Faddeev equations: a view of baryon properties

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We present a calculation of the three-quark core contribution to the mass of the Δ-resonance in a Poincaré-covariant Faddeev framework. A consistent setup for the dressed-quark propagator, the quark-quark and quark-'diquark' interactions is used, where all the ingredients are solutions of their respective Dyson-Schwinger or Bethe-Salpeter equations in rainbow-ladder truncation. We discuss the evolution of the Δ mass with the current-quark mass and compare to the previously obtained mass of the nucleon.

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1. Introduction

Present advanced experimental facilities offer a precise test of the composite structure of the nucleon. While its basic properties are satisfactorily described within various approaches (e.g. quark models, chiral effective field theory methods, lattice QCD), a correct understanding of the rich structure of the nucleon shall be extracted in terms of QCD's quark and gluon degrees of freedom. Dynamical chiral symmetry breaking and confinement, two prominent phenomena tightly connected with the formation of bound states, can be addressed only within a non-perturbative approach to QCD. In this respect Dyson-Schwinger equations (DSEs) represent a continuum tool to access these aspects (for recent reviews see [1, 2]). They form an infinite set of coupled integral equations for the Green functions. Bound states (mesons and baryons) correspond to poles in the respective n-point Green function. The residue defines the bound state amplitude. It is these amplitudes that contain all the necessary information to achieve a correct theoretical understanding of the hadron properties. Hadron bound-state equations provide these amplitudes: for example, mesons ($q\bar{q}$ bound-states) have been successfully explored in Bethe-Salpeter equation (BSE) approaches, while baryons (poles in a 6-quark Green function) are studied by means of a covariant equation of the Faddeev type (a quark-'diquark' bound-state BSE describing three-particle kernels). Such an approach to baryons was motivated by the observation that the attractive nature of quark-antiquark correlations in a color-singlet meson is also attractive for $\bar{3}_C$ quark-quark correlations within a color-singlet baryon [3, 4]. Presently, the baryon analysis has to some extent reached the level of sophistication in the mesonic sector by fully solving the DS (quark propagator) and BS (diquark amplitudes) equations in a simple consistent setup: a rainbow-ladder (RL) truncation which correctly implements chiral symmetry and its spontaneous breaking. The RL truncated DSEs have been widely used with the aim to reproduce hadron observables: masses and electromagnetic properties of mesons and baryons, e.g. [5, 6, 7, 8, 9, 10]. However, evidence has been provided that important corrections to RL truncation are mesonic effects in the quark and quark-gluon-vertex DSEs, see e.g. [11], expected to contribute a significant amount of attraction in the chiral limit while decreasing in the heavy-quark limit. Non-resonant terms beyond RL are assumed to provide further attraction in the pseudoscalar and vector mesons sector [12, 13]. Moreover, contributions associated with an infrared-divergent quark-gluon vertex [14, 15] are expected to be small for most hadronic observables. Therefore in the present study we employ the recent prescription of [16] and identify RL results with the hadron quark-core contributions (for applications to the N mass and form factors and the Δ mass, see [16, 17, 18]).

2. A Poincaré covariant approach to baryons

In a quark-'diquark' scenario, a color-singlet baryon emerges as a bound state of a color-triplet quark and a color-antitriplet diquark correlation. The baryon amplitude and mass are obtained as exact solutions of the baryon quark-diquark BSE (diagramatically represented in Fig. 1) if all its ingredients are fully specified: the dressed-quark propagator (single line), the diquark propagator (double line), and the diquark amplitude Γ^ν and its charge-conjugate $\bar{\Gamma}^\nu$ which appear in the quark-diquark kernel. The renormalized dressed-quark propagator is obtained from its DSE. Its kernel includes the dressed gluon propagator, one bare and one dressed quark-gluon vertex. Upon
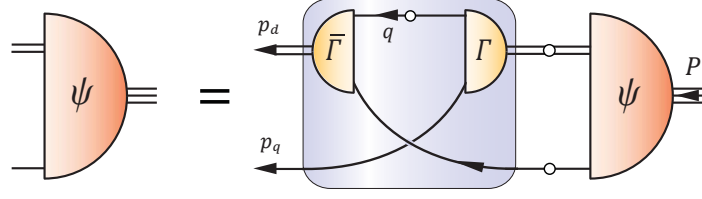
Figure 1: The quark-diquark BSE . Ψ is the covariant amplitude for a baryon of total momentum P. p_q and p_d label the dressed-quark and diquark momenta, respectively. The binding mechanism in the baryons is realized via an iterated exchange of roles between the bystander quark and any of the quarks contained in the diquark.

operating a RL truncation on this kernel, the dressed quark-gluon vertex retains only the vector part $\sim \gamma_\mu$ out of the twelve possible tensors in its general structure, while the dressings of the gluon propagator and of the quark-gluon vertex are absorbed into an effective coupling $\alpha(k^2)$. In this setup the axial-vector Ward Takahashi identity is preserved: the pion remains massless in the chiral limit, while at finite quark masses one obtains a generalized Gell-Mann-Oakes-Renner formula for pseudoscalar mesons [19, 20]. A convenient choice for the effective coupling is given by [21] :

$$\alpha(k^2,\hat{m},\omega) = c(\hat{m})\,\alpha_{IR}(k^2,\omega) + \alpha_{UV}(k^2), \tag{2.1}$$

where k^2 is the gluon momentum, $\hat{m}$ is the renormalization-point independent current-quark mass. The second term $\alpha_{UV}(k^2)$ decreases logarithmically for large gluon momenta and reproduces QCD's perturbative running coupling. The first term of the interaction is the seed of the model and characterized by a width parameter ω. It accounts for the non-perturbative enhancement at small and intermediate gluon momenta which provides the necessary strength to allow for dynamical chiral symmetry breaking and the dynamical generation of a constituent-quark mass scale. The method of [16] relies upon a parametrization of the BSE solutions of the core $m_\rho(m_\pi)$:

$$x_\rho^2 = 1 + x_\pi^4/(0.6 + x_\pi^2), \ x_\rho = m_\rho/m_\rho^0, \ x_\pi = m_\pi/m_\rho^0, \tag{2.2}$$

with the chiral-limit value $m_\rho^0 = 0.99$ GeV. The above parametrization imposes a dependence of the coupling strength $c(\hat{m})$ of Eq. (2.1) with the current-quark mass determined in [16].

In the bound-state equation of Fig. 1 diquarks were implemented as pseudoparticle poles in the quark-quark scattering matrix. Their on-shell amplitudes Γ^ν are obtained from the corresponding scalar and axial-vector diquark BSE, whose kernel is (consistently) RL truncated, while their off-shell behavior is parametrized [17] (as imposed by the UV behavior of the two-quark scattering matrix). The nucleon is well described by scalar and axial-vector diquark correlations, while only the axial-vector diquarks contribute to the spin$-\frac{3}{2}$ and isospin$-\frac{3}{2}$ flavor symmetric Δ.

3. Nucleon and Δ masses

To compute the amplitudes and the masses of the Δ and N baryons numerically, the on-shell quark-diquark amplitudes Ψ of Fig. 1 are decomposed into an orthogonal set of Dirac covariants

(for details see [17, 22]). The corresponding partial-wave decomposition in terms of eigenfunctions of the quark-diquark total spin and orbital angular momentum in the respective baryon's rest frame reveals an admixture of p and $d-$waves in the nucleon amplitude and p, d and $f-$wave components in the Δ amplitude, which can be linked to the deviation from sphericity of both nucleon and delta.

The nucleon and Δ masses emerge upon solving the quark DSE and the diquark and quark-diquark BSE. We depict the results in Fig. 2. The solid curve for m_ρ represents the chosen core dependence of Eq. (2.2) and fixes the parameters in our interaction. The bands represent the results for M_N and M_Δ and show the sensitivity on the width parameter ω. At the physical pion mass we obtain $M_\Delta = 1.73(5)$ GeV [18] and $M_N = 1.26(2)$ GeV [17]. We observe that the core model uniformly overestimates these values by $\sim 30\%$ in the chiral limit. At larger quark masses the deviation from the lattice data decreases, which is in accordance with the core model of Eq. (2.2): beyond-RL corrections to most hadronic observables become negligible in the heavy-quark limit. It is evident from Fig. 2 that the experimental values (filled circles) of the ϕ meson and the Ω baryon are overestimated by the same percentage, namely $\sim 8\%$ (the pseudoscalar value $m_{s\bar{s}} = 0.69$ GeV corresponds to the BSE solution at a strange quark mass $\hat{m}_s = 150$ MeV [23]).

Hadron Dyson-Schwinger studies and chiral analyses of lattice results (e.g. [24, 25]) predict pseudoscalar meson corrections of about $200 - 400$ MeV to the nucleon mass and of a smaller or similar size to the delta mass. Our value for M_N in the chiral region is consistent with a pseudoscalar-meson dressing as dominant correction to the quark-diquark core, but the result for M_Δ is not. The larger corrections needed in this case could be provided via an improved treatment of the quark-core: inserting further diquark channels, employing the full qq scattering kernel instead of the diquark pole or including irreducible 3-body interactions. Also the sizeable ω dependence of M_Δ could suggest that taking into account only an axial-vector diquark might not be sufficient to describe the delta baryon, and further diquark components could diminish the Δ 'core' mass. We observe that although the scalar and axial-vector diquark masses display a significant dependence with ω (second panel of Fig. 2), they sensitively cancel when constituting the nucleon mass. Finally, a comparison of the splitting $M_\Delta - M_N$ with the diquark mass splitting $M_{\text{av}} - M_{\text{sc}}$ (third panel Fig. 2) shows that they both decrease with increasing current-quark mass although there is no direct relationship between the two quantities [18].

We conclude that in the current Poincaré covariant setup, the RL truncation of the QCD DSEs delivers inflated 'core' contributions. A full understanding of the nucleon structure ultimately relies upon implementing chiral corrections to RL. Their overall attractive effect is expected to shift the core results to the experimental values.

Acknowledgements

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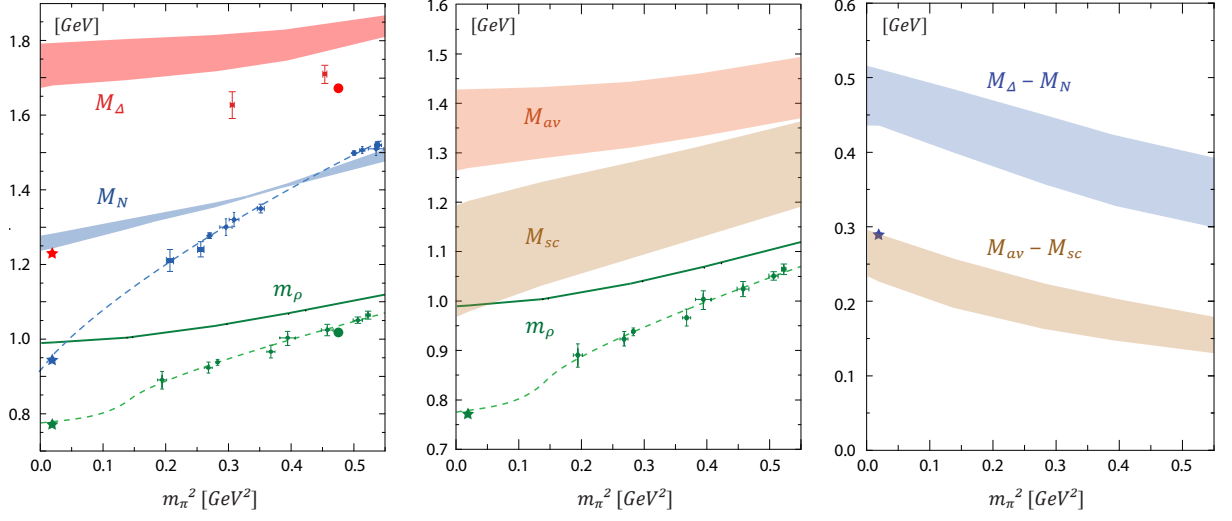[3] R. T. Cahill, C. D. Roberts, J. Praschifka, Phys. Rev. D 36 (1987) 2804.

Figure 2: Evolution of N and Δ masses (*left panel*), evolution of m_ρ, scalar and axial-vector diquark masses (*center panel*), and the mass splittings $M_\Delta - M_N$ and $M_{av} - M_{sc}$ (*right panel*) vs. m_π^2 as obtained from the pseudoscalar $q\bar{q}$ BSE. We present a selection of lattice data and their chiral extrapolations (*dashed lines*) for m_ρ [26, 27], M_N [28, 29, 30] and M_Δ [31]. Experimental values: stars for the N and Δ, filled circles for the $\phi(1020)$ and $\Omega(1672)$.

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