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Magnetic monopole loops supported by a meron pair as the quark confiner

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We give a definition of gauge-invariant magnetic monopoles in Yang-Mills theory without using the Abelian projection due to 't Hooft. They automatically appear from the Wilson loop operator. This is shown by rewriting the Wilson loop operator using a non-Abelian Stokes theorem. The magnetic monopole defined in this way is a topological object of co-dimension 3, i.e., a loop in four-dimensions. We show that such magnetic loops indeed exist in four-dimensional Yang-Mills theory. In fact, we give an analytical solution representing circular magnetic monopole loops joining a pair of merons in the four-dimensional Euclidean SU(2) Yang-Mills theory. This is achieved by solving the differential equation for the adjoint color (magnetic monopole) field in the two–meron background field within the recently developed reformulation of the Yang-Mills theory. Our analytical solution corresponds to the numerical solution found by Montero and Negele on a lattice. This result strongly suggests that a meron pair is the most relevant quark confiner in the original Yang-Mills theory, as Callan, Dashen and Gross suggested long ago.

8th Conference Quark Confinement and the Hadron Spectrum
September 1-6 2008
Mainz, Germany

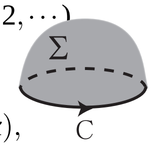
[*]This work is financially supported by Grant-in-Aid for Scientific Research (C) 18540251 from Japan Society for the Promotion of Science (JSPS).

1. Wilson loop and magnetic monopole

For a closed loop C, the Wilson loop operator for SU(2) Yang-Mills connection is defined by

$$W_C[\mathbf{A}] := \mathrm{tr}\left[\mathscr{P}\exp\left\{ig\oint_C dx^\mu \mathbf{A}_\mu(x)\right\}\right]/\mathrm{tr}(\mathbf{1}), \quad \mathbf{A}_\mu(x) = \mathbf{A}^A_\mu(x)\sigma^A/2. \tag{1.1}$$

The path-ordering $\mathscr{P}$ is removed by using the Diakonov-Petrov version [1] of a non-Abelian Stokes theorem for the Wilson loop operator: in the J representation of SU(2) ($J = 1/2, 1, 3/2, 2, \cdots$)



$$\begin{aligned} W_C[\mathbf{A}] &:= \int d\mu_\Sigma(U)\exp\left\{iJg\int_{\Sigma:\partial\Sigma=C} dS^{\mu\nu} f_{\mu\nu}\right\}, \text{ no path-ordering} \\ f_{\mu\nu}(x) &:= \partial_\mu[\mathbf{A}^A_\nu(x)\boldsymbol{n}^A(x)] - \partial_\nu[\mathbf{A}^A_\mu(x)\boldsymbol{n}^A(x)] - g^{-1}\varepsilon^{ABC}\boldsymbol{n}^A(x)\partial_\mu\boldsymbol{n}^B(x)\partial_\nu\boldsymbol{n}^C(x), \\ n^A(x)\sigma^A &:= U^\dagger(x)\sigma^3 U(x), \quad U(x)\in SU(2) \quad (A,B,C\in\{1,2,3\}), \end{aligned} \tag{1.2}$$

where $d\mu_\Sigma(U)$ is the product measure of an invariant measure on SU(2)/U(1) over Σ:

$$d\mu_\Sigma(U) := \prod_{x\in\Sigma} d\mu(U(x)), \quad d\mu(U(x)) = \frac{2J+1}{4\pi}\delta(\boldsymbol{n}^A(x)\boldsymbol{n}^A(x)-1)d^3\boldsymbol{n}(x), \tag{1.3}$$

where we have introduced a unit vector field $\boldsymbol{n}(x)$.

The geometric and topological meaning of the Wilson loop operator was given in [2]:

$$W_C[\mathscr{A}] = \quad \textstyle\int d\mu_\Sigma(U)\exp\left\{iJg(\Xi_\Sigma,k)+iJg(N_\Sigma,j)\right\}, \quad C=\partial\Sigma \tag{1.4}$$

$$k := \delta^* f = {}^*df, \quad \Xi_\Sigma := \delta^*\Theta_\Sigma\triangle^{-1} \leftarrow \quad \text{(D-3)-forms} \tag{1.5}$$

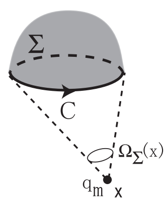
$$j := \delta f, \quad N_\Sigma := \delta\Theta_\Sigma\triangle^{-1} \leftarrow \quad \text{1-forms (D-indep.)} \tag{1.6}$$

$$\Theta^{\mu\nu}_\Sigma(x) = \textstyle\int_\Sigma d^2S^{\mu\nu}(x(\sigma))\delta^D(x-x(\sigma)), \tag{1.7}$$

where k and j are gauge invariant and conserved currents, $\delta k = 0 = \delta j$. Thus, **we do not need to use the Abelian projection proposed by 't Hooft [3] to define magnetic monopoles in Yang-Mills theory! The Wilson loop operator knows the (gauge-invariant) magnetic monopole!** Then the magnetic monopole is a topological object of co-dimension 3. In D dimensions,
D=3: 0-dimensional point defect → magnetic monopole of Wu-Yang type
D=4: 1-dimensional line defect → magnetic monopole loop (closed loop)

For $D = 3$,

$$k(x) = \frac{1}{2}\varepsilon^{jk\ell}\partial_\ell f_{jk}(x) = \rho_m(x) \tag{1.8}$$



denotes the magnetic charge density at x, and

$$\Xi_\Sigma(x) = \Omega_\Sigma(x)/(4\pi) \tag{1.9}$$

agrees with the (normalized) solid angle at the point x subtended by the surface Σ bounding the Wilson loop C. Then the magnetic part $W^m_{\mathscr{A}}$ is written as

$$W^m_{\mathscr{A}} := \exp\left\{iJg(\Xi_\Sigma,k)\right\} = \exp\left\{iJg\int d^3x\rho_m(x)\frac{\Omega_\Sigma(x)}{4\pi}\right\}. \tag{1.10}$$

The magnetic charge q_m obeys the Dirac-like quantization condition:

$$q_m := \int d^3x\rho_m(x) = 4\pi g^{-1}n \quad (n\in\mathbb{Z}). \tag{1.11}$$

The proof follows from a fact that the non-Abelian Stokes theorem does not depend on the surface Σ chosen for spanning the surface bounded by the loop C. See [2].

For an ensemble of point-like magnetic charges: $k(x) = \sum_{a=1}^{n} q_m^a \delta^{(3)}(x - z_a)$, we have

$$W_{\mathscr{A}}^m = \exp\left\{ iJ \frac{g}{4\pi} \sum_{a=1}^{n} q_m^a \Omega_\Sigma(z_a) \right\} = \exp\left\{ iJ \sum_{a=1}^{n} n_a \Omega_\Sigma(z_a) \right\}, \quad n_a \in \mathbb{Z}. \tag{1.12}$$

The magnetic monopoles in the neighborhood of the Wilson surface Σ ($\Omega_\Sigma(z_a) = \pm 2\pi$) contribute to the Wilson loop

$$W_{\mathscr{A}}^m = \prod_{a=1}^{n} \exp(\pm i 2\pi J n_a) = \begin{cases} \prod_{a=1}^{n} (-1)^{n_a} & (J = 1/2, 3/2, \cdots) \\ = 1 & (J = 1, 2, \cdots) \end{cases}. \tag{1.13}$$

This enables us to explain the N-ality dependence of the asymptotic string tension. See, [4].

For $D = 4$, $\Omega_\Sigma^\mu(x)$ is the $D = 4$ solid angle and the magnetic part reads

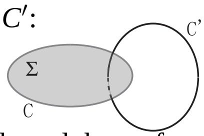
$$W_{\mathscr{A}}^m = \exp\left\{ iJg \int d^4x \Omega_\Sigma^\mu(x) k^\mu(x) \right\}. \tag{1.14}$$

Suppose the existence of an ensemble of magnetic monopole loops C_a' in $D = 4$ Euclidean space, $k^\mu(x) = \sum_{a=1}^{n} q_m^a \oint_{C_a'} dy_a^\mu \delta^{(4)}(x - x_a)$, $\quad q_m^a = 4\pi g^{-1} n_a$. Then the Wilson loop operator reads

$$W_{\mathscr{A}}^m = \exp\left\{ iJg \sum_{a=1}^{n} q_m^a L(\Sigma, C_a') \right\} = \exp\left\{ 4\pi J i \sum_{a=1}^{n} n_a L(\Sigma, C_a') \right\}, \quad n_a \in \mathbb{Z}, \tag{1.15}$$

where $L(\Sigma, C')$ is the linking number between the surface Σ and the curve C':

$$L(\Sigma, C') := \oint_{C'} dy^\mu(\tau) \Xi_\Sigma^\mu(y(\tau)). \tag{1.16}$$

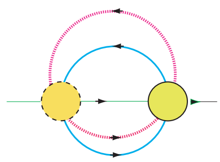


Here the curve C' is identified with the trajectory k of a magnetic monopole and the surface Σ with the world sheet of a hadron (meson) string for a quark-antiquark pair.

The Wilson loop operator is a probe of the gauge-invariant magnetic monopole defined in our formulation. Thus, calculating the Wilson loop average reduces to the summation over the magnetic monopole charge (D=3) or current (D=4) with a geometric factor, the solid angle (D=3) or linking number (D=4).

2. Main results (Magnetic loops indeed exist in YM$_4$)

We can show that **the gauge-invariant magnetic loop (assumed in the above) indeed exists in SU(2) Yang-Mills theory in $D = 4$ Euclidean space**: we give a first* (exact) analytical solution representing **circular magnetic monopole loops joining two merons** [5].[1]



Our method reproduces also the previous results based on MAG (MCG) and LAG:
(i) The magnetic straight line can be obtained in the one-instanton or one-meron background. [6, 7]
(ii) The magnetic closed loop can NOT be obtained in the one-instanton background. [8, 9]

[1] There is an exception: Bruckmann & Hansen, hep-th/0305012, Ann.Phys.**308**, 201 (2003). However, it has $Q_P = \infty$

3. Reformulating Yang-Mills theory in terms of new variables

SU(2) Yang-Mills theory written in terms of $\mathbf{A}_\mu^A(x)$ $(A=1,2,3)$	$\Longleftrightarrow$ change of variables	A reformulated Yang-Mills theory written in terms of new variables: $\boldsymbol{n}^A(x), c_\mu(x), \mathbf{X}_\mu^A(x)$ $(A=1,2,3)$

We introduce a "color field" $\mathbf{n}(x)$ of unit length with three components

$$\mathbf{n}(x) = (n_1(x), n_2(x), n_3(x)), \quad \mathbf{n}(x)\cdot\mathbf{n}(x) = n_A(x)n_A(x) = 1 \tag{3.1}$$

The color field $\mathbf{n}(x)$ is identified with $\mathbf{n}(x)$ in (1.2). New variables $\boldsymbol{n}^A(x), c_\mu(x), \mathbf{X}_\mu^A(x)$ should be given as functionals of the original $\mathbf{A}_\mu^A(x)$. The off-shell Cho-Faddeev-Niemi-Shabanov decomposition [10] is reinterpreted as change of variables from $\mathbf{A}_\mu^A(x)$ to $\boldsymbol{n}^A(x), c_\mu(x), \mathbf{X}_\mu^A(x)$ via the reduction of an enlarged gauge symmetry. See [11, 12]. Expected role of the color field: 1) The color field $\boldsymbol{n}(x)$ plays the role of recovering color symmetry which will be lost in the conventional approach, e.g., in the MA gauge. 2) The color field $\boldsymbol{n}(x)$ carries topological defects responsible for non-perturbative phenomena, e.g., quark confinement.

4. Bridge between $\mathbf{A}_\mu(x)$ and $\mathbf{n}(x)$

For a given Yang-Mills field $\mathbf{A}_\mu(x)$, the color field $\mathbf{n}(x)$ is obtained by solving the reduction differential equation (RDE): [12]

$$\mathbf{n}(x) \times D_\mu[\mathbf{A}]D_\mu[\mathbf{A}]\mathbf{n}(x) = \mathbf{0}. \tag{4.1}$$

For a given SU(2) Yang-Mills field $\mathbf{A}_\mu(x) = \mathbf{A}_\mu^A(x)\frac{\sigma_A}{2}$, look for unit vector fields $\mathbf{n}(x)$ such that $-D_\mu[\mathbf{A}]D_\mu[\mathbf{A}]\mathbf{n}(x)$ is proportional to $\mathbf{n}(x)$: an eigenvalue-like form:

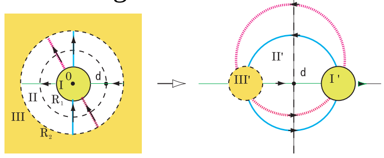
$$-D_\mu[\mathbf{A}]D_\mu[\mathbf{A}]\mathbf{n}(x) = \lambda(x)\mathbf{n}(x) \quad (\lambda(x) \geq 0). \tag{4.2}$$

The solution is not unique. We choose the solution giving the smallest value of the reduction functional $F_{\rm rc}$ which agrees with the integral of the scalar function $\lambda(x)$ over $\mathbb{R}^D$:

$$\begin{aligned} F_{\rm rc} &= \int d^Dx \frac{1}{2}(D_\mu[\mathbf{A}]\mathbf{n}(x))\cdot(D_\mu[\mathbf{A}]\mathbf{n}(x)) = \int d^Dx \frac{1}{2}\mathbf{n}(x)\cdot(-D_\mu[\mathbf{A}]D_\mu[\mathbf{A}]\mathbf{n}(x)) \\ \Longrightarrow F_{\rm rc}^* &= \int d^Dx \frac{1}{2}\mathbf{n}(x)\cdot\lambda(x)\mathbf{n}(x) = \int d^Dx \frac{1}{2}\lambda(x). \end{aligned} \tag{4.3}$$

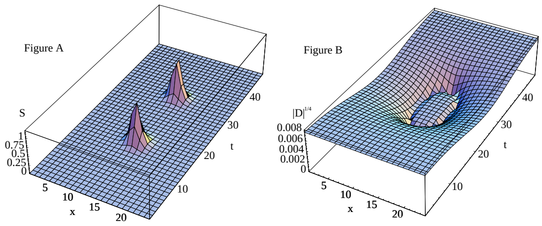
5. Conclusion and discussion

For given one-instanton and two-meron background $\mathbf{A}_\mu(x)$, we have solved the RDE for the color field $\mathbf{n}(x)$ [12]. In the four-dimensional Euclidean SU(2) Yang-Mills theory, we have given a first analytical solution representing circular magnetic monopole loops k_μ which go through a pair of merons (with a unit topological charge) with non-trivial linking with the Wilson surface Σ.

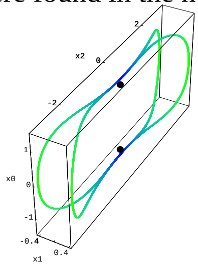


This is achieved by solving the reduction differential equation for the adjoint color (magnetic monopole) field in the two–meron background field using the recently developed reformulation of the Yang-Mills theory [11, 12] and a non-Abelian Stokes theorem [2].

Our analytical solution corresponds to a numerical solution found on a lattice by Montero and Negele [13].



We have not yet obtained the analytic solution representing magnetic loops connecting 2-instantons, which were found in the numerical way by Reinhardt & Tok [7].



Thus we are lead to a conjecture: A meron pair is the most relevant quark confiner in the original Yang-Mills theory, as Callan, Dashen and Gross suggested long ago [14]. This means a duality relation:

$$\text{dual Yang-Mills: magnetic monopole loops} \Longleftrightarrow \text{original Yang-Mills: merons}$$

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