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Temperature effects in the Nambu–Jona-Lasinio model with six and eight quark interactions

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The extension of the Nambu–Jona-Lasinio Model (NJL) [1] with 't Hoof determinant [2, 3, 4] to include eight quark interactions [5] has been proposed as a solution for the vacuum stability problems which have been shown to afflict this model [6]. While the mesonic spectra, which can now be built on the spontaneously broken vacuum induced by the 't Hooft interaction strength, as opposed to the commonly considered case driven by the four-quark coupling, remains relatively unchanged (except for the sigma meson mass, which decreases), the study of temperature effects [7, 8] ($SU_f(3)$ massless case and realistic massive cases) has shown interesting new features: chiral symmetry restoration can occur through a rapid crossover to the unbroken phase with a slope and critical temperature (T_c) which are regulated by the OZI violating part of the added eight quark interactions. As a consequence, T_c can be considerably reduced as compared to the standard approach, in accordance with recent lattice calculations [9]. A first order transition behavior is also a possible solution within this approach.

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1. Introduction

Dynamical chiral symmetry breaking is expected to play a major role in hadron dynamics in the non-perturbative regime of QCD. The NJL Model ([1], for reviews see e.g. [10]), shares the global symmetries with QCD and incorporates by construction a mechanism for chiral symmetry breaking and has thus proven to be a very useful tool for the study of low energy hadron phenomenology. Writing this model in terms of quark degrees of freedom we can regard the quark condensates as order parameters and the light pseudo-scalar mesons as quasi-Goldstone bosons. In the extended version of the model, the inclusion of a $2N_f$ 't Hooft determinant interaction (where N_f is the number of considered flavors) eliminates the unwanted $U_A(1)$ symmetry [2, 3, 4]. The inclusion of higher order multiquark interactions can be motivated in analogy with the instanton gas model where an infinite tower of interactions appears beyond the zero mode approximation [11]. One has to assume however that some hierarchy, which could possibly stem from hierarchies in gluon field correlators, induces a similar hierarchy in multi-quark interactions (which does not happen in the instanton gas model). Here we present results obtained with an extended version of the model which includes 6 and 8 quark interactions in the light quark sector (u, d and s). The terms considered constitute the most general spin zero chirally symmetric non-derivative combinations. The two flavor NJL model with inclusion of 8q interactions has been considered in [12].

The model lagrangian can be decomposed in the following terms: the free Dirac Lagrangian $\mathscr{L}_D$, the NJL 4-quark interaction ($\mathscr{L}_{NJL}$) which, if strong enough, results in a double well potential with the resulting non-vanishing quark condensates in the physical vacuum, the 6-quark flavor determinant 't Hooft term ($\mathscr{L}_H$) and an 8 quark interaction $\mathscr{L}_{8q} = \mathscr{L}_{8q}^{(1)} + \mathscr{L}_{8q}^{(2)}$ (where $\mathscr{L}_{8q}^{(1)}$ violates the Okubo-Zweig-Iizuka (OZI) rule). Chiral symmetry is explicitly broken by the inclusion of the current quark masses (here we will consider the chiral symmetric case $m_u = m_d = m_s = 0$ and a more realistic $m_u = m_d \neq m_s \neq 0$). The Lagrangian is written as (q are the quark fields, λ_a are Gell-Mann flavor matrices, $P_{L,R}$ are chiral projectors and det is a determinant in flavor space):

$$\mathscr{L}_{eff} = \mathscr{L}_D + \mathscr{L}_{NJL} + \mathscr{L}_H + \mathscr{L}_{8q}$$

$$\mathscr{L}_D = \bar{q}\left(\imath\gamma^\mu\partial_\mu - \hat{m}\right)q \qquad \mathscr{L}_{8q} = \mathscr{L}_{8q}^{(1)} + \mathscr{L}_{8q}^{(2)}$$

$$\mathscr{L}_{NJL} = \frac{G}{2}\left[(\bar{q}\lambda_a q)^2 + (\bar{q}\imath\gamma 5\lambda_a q)^2\right] \qquad \mathscr{L}_{8q}^{(1)} = 8g_1\left[(\bar{q}_i P_R q_m)(\bar{q}_m P_L q_i)\right]^2$$

$$\mathscr{L}_H = \kappa\left(\det\bar{q}P_L q + \det\bar{q}P_R q\right) \qquad \mathscr{L}_{8q}^{(2)} = 16g_2\left(\bar{q}_i P_R q_m\right)\left(\bar{q}_m P_L q_j\right)\left(\bar{q}_j P_R q_K\right)\left(\bar{q}_K P_L q_i\right)$$

2. Functional Integral Bosonization

For $N_f > 2$ the bosonization of the vacuum-to-vacuum amplitude can be done with the aid of a functional identity which introduces two sets of flavour nonet bosonic fields: $\{\sigma_a, \phi_a\}$, related to the physical scalar and pseudo-scalar mesons, and $\{s_a, p_a\}$ which are auxiliary fields [4]. Through this procedure we can separate the functional integral in two parts:

$$Z = \int \prod_a \mathscr{D}\sigma_a \mathscr{D}\phi_a \mathscr{D}\bar{q}\mathscr{D}q e^{\imath\int \mathrm{d}x^4 \mathscr{L}_q(\bar{q},q,\sigma,\phi)} \int \prod_a \mathscr{D}s_a \mathscr{D}p_a e^{\imath\int \mathrm{d}x^4 \mathscr{L}_r(\sigma,\phi,s_a,p_a)} \tag{2.1}$$

where $\mathscr{L}_q$ contains all the fermionic dependence and is now quadratic in the quark fields. It can be evaluated in a symmetry preserving heat kernel scheme (generalized to deal with non-degenerate quark current masses [13]). The unphysical auxiliary fields can be integrated out of $\mathscr{L}_r$ using the stationary phase approximation (SPA) which for the present results was done to leading order. Introducing an expansion of the auxiliary variables in powers of the external fields, in the stationary phase condition, we obtain recursion relations that completely specify the expansion. The lowest order terms of the stationary phase conditions are independent of the fields and given by:

$$\begin{cases} Gh_u + \Delta_u + \frac{\kappa}{16} h_u h_s + \frac{g_1}{4} h_u \left(2h_u^2 + h_s^2\right) + \frac{g_2}{2} h_u^3 = 0 \\ Gh_s + \Delta_s + \frac{\kappa}{16} h_u^2 + \frac{g_1}{4} h_s \left(2h_u^2 + h_s^2\right) + \frac{g_2}{2} h_s^3 = 0 \end{cases} \tag{2.2}$$

in the isotopic limit $(m_u = m_d \neq m_s)$; here quark condensates are expressed as $< \bar{q}q >_i = h_i/2$, $(i = u, s)$ and $\Delta_i = M_i - m_i$, represent the shifts of the scalar fields σ_i, obtained by demanding that their expectation values in the vacuum are zero. Equations 2.2 must be solved self-consistently with the gap equations 2.3

$$\begin{cases} h_u + \frac{N_c}{6\pi^2} M_u \left(3I_0 - \left(M_u^2 - M_s^2\right) I_1\right) = 0 \\ h_s + \frac{N_c}{6\pi^2} M_s \left(3I_0 + 2\left(M_u^2 - M_s^2\right) I_1\right) = 0 \end{cases} \tag{2.3}$$

through which the quarks acquire their dynamical masses M_i. Here I_0, I_1 denote quark loop integrals.

3. Results

It has been shown that to insure global stability of the vacuum the following conditions must be fulfilled [14]: $g_1 > 0$, $g_1 + 3g_2 > 0$ and $G > \frac{1}{g_1}\left(\frac{\kappa}{16}\right)^2$. Thus the inclusion of the 't Hooft term without the 8-quark interaction leads to global instability (metastability in mean field approximation (MF))[6] as can be seen in the analysis of the effective potential as a function of the quark condensate Fig. 1 (a, c in SPA, b in MF).

The model parameters (current quark masses, couplings and cutoff used in the regularization of the quark loop) are fixed using chosen scalar and pseudo-scalar mesons masses and weak decay constants (f_π and f_K) [14]. Despite the dramatic effect on vacuum stability the meson spectra at $T = 0$ are left basically unchanged upon the addition of the eight quark interactions due to the interplay of the parameters (mainly a decrease in G when g_1 increases, accompanied by a decrease in the σ meson mass). Contrarily to the standard case where the role the 6q term is restricted to a perturbation of the vacuum, with the inclusion of the eight quark term the mesonic spectra can be built on a 't Hooft 6q-interaction induced vacuum.

At finite temperature additional effects associated with the eight quark terms become evident. The effective potential as a function of the condensate, for example, changes its curvature at the origin raising the potential with increasing temperature (see Fig. 2). For an effective potential with curvature $\tau < 1$ at $T = 0$ with a global minimum induced by the 't Hooft term coexisting with the local minimum at the origin, a significant decrease of T_c can be achieved. In the case of realistic quark masses the minimum is shifted away from the origin and depending on the choice of parameters we can get one or three sets of solutions $\{M^{(i)}, M_s^{(i)}\}$ which correspond to the extrema of the thermodynamic potential as can be seen in Fig. 3. For an OZI violating interaction strength, g_1, greater than a critical value the three solutions are physical and when two of them meet there

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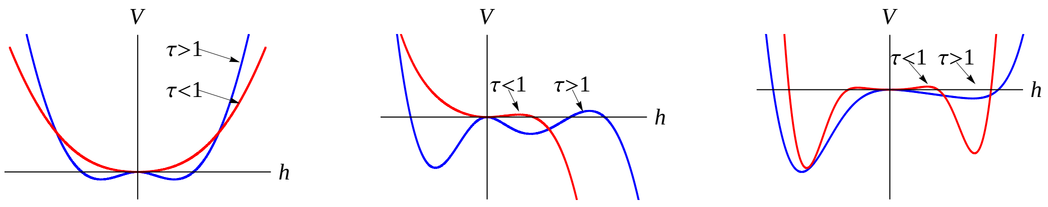
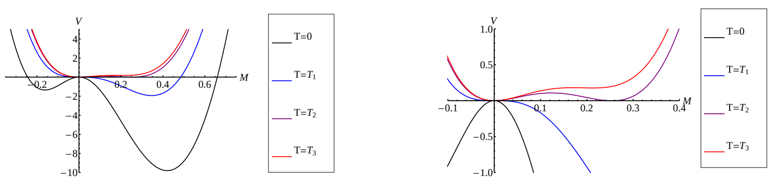


(a) 4 q (b) 4 q + 6 q (c) 4 q + 6 q + 8 q

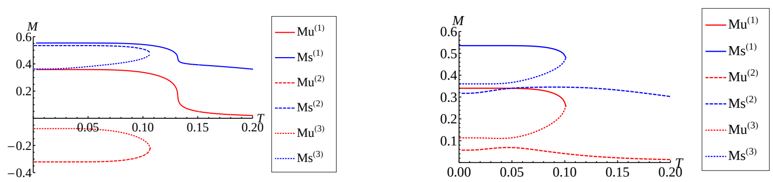
Figure 1: Shapes of the effective potential,V , at $T = 0$ for the cases with 4q, 4q + 6 q and 4q + 6q + 8q interactions as functions of the quark condensate, $\langle \bar{q}q \rangle = h/2$, in the $SU(3)$ chiral limit for different curvatures $\tau = NcG\frac{\Lambda^2}{2\pi^2}$ at the origin.



(a) (b)

Figure 2: a) Effective potential, V, (in $(10 \text{ GeV})^{-4}$) for several temperatures in the $SU(3)$ chiral limit shown as a function of the quark mass, M, (in GeV). The three temperatures $T_1 < T_2 < T_3$ correspond to: appearance of a saddle point at the origin and after that a local mininum (T_1), degenerate two minima (T_2), and disappearance of the minimum corresponding to spontaneously broken symmetry (T_3). b) Close-up view of a).

is a a first order transition to the lower one; for lower values only one of the solution branches is physical and there is a rapid crossover when the other two cease to exist. Recent developments on this work have shown that the critical end point position can also be regulated by g_1[15]. The temperature dependence of the meson mass spectra reflects the character of this transition as can be seen in Fig. 4 for the rapid crossover case [8].



(a) (b)

Figure 3: Quark mass solution pairs, $M^{(i)}$, $i = 1,2,3$, (in GeV) as a function of the temperature, T (in GeV), for details see [8]: crossover case (a) and the first order transition case (b).

4. Summary

The introduction of the eight quark interactions needed to ensure vacuum stability lowers the

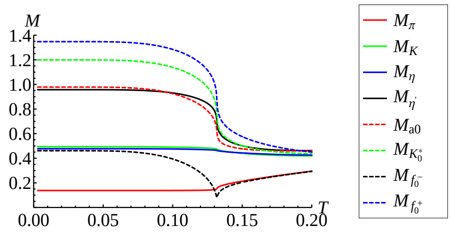


Figure 4: Temperature dependence of the meson masses in the crossover case (all in units GeV).

critical temperature for chiral transition while leaving the meson spectra at $T = 0$ unaffected and introduces the possibility of first order or crossover transitions. They act as a chiral thermometer as it is their strength that determines the type of transition, its slope and the temperature at which it occurs.

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