ_t and ξ_ϕ.

Adding the extra six flat dimension, the metric change to $ds^2 + -f(dt+\Omega)^2 + ds^2_{\text{base}} + \sum_{i=1}^6 dz_i^2$. This metric is also a solution of ten dimensional vacuum Einstein equation $R_{\mu\nu} = 0$. To add first charge (D3-brane), we apply a sequence in below;

$$B_{\alpha_2}(z_1) \to T(z_1) \to S \to T(z_2) \to T(z_3)\,,$$

then we find a $D3$-brane solution in type IIB supergravity. We denote $B_\alpha(z_1)$ is boost for z_i direction with boost parameter α, $T(z_i)$ is T-dual for z_i direction, and S is S-dual. To add second and third charges, we take next sequence in below,

$$T(z_4) \to B_{\alpha_1}(z_1) \to T(z_1) \to S \to T(z_5) \to T(z_6) \to T(z_2) \to B(z_6) \to T(z_6) \to S \to T(z_5)\,,$$

then we find a $D2 \perp D2 \perp D2$ intersecting brane solution as

$$ds^2 = -\xi^{-1/2} f(dt + c_1c_2c_3\Omega)^2 + \xi^{1/2} ds^2_{\text{base}} + \xi^{1/2}\left[h_1^{-1}\sum_{i=1}^{2} dz_i^2 + h_2^{-1}\sum_{i=3}^{4} dz_i^2 + h_3^{-1}\sum_{i=5}^{6} dz_i^2\right]\,,$$

where $\xi = h_1h_2h_3 - \beta_t^2$, $\beta_t = s_1s_2s_3\frac{ma\cos\theta}{\Sigma^2}$. The dilaton field is $e^{-2\varphi} = \xi^{-3/2}h_1h_2h_3$ and the gauge fields are

$$B_{z_iz_j} = h_\alpha^{-1}\frac{c_\alpha}{s_\alpha}\beta_t\,,\ \tilde{A}_t = -\xi^{-1}\beta_t f\,,\ \tilde{A}_\phi = -\xi^{-1}\beta_t f\omega$$
$$\tilde{C}_{z_iz_jt} = \frac{2}{3}h_\alpha^{-1}s_\alpha c_\alpha(f-1)\,,\ \tilde{C}_{z_iz_j\phi} = \frac{2}{3}h_\alpha^{-1} f c_\alpha^{-1} s_\alpha\omega\,.$$

where the pair of indices $(i,j) = (1,2),(3,4),(5,6)$ are corresponding to $\alpha = 1,2,3$, and $\omega = c_1c_2c_3\Omega$.

For the first example of a black hole solution, we continue to apply the U-duality for the charging up the four-dimensional Kerr solutions. Next we try to apply another example related to the rotating black string, which can be described by the Kerr metric with another one extra dimension, and these solutions must possess the Gregory-Laflamme instability [6].

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2.1 Kerr solutions in String/M-theory

In order to get M5⊥M5⊥M5 brane solution, we must apply the sequence in following as

$$T(z_1) \to T(z_3) \to T(z_5) \to T(z_2) \to T(z_4) \to T(z_6)\,,$$

then we find the $D4 \perp D4 \perp D4$-brane solutions and we lift up the z^7 direction and boost for the same direction z^7, then we find the $M5 \perp M5 \perp M5$-brane with pp-wave solutions in M-theory;

$$\begin{aligned} ds^2 = {} & \Xi^{1/3}\left[\bar{h}_1^{-1}\sum_{i=1}^{2} dz_i^2 + \bar{h}_2^{-1}\sum_{i=3}^{4} dz_i^2 + \bar{h}_3^{-1}\sum_{i=5}^{6} dz_i^2\right] \\ & + \Xi^{1/3}\left[-\xi^{-1} f(dt+\omega d\phi)^2 + ds^2_{\rm base}\right] + \Xi^{-2/3}\xi\left(dz_7 + \hat{A}_t dt + \hat{A}_\phi d\phi\right)^2, \end{aligned} \tag{2.1}$$

where the conformal factor can be written by $\Xi = \bar{h}_1\bar{h}_2\bar{h}_3$ and $\bar{h}_i = \xi\hat{h}_i^{-1}$ with $\hat{h}_i = -s_i^2 f + c_i^2 g_i^{-1}$. The function g_i is determined by $g_i = 1 + (h_1h_2h_3)^{-1}h_i^{-1}s_2^{-2}\beta_t^2$. The three-form fields related to $M5$-brane are given by

$$\hat{C}_7^{(\alpha)} \equiv C_{z_iz_jz_7} = \frac{2}{3}\bar{h}_\alpha^{-1}s_\alpha^{-1}c_\alpha\beta_t\,,\ \ \hat{C}_a^{(\alpha)} \equiv \tilde{C}_{z_iz_ja} = \frac{8}{3}\hat{D}_a^{(\alpha)} + \bar{h}_\alpha^{-1}s_\alpha^{-1}c_\alpha\beta_t\hat{A}_a\,,$$

where the component of M5-brane fields and the metric components are given by

$$\begin{aligned} \hat{D}_t^{(1)} &= -\xi^{-1}h_2s_1c_2c_3\tilde{D}_t\,,\ \ \hat{D}_\phi^{(1)} = s_2c_2\left[\tilde{D}_\phi + (1-\xi^{-1}h_2c_1^2c_3^2)\tilde{D}_t\Omega_\phi\right] \\ \hat{D}_t^{(2)} &= -\xi^{-1}h_1c_1s_2c_3\tilde{D}_t\,,\ \ \hat{D}_\phi^{(2)} = s_1c_1\left[\tilde{D}_\phi + (1-\xi^{-1}h_1c_2^2c_3^2)\tilde{D}_t\Omega_\phi\right] \\ \hat{D}_t^{(3)} &= -\xi^{-1}h_3c_1c_2s_3\tilde{D}_t\,,\ \ \hat{D}_\phi^{(3)} = s_3c_3\left[\tilde{D}_\phi + (1-\xi^{-1}h_3c_1^2c_2^2)\tilde{D}_t\Omega_\phi\right] \\ \hat{A}_t &= s_1c_1s_2c_2s_3c_3\frac{ma^2\cos^2\theta}{\Sigma^4} \\ \hat{A}_\phi &= -s_1s_2s_3\left(c_1^2c_2^2c_3^2 + \frac{2r\Sigma^2h_1h_2h_3}{r^2-a^2\cos^2\theta}\right)\frac{ma^3\sin^2\theta\cos^2\theta}{\Sigma^4}\,. \end{aligned}$$

We note that Kerr metric with flat dimensions has no Chern-Simons term, but there are exist in eleven dimension after the charging up sequence, e.g., $\hat{C}_7^{(1)} \wedge d\hat{C}_t^{(2)} \wedge \hat{C}_\phi^{(3)}$ or like that combinations. The Chern-Simons terms gives the non-trivial effect of the topology for BPS black ring solutions, and this effect change the Laplace equation for the harmonic function h_i to the Poisson equation for the non-harmonic function $\bar{h}_i$. Because of non conformally flat base space, supersymmetrie breaks for ϕ direction, thus in the ordinary base space configuration we took maximally charge for three.

Finally we apply the boost for the z_7 direction, then we find non-BPS four charge solution. However if we took same special value for physical parameters, we find supersymmetric solution with conformally flat base space. To Compactify extra seven dimensions $(z_1,\cdots,z_7)$ on torus, we find the four dimensional charged solution as

$$ds^2 = -\Upsilon f(dt+\bar{\omega}d\phi)^2 + \Upsilon^{-1}ds^2_{\rm base}\,, \tag{2.2}$$

where $\bar{\omega}$ and Υ are determined by

$$\begin{aligned} \bar{\omega} &= c_4\omega + s_4(\omega\hat{A}_t - \hat{A}_\phi) \\ \Upsilon^{-2} &= \Xi\left(-\xi^{-1}fs_4^2 + \Xi^{-1}\xi(c_4+s_4\hat{A}_t)^2\right)\,, \end{aligned}$$

In the Kerr metric case, the regularity condition for the rotating axis are the same as before, thus the metric has no conical singularity at the ordinary event horizon $r_+ = m + \sqrt{m^2 - a^2}$.

In the asymptotic region ($r \to \infty$) the metric becomes flat and the ADM mass $M = m(1 + \sum_{i=1}^4 s_i^2/2)$, the conserved charge $Q = \sqrt{2}m\sum_{i=1}^4 s_i c_i/2$ and the angular momentum $J = c_1c_2c_3c_4a$ are given in the asymptotic metric form. The surface gravity change as below

$$\kappa = \frac{1}{\beta_{t+}^2(c_4 + s_4\hat{A}_{t+})}\frac{r_+^2 - a^2}{4mr_+^2}, \tag{2.3}$$

where β_{t+} and $\hat{A}_{t+}$ are defined by the substitution for $r = r_+$. The surface area of the outer event horizon

$$\begin{aligned}\mathscr{A} &= \int d\theta d\phi \left.\sqrt{\Sigma^2\left(\Upsilon^{-2}\gamma_{\phi\phi} - f\bar{\omega}^2\right)}\right|_{r=r_+} \\ &= 8\pi m r_+ c_1c_2c_3\left[c_4 - \frac{1}{2}a^3 s_4\left(\frac{\pi}{4} - \frac{a}{r_+}\arctan\frac{r_+}{a}\right)\right],\end{aligned}$$

and we can show the thermodynamics with the physical parameter as the charge and angular momentum and the temperature, but the dilaton fields does not contribute.

The extremal solution ($\kappa = 0$) is given by $m = a$, which is as the same as ordinary Kerr metric. The area surface is vanishing in the extremal limit of the ordinary Kerr metric, however the area surface in M-theoretical Kerr metric doesn't vanishing ($\mathscr{A} = 8\pi m^2 c_1c_2c_3c_4$). Since c_α is related to the charge of M-brane (or D-brane), this non-vanishing area surface gives the microstate of M-brane.

Sen gave the rotating charged black hole solution [7], which metric is given in

$$ds^2 = -h_\alpha^{-1} f\left(dt + c_\alpha^2\Omega\right)^2 + h_\alpha ds_{\text{base}}^2, \tag{2.4}$$

where $h_\alpha = -s_\alpha^2 f + c_\alpha^2$. Sen's solution is included in our solution with the parameter $c_1 = c_2 = c_\alpha$ and $c_3 = c_4 = 1$, and this case the action is changing as $\phi_1 = \phi_2$ and $\phi_3 = \phi_4 = 0$ and $\rho_\alpha = 0$ and only $\mathscr{A}_a^{(1)} = \mathscr{A}_a^{(2)}$ are exist.

3. Concluding remark

In this paper we have presented the charging up Kerr solution using the U-duality method. Black hole solutions are presented by intersecting M-brane, and we have construct the intersecting M-brane solution, which consistent to the supersymmetrical black hole solution given by our previous work [4]. The four-dimensional black hole solution is given by M5$\perp$M5$\perp$M5 brane solution with traveling wave, and the five-dimensional black string solution is given by M2$\perp$M2$\perp$M2 brane solution. Both of the solution exist the non-vanishing Chern-Simons term in M-theory, although the Chern-Simons term in the ordinary Kerr metric with additional flat extra dimensions must be vanishing. The Chern-Simons terms change the metric function given by Laplece equation to the function given by Poisson equation.

The four-dimensional solution is represented by the charged dilaton black hole. The Maxwell charge and the dilatons are coupled to each other, and the angular momentum are represented by the charges. Thus we are only possible to take the limit to vanishing the charges with the trivial dilaton, and this limit gives static black brane solution [**?**]. The black hole has the regular extremal limit to the BPS solution in four dimension with non-vanishing are surface, and the area surface are represented by the micro state of M- or D-branes.

The five-dimensional solution is fully understood as the charged dilaton Kerr black hole with flat extra one dimension with traveling wave along this extra direction. The charged-rotating black string solution with regular event horizon is given in a higher dimensional Einstein-Maxwell theory with a positive cosmological constant [9] , however we gave the another solution with regular event horizon coupled to dilaton. The representation for $M2 \perp M2 \perp M2$-brane is the same configuration as the non-BPS black ring solution given by [3], and tihs solution include the limit for $R \to \infty$ for the non-BPS two rotating charged black ring solution. The black string solution must have the Gregory-Laflamme instability [10].

Since there are kk-wave mode in the ten-dimensional metric, the D-brane configuration of this metric is suitable to apply the AdS/CFT correspondence. We will show the micro state of these solutions in the context of AdS/CFT correspondence, and we will also show the regular solutions with the specific physical parameters in subsequent paper, witch we are writing now. In the limit for the CFT, we compare the micro state of Kerr black hole by $D0 \perp D6$-brane solutions given by Horowitz et. al., [5].

By the way of this paper we only consider from the vacuum solutions, but adding the extra dimension we can extend to the Einstein manifold with the constant gauge field, which satisfy the Einstein and Maxwell equation in lower and higher dimension. In this formalism, including the one rotating black ring case, we can apply the more interesting case, especially cosmology and black hole dynamics.

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