

The chiral partner of the nucleon in the mirror assignment with global symmetry

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We calculate the pion-nucleon scattering lengths $a_0^{(\pm)}$ and the mass parameter m_0 , which describes the nucleon mass in the chiral limit, at tree-level in the framework of a globally symmetric linear sigma model with parity-doubled nucleons. When recent lattice results [1] are used, we obtain $m_0 \simeq 300 - 600$ MeV. While $a_0^{(-)}$ is in fair agreement with experimental data, $a_0^{(+)}$ is too small because of the employed large scalar meson mass. This indicates the need to account for additional scalar degrees of freedom.

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1. Introduction

Effective models which embody chiral symmetry and its spontaneous breakdown at low temperatures and densities are widely used to understand the properties of light hadrons. Viable candidates obey a well-defined set of low-energy theorems [2, 3] but they still differ in some crucial and interesting aspects such as the mass generation of the nucleon and the behavior at non-zero T and μ .

Here we concentrate on a linear sigma model with $U(2)_R \times U(2)_L$ symmetry and parity-doubled nucleons. The mesonic sector involves scalar, pseudoscalar, vector, and axial-vector mesons. In the baryonic sector, besides the usual nucleon doublet field N , a second baryon doublet N^* with $J^P = \frac{1}{2}^-$ is included. As first discussed in Ref. [4] and extensively analyzed in Ref. [5], in the so-called mirror assignment the nucleon fields N and N^* have a mass $m_0 \neq 0$ in the chirally symmetric phase. The chiral condensate φ increases the masses and generates a mass splitting of N and N^* , but is no longer solely responsible for generating the masses. Such a theoretical set-up has been used in Ref. [6] to study the properties of cold and dense nuclear matter. The experimental assignment for the chiral partner of the nucleon is still controversial: the well-identified resonances $N^*(1535)$ and $N^*(1650)$ are two candidates with the right quantum numbers listed in the PDG [7], but we shall also investigate the possibility of a very broad and not yet discovered resonance centered at about 1.2 GeV, which has been proposed in Ref. [6].

The aim of the present work is the development of an effective model which embodies the chiral partner of the nucleon in the mirror assignment within the context of global chiral symmetry involving also vector and axial-vector mesons. In this way more terms appear than those originally proposed in Ref. [4]. Moreover, we restrict our study to operators up to fourth order (thus not including Weinberg-Tomozawa interaction terms). The axial coupling constant of the nucleon can be correctly described. Using recent information about the axial coupling of the partner [1] and experimental knowledge about its decay width [7] we can evaluate the mass parameter m_0 which describes the nucleon mass in the chiral limit. Then, we further evaluate pion-nucleon scattering lengths and we compare them with the experimental values [8].

2. The model and its implications

The scalar and pseudoscalar fields are included in the matrix $\Phi = (\sigma + i\eta)t^0 + (\vec{a}_0 + i\vec{\pi}) \cdot \vec{t}$ and the (axial-)vector fields are represented by the matrices $R^\mu = (\omega^\mu - f_1^\mu)t^0 + (\vec{\rho}^\mu - \vec{a}_1^\mu) \cdot \vec{t}$ and $L^\mu = (\omega^\mu + f_1^\mu)t^0 + (\vec{\rho}^\mu + \vec{a}_1^\mu) \cdot \vec{t}$ ($\vec{t} = \frac{1}{2}\vec{\tau}$, where $\vec{\tau}$ are the Pauli matrices and $t^0 = \frac{1}{2}1_2$). The corresponding Lagrangian describing only mesons reads

$$\begin{aligned} \mathcal{L}_{mes} = & \text{Tr} \left[(D_\mu \Phi)^\dagger (D^\mu \Phi) - m^2 \Phi^\dagger \Phi - \lambda_2 (\Phi^\dagger \Phi)^2 \right] - \lambda_1 (\text{Tr}[\Phi^\dagger \Phi])^2 + c(\det \Phi^\dagger + \det \Phi) \\ & + \text{Tr}[H(\Phi^\dagger + \Phi)] - \frac{1}{4} \text{Tr} [(L^{\mu\nu})^2 + (R^{\mu\nu})^2] + \frac{m_1^2}{2} \text{Tr} [(L^\mu)^2 + (R^\mu)^2] \\ & + h_2 \text{Tr}(\Phi^\dagger L_\mu L^\mu \Phi + \Phi R_\mu R^\mu \Phi^\dagger) + h_3 \text{Tr}(\Phi R_\mu \Phi^\dagger L^\mu) + \mathcal{L}_{three} + \mathcal{L}_{four} , \end{aligned} \tag{2.1}$$

where $D^\mu \Phi = \partial^\mu + ig_1(\Phi R^\mu - L^\mu \Phi)$ and $L^{\mu\nu} = \partial^\mu L^\nu - \partial^\nu L^\mu$, $R^{\mu\nu} = \partial^\mu R^\nu - \partial^\nu R^\mu$ are the field strength tensors of the (axial-)vector fields. \mathcal{L}_{three} and \mathcal{L}_{four} describe 3- and 4-particle interactions of (axial-)vector fields, which are irrelevant for this work, see Ref. [9].

The baryon sector involves the baryon doublets Ψ_1 and Ψ_2 , where Ψ_1 has positive parity and Ψ_2 negative parity. In the so-called mirror assignment, Ψ_1 and Ψ_2 transform in the opposite way under chiral symmetry, namely:

$$\Psi_{1R} \longrightarrow U_R \Psi_{1R}, \bar{\Psi}_{1R} \longrightarrow \bar{\Psi}_{1R} U_R^\dagger, \quad \Psi_{2R} \longrightarrow U_L \Psi_{2R}, \bar{\Psi}_{2R} \longrightarrow \bar{\Psi}_{2R} U_L^\dagger, \quad (2.2)$$

and similarly for the left-handed fields. Such field transformations allow to write down a baryonic Lagrangian with a chirally invariant mass term for the fermions, parametrized by m_0 :

$$\begin{aligned} \mathcal{L}_{bar} = & \bar{\Psi}_{1L} i\gamma_\mu D_{1L}^\mu \Psi_{1L} + \bar{\Psi}_{1R} i\gamma_\mu D_{1R}^\mu \Psi_{1R} + \bar{\Psi}_{2L} i\gamma_\mu D_{2L}^\mu \Psi_{2L} + \bar{\Psi}_{2R} i\gamma_\mu D_{2R}^\mu \Psi_{2R} \\ & - \hat{g}_1 (\bar{\Psi}_{1L} \Phi \Psi_{1R} + \bar{\Psi}_{1R} \Phi^\dagger \Psi_{1L}) - \hat{g}_2 (\bar{\Psi}_{2L} \Phi^\dagger \Psi_{2R} + \bar{\Psi}_{2R} \Phi \Psi_{2L}) \\ & - m_0 (\bar{\Psi}_{1L} \Psi_{2R} - \bar{\Psi}_{1R} \Psi_{2L} - \bar{\Psi}_{2L} \Psi_{1R} + \bar{\Psi}_{2R} \Psi_{1L}), \end{aligned} \quad (2.3)$$

where $D_{1R}^\mu = \partial^\mu - ic_1 R^\mu$, $D_{1L}^\mu = \partial^\mu - ic_1 L^\mu$ and $D_{2R}^\mu = \partial^\mu - ic_2 R^\mu$, $D_{2L}^\mu = \partial^\mu - ic_2 L^\mu$ are the covariant derivatives for the nucleonic fields. The coupling constants \hat{g}_1 and \hat{g}_2 parametrize the interaction of the baryonic fields with scalar and pseudoscalar mesons and $\varphi = \langle 0 | \sigma | 0 \rangle = Z f_\pi$ is the chiral condensate emerging upon spontaneous chiral symmetry breaking in the mesonic sector. The parameter $f_\pi = 92.4$ MeV is the pion decay constant and Z is the wavefunction renormalization constant of the pseudoscalar fields [11].

The term proportional to m_0 generates also a mixing between the fields Ψ_1 and Ψ_2 . The physical fields N and N^* , referring to the nucleon and to its chiral partner, arise by diagonalizing the baryonic part of the Lagrangian. As a result we have:

$$\begin{pmatrix} N \\ N^* \end{pmatrix} = \hat{M} \begin{pmatrix} \Psi_1 \\ \Psi_2 \end{pmatrix} = \frac{1}{\sqrt{2 \cosh \delta}} \begin{pmatrix} e^{\delta/2} & \gamma_5 e^{-\delta/2} \\ \gamma_5 e^{-\delta/2} & -e^{\delta/2} \end{pmatrix} \begin{pmatrix} \Psi_1 \\ \Psi_2 \end{pmatrix}. \quad (2.4)$$

The masses of the nucleon and its partner are obtained upon diagonalizing the mass matrix \hat{M} :

$$m_{N,N^*} = \frac{1}{2} \sqrt{4m_0^2 + (\hat{g}_1 + \hat{g}_2)^2 \varphi^2} \pm \frac{(\hat{g}_1 - \hat{g}_2) \varphi}{2}, \quad (2.5)$$

i.e., the nucleon mass is not only generated by the chiral condensate φ but also by m_0 . The parameter δ in Eq. (2.4) is related to the masses and the parameter m_0 by the expression: $\delta = \text{Arcosh} \left[\frac{m_N + m_{N^*}}{2m_0} \right]$. Let us consider two important limiting cases. (i) When $\delta \rightarrow \infty$, corresponding to $m_0 \rightarrow 0$, no mixing is present and $N = \Psi_1$, $N^* = \Psi_2$. In this case $m_N = \hat{g}_1 \varphi / 2$ and $m_{N^*} = \hat{g}_2 \varphi / 2$, thus the nucleon mass is generated solely by the chiral condensate as in the linear sigma model. (ii) In the chirally restored phase where $\varphi \rightarrow 0$, one has mass degeneracy $m_N = m_{N^*} = m_0$. When chiral symmetry is broken, $\varphi \neq 0$, a splitting is generated. By choosing $0 < \hat{g}_1 < \hat{g}_2$ the inequality $m_N < m_{N^*}$ is fulfilled.

Note also that, when $g_1 = c_1 = c_2$ and $h_1 = h_2 = h_3 = 0$, the chiral symmetry becomes local [10, 12]. The corresponding model has been studied in Ref. [13]. It was not possible to make a clear-cut prediction as to whether the mass of the nucleon is dominantly generated by the chiral condensate or by mixing with its chiral partner. In addition to this, the description of the mesonic decays was

not correct in a locally symmetric framework as shown in Ref. [14]. Also, the expression for the axial charge reads $g_A^N = \frac{\tanh \delta}{Z^2}$. Since $|\tanh \delta| < 1$ for all δ and $Z > 1$, we obtain $g_A^N < 1$, at odds with the experimental value $g_A^N = 1.267 \pm 0.004$. Thus, in the context of local symmetry one is obliged to introduce terms of higher order such as the Weinberg-Tomozawa one. As a final remark, note that by setting $Z = 1$ (which in turn means $g_1 = 0$) the vector mesons and axial-vector mesons drop out and only the (pseudo-)scalar and nucleonic terms survive in the Lagrangian. Then, for $g_A^{N^*}$ a value much larger than 0.5 is predicted, which is in disagreement with lattice data [1].

In Ref. [6] a large value of the parameter m_0 (~ 800 MeV) is claimed to be needed for a correct description of nuclear matter properties, thus pointing to a small contribution of the chiral condensate to the nucleon mass. Validating this claim through the evaluation of pion-nucleon scattering at zero temperature and density is the subject of the present paper.

The expressions for the axial coupling constants of the nucleon and the partner are given by:

$$g_A^N = \frac{e^\delta}{2 \cosh \delta} g_A^{(1)} + \frac{e^{-\delta}}{2 \cosh \delta} g_A^{(2)}, \quad g_A^{N^*} = -\frac{e^{-\delta}}{2 \cosh \delta} g_A^{(1)} + \frac{e^\delta}{2 \cosh \delta} g_A^{(2)}, \quad (2.6)$$

where

$$g_A^{(1)} = 1 - \frac{c_1}{g_1} \left(1 - \frac{1}{Z^2}\right), \quad g_A^{(2)} = -1 + \frac{c_2}{g_1} \left(1 - \frac{1}{Z^2}\right) \quad (2.7)$$

refer to the axial coupling constants of the bare, unmixed fields Ψ_1 and Ψ_2 . Note that when $\delta \rightarrow \infty$ one has $g_A^N = g_A^{(1)}$ and $g_A^{N^*} = g_A^{(2)}$. Also, when $c_1 = c_2 = 0$ (or $Z = 1$) we obtain the results of Ref. [4]: $g_A^N = \tanh \delta$ and $g_A^{N^*} = -\tanh \delta$, which in the limit $\delta \rightarrow \infty$ reduces to $g_A^N = 1$ and $g_A^{N^*} = -1$. However, in our model the interaction with the (axial-)vector mesons generates additional contributions to g_A^N and $g_A^{N^*}$, which are fixed via the experimental result for g_A^N and the lattice result for $g_A^{N^*}$, see the next section. From the Lagrangian (2.3) one can compute the decay $N^* \rightarrow N\pi$ and the scattering amplitudes $a_0^{(\pm)}$ [15].

3. Results and discussion

We consider three possible assignments for the partner of the nucleon. (1) The resonance $N^*(1535)$, with mass $M_{N^*(1535)} = 1535$ MeV and $\Gamma_{N^*(1535) \rightarrow N\pi} = (67.5 \pm 23.6)$ MeV, which – being the lightest baryonic resonance with the correct quantum numbers – surely represents one of the viable and highly discussed candidates for the nucleon partner. (2) The resonance $N^*(1650)$, with a mass lying just above, $M_{N^*(1650)} = 1650$ MeV and $\Gamma_{N^*(1650) \rightarrow N\pi} = (92.5 \pm 37.5)$ MeV. (3) A *speculative* candidate $N^*(1200)$ with a mass $M_{N^*(1200)} \sim 1200$ MeV and a very broad width $\Gamma_{N^*(1650) \rightarrow N\pi} \gtrsim 800$ MeV, such to have avoided experimental detection up to now [6].

For all these scenarios, we want to determine the values of the parameters c_1 , c_2 , and m_0 . Beyond the width, which is different in the three cases mentioned above, we also use $g_A^{N^*} = 0.2 \pm 0.3$, as predicted by lattice QCD [1], and $g_A^N = 1.26$. We repeat the evaluation for different values of Z . Remarkably, m_0 does not depend on Z .

Figure 1 shows the mass parameter m_0 as a function of the axial coupling constant of N^* for different masses of N^* . For the range of $g_A^{N^*}$ given by Ref. [1], $m_0 = 300 - 600$ MeV for $N^*(1535)$ and $N^*(1650)$, meaning that half of the nucleonic mass survives in the chirally restored phase. On the contrary, for $N^*(1200)$ the value for m_0 lies above 1000 MeV. This result suggests that the

contribution of the chiral condensate to the nucleonic mass should be negative, which is rather unnatural. We can then discard the possibility that a hypothetical, not yet discovered $N^*(1200)$ is the chiral partner of the nucleon.

According to Ref. [16], when the fields N and N^* belong to a parity doublet, $g_A^N \sim 1$ and $g_A^{N^*} \sim -1$. Then, in Ref. [16] the lattice result of [1] is used against the identification of $N^*(1535)$ as the partner of the nucleon. However, within our model we can *still* accommodate $N^*(1535)$ (or also $N^*(1650)$) as the partner of the nucleon. The small value of $g_A^{N^*}$ arises because of interactions of the partner with the axial-vector mesons.

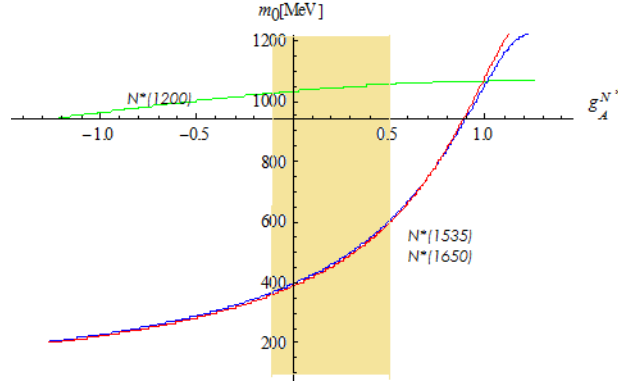


Figure 1: m_0 as a function of $g_A^{N^*}$.

In Figure 2 we plot the isospin-odd and isospin-even scattering lengths as a function of the axial charge $g_A^{N^*}$ for $Z = 1.5$. A comparison of our results to the experimental data on πN scattering lengths, as measured in Ref. [8] by precision X-ray experiments on pionic hydrogen and pionic deuterium, yields the following: (a) the isospin-odd scattering length $a_0^{(-)}$ is close to the experimental range, but we expect an even better result with the introduction of the Δ resonance. (b) The isospin-even scattering length $a_0^{(+)}$ is an order of magnitude smaller than the experimental band $a_{0,\text{exp}}^{(+)} = (-8.85783 \pm 7.16)10^{-6}$ MeV. The reason for this is the strong dependence of $a_0^{(+)}$ on the scalar mesons. Here we use $m_\sigma = 1370$ MeV. A smaller mass of the sigma meson may be favored, however this result seems to be excluded [9].

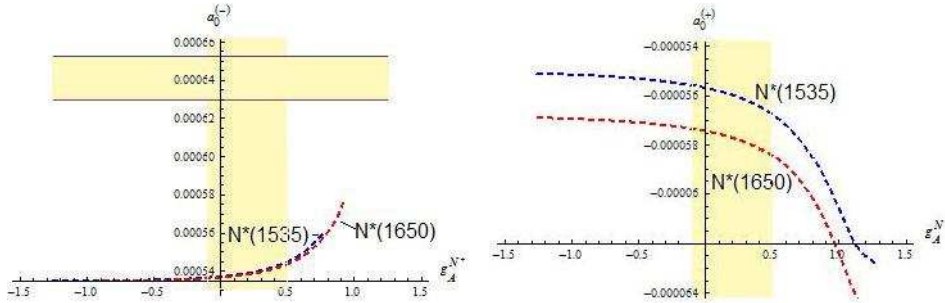


Figure 2: The scattering lengths (a) $a_0^{(-)}$ and (b) $a_0^{(+)}$ as a function of $g_A^{N^*}$.

4. Summary and outlook

We have computed the pion-nucleon scattering lengths at tree-level in the framework of a globally symmetric linear sigma model with parity-doubled nucleons. Within the mirror assignment the mass of the nucleon originates only partially from the chiral condensate, but also from the mass parameter m_0 . Using the lattice results of Ref. [1] we find that $m_0 \simeq 300 - 600$ MeV. Approximately half of the nucleon mass survives in the chirally restored phase. The isospin-odd scattering length lies close to the experimental band, but could be improved in further studies. The isospin-even scattering length is too small: future inclusion of a light tetraquark state gives rise to a large contribution, and thus is expected to improve the results. We also plan to extend our model by including the Δ resonance, necessary to correctly reproduce the p-wave scattering lengths [17] and to evaluate the radiative η -photoproduction.

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References

- [1] T. T. Takahashi and T. Kunihiro, Phys. Rev. D **78** (2008) 011503
- [2] U. G. Meissner, Phys. Rept. **161** (1988) 213.
- [3] S. Gasiorowicz and D. A. Geffen, Rev. Mod. Phys. **41** (1969) 531.
- [4] C. DeTar and T. Kunihiro, Phys. Rev. D **39** (1989) 2805.
- [5] D. Jido, M. Oka, and A. Hosaka, Prog. Theor. Phys. **106** (2001) 873 [arXiv:hep-ph/0110005].
- [6] D. Zschesche, L. Tolos, J. Schaffner-Bielich, and R. D. Pisarski, arXiv:nucl-th/0608044.
- [7] C. Amsler et al. Phys. Lett. B **667** (2008)
- [8] H. C. Schroder *et al.*, Eur. Phys. J. C **21** (2001) 473.
- [9] D. Parganlija, F. Giacosa, and D. Rischke, these proceedings.
- [10] P. Ko and S. Rudaz, Phys. Rev. D **50** (1994) 6877.
- [11] S. Strüber and D. H. Rischke, gauged linear Phys. Rev. D **77** (2008) 085004
- [12] R. D. Pisarski, arXiv:hep-ph/9503330.
- [13] S. Wilms, F. Giacosa, and D. H. Rischke, *Proceedings of 19th International Conference on Ultra-Relativistic Nucleus-Nucleus Collisions: Quark Matter 2006 (QM2006), Shanghai, China, 14-20 Nov 2006*
- [14] D. Parganlija, F. Giacosa, and D. H. Rischke, AIP Conf. Proc. **1030** (2008) 160 [arXiv:0804.3949 [hep-ph]].
- [15] S. Gallas, F. Giacosa, and D. H. Rischke, *in preparation*
- [16] L. Y. Glozman, Phys. Rept. **444** (2007) 1 [arXiv:hep-ph/0701081].
- [17] M. Procura, B. U. Musch, T. R. Hemmert, and W. Weise, freedom,” Phys. Rev. D **75** (2007) 014503 [arXiv:hep-lat/0610105].