

## Quark distributions in nucleons and nuclei

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We discuss the medium modifications of quark distributions and structure functions in the framework of a chiral effective quark theory. Particular emphasis is put on the isospin dependence of the in-medium quark distributions. As an interesting application, we discuss a possible solution of the so called NuTeV anomaly. Possible extensions of the model to describe fragmentation functions are also discussed.

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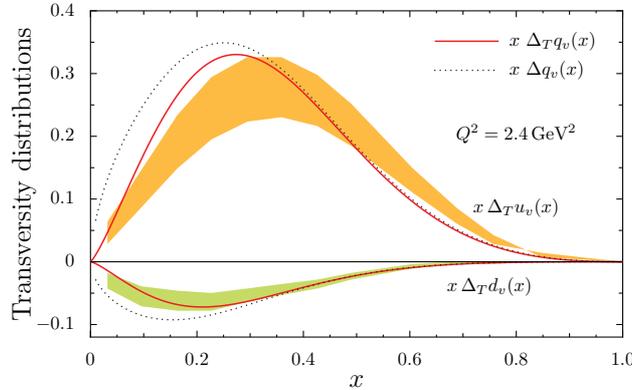
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Effective chiral quark theories are powerful tools to assess the properties of free hadrons and the medium modifications due to the presence of other hadrons. Much progress has been achieved recently by using the Nambu-Jona-Lasinio (NJL) model to describe single nucleons in the Faddeev framework, nuclear matter and finite nuclei in the mean field approximation, and the properties of bound nucleons by combining these two aspects. In this paper we present some recent results obtained within this approach, and discuss an interesting application to neutrino deep inelastic scattering (DIS) on nuclear targets.

In the Faddeev approach to the NJL model[1], the nucleon is described as a bound state of a quark and a diquark, where the scalar and axial vector diquark channels are most important and will be included in all results presented in this paper. In the framework of the mean field approximation, the NJL model also gives a successful description of the nuclear matter equation of state, which includes important effects of the quark substructure of the nucleons[2]. By using the resulting density dependent effective masses of the quark, the diquark and the nucleon, the model can be used to calculate the properties of a nucleon in the medium[3]. The mean field approach can be extended to describe also finite nuclei[4].

As an example of recent calculations[5] carried out in this approach, we first show the transversity quark distributions in the proton in Fig.1, in comparison to recent fits[6] to Hermes, Compass and Belle data. The calculated first moments (tensor charges) at  $Q^2 = 0.8 \text{ GeV}^2$  are  $\Delta_T u = 0.7$  and  $\Delta_T d = -0.15$ , which are within the limits deduced from experiment. Together with earlier results



**Figure 1:** Calculated transversity quark distributions (solid lines, from Ref.[5]) in comparison to recent fits to data (shaded areas, from Ref.[6]). The dotted lines show the calculated helicity distributions for comparison.

obtained for the spin independent and helicity distributions[7], we can say that the model gives an excellent description of the quark distribution functions in the free nucleon.

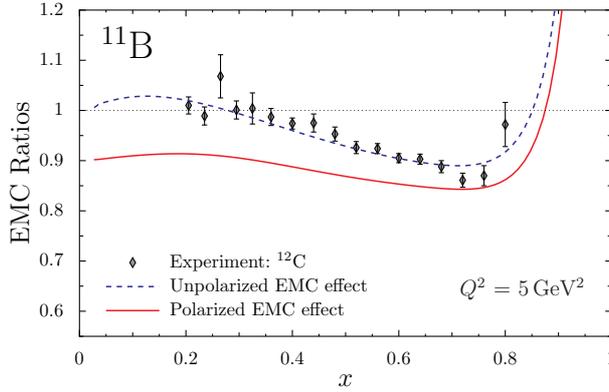
The medium modifications of spin independent and spin dependent structure functions are

usually expressed by the following EMC ratios <sup>1</sup>:

$$R(x) = \frac{F_{2A}(x_A)}{ZF_{2p}(x) + NF_{2n}(x)},$$

$$R_s(x) = \frac{g_{1A}(x_A)}{P_p g_{1p}(x) + P_n g_{1n}(x)}.$$

Here we present two examples of recent calculations: Fig.2 shows the EMC ratios for the nucleus <sup>11</sup>B, together with experimental data for the spin independent ratio. It is clearly seen that the predicted polarized EMC effect is more pronounced than the unpolarized one, and the experimental verification of this interesting prediction is a challenging problem. As a second example[9], we



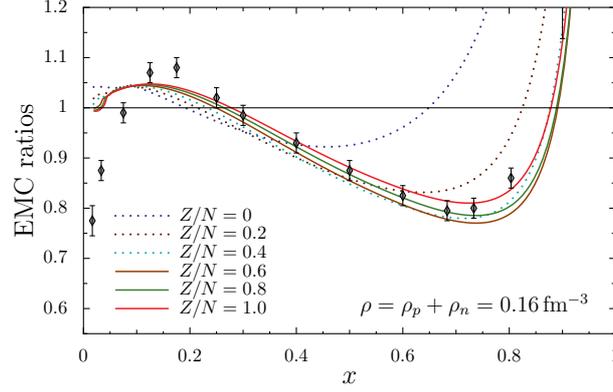
**Figure 2:** EMC ratios for the nucleus <sup>11</sup>B from Ref.[4]. The experimental data for <sup>12</sup>C are from Ref.[8].

show in Fig. 3 the unpolarized EMC ratios in isospin asymmetric infinite matter for several values of  $Z/N$  at fixed baryon density. It is seen that the EMC effect in the valence quark region first increases as the system becomes neutron rich, and then decreases. The reason for this behavior is that, because of the symmetry energy, the binding - and therefore also the medium modifications - of the  $u$  quarks increases, which leads to a more pronounced EMC effect as long as the  $u$  quarks dominate the structure function on account of their bigger charge. This is in contrast to the case of proton rich matter, where the EMC effect always decreases with increasing isospin asymmetry. As an interesting application of this result, we discuss the following “Paschos-Wolfenstein (PW) ratio”[11] for inclusive DIS of neutrinos and antineutrinos from iron, which was measured in 2002 by the NuTeV collaboration[12]:

$$R_{\text{PW}} = \frac{\sigma(\nu\text{Fe} \rightarrow \nu\text{X}) - \sigma(\bar{\nu}\text{Fe} \rightarrow \bar{\nu}\text{X})}{\sigma(\nu\text{Fe} \rightarrow \mu^-\text{X}) - \sigma(\bar{\nu}\text{Fe} \rightarrow \mu^+\text{X})} = \frac{\text{NC}}{\text{CC}}.$$

Here it is understood that all cross sections are integrated over the Bjorken variable  $x$ , and the notations NC and CC stand for the neutral current and charged current weak processes. This ratio

<sup>1</sup>Here  $F_{2A}$ ,  $F_{2p}$ , and  $F_{2n}$  are the spin independent structure functions of the nucleus with  $A = Z + N$ , the proton, and the neutron, and  $g_{1A}$ ,  $g_{1p}$ , and  $g_{1n}$  are the corresponding spin dependent quantities.  $x$  is the Bjorken variable for the nucleon, and  $x_A = x \frac{AM_N}{M_A}$  is the Bjorken variable for the nucleus, multiplied by the mass number  $A$ . The quantities  $P_p$  and  $P_n$  are the polarization factors (expectation values of the spin operator) of protons and neutrons in the nucleus.



**Figure 3:** Spin independent EMC ratios for infinite matter at fixed baryon density for several ratios  $Z/N$ . The data points are the extrapolations of nuclear DIS to the limit of isospin symmetric nuclear matter[10].

can be expressed in terms of nuclear valence quark distributions as follows[13]:

$$R_{PW} = \frac{\int_0^A dx_A x_A (\alpha u_A(x_A) + \beta d_A(x_A))}{\int_0^A dx_A x_A (d_A(x_A) - \frac{1}{3}u_A(x_A))}, \quad (1)$$

where  $\alpha = \frac{2}{3}(\frac{1}{4} - \frac{2}{3}\sin^2\Theta_W)$  and  $\beta = \frac{2}{3}(\frac{1}{4} - \frac{1}{3}\sin^2\Theta_W)$  depend on the Weinberg angle  $\Theta_W$ .

If  $u_A(x) = d_A(x)$ , the PW ratio measures the Weinberg angle  $\Theta_W$ :

$$R_{PW} \xrightarrow{N=Z} \frac{1}{2} - \sin^2\Theta_W. \quad (2)$$

After applying the standard ‘‘isoscality corrections’’ on the level of parton distributions in free nucleons, the value deduced from the experiment was  $R_{PW} = 0.272$  [12], which would translate into  $\sin^2\Theta_W = 0.228$  on account of Eq.(2). This is different from the Standard Model value[13] of  $\sin^2\Theta_W = 0.223$ , and this discrepancy is often called the ‘‘NuTeV anomaly’’<sup>2</sup> [14].

It is important to note, however, that because of the isospin asymmetry of the target nucleus the simple connection between  $R_{PW}$  and the Weinberg angle is lost. Because the difference  $u_A - d_A$  is in general medium dependent, the standard ‘‘isoscality corrections’’, which are based on free parton distributions, may be insufficient. In fact, if we use our medium modified quark distributions, which led to the results of Fig.3, for  $Z/N = 26/30$ , apply the same naive isoscality corrections as in the NuTeV analysis, and use Eq.(1) with the Standard Model value of the Weinberg angle, we obtain the ratio  $R_{PW} = 0.273$ . This is very close to the value deduced from the NuTeV experiment, which indicates that the measured PW ratio is actually consistent with the Standard Model value of  $\sin^2\Theta_W$ . We can thus conclude that the isospin dependence of the in-medium quark distributions largely removes the ‘‘anomaly’’.

Finally, we wish to discuss an extension of the model to describe also fragmentation functions. By using crossing symmetry and charge conjugation, it is possible to establish a formal relation between the spin independent distribution function of a quark  $q$  in a hadron  $h$

$$f_q^h(x) = \frac{1}{2} \sum_n \delta(p_- x - p_- + p_{n-}) \langle p | \bar{\psi} | p_n \rangle \gamma^+ \langle p_n | \psi | p \rangle \equiv \Theta(1-x) F(x),$$

<sup>2</sup>The precise values are  $\sin^2\Theta_W = 0.2277 \pm 0.0013 \pm 0.0009$  determined by the NuTeV group, compared to  $\sin^2\Theta_W = 0.2227 \pm 0.0004$  of the Standard Model, which indicates a  $3\sigma$  discrepancy.

and the spin independent fragmentation function of a quark  $q$  into a hadron  $h$

$$D_q^h(z) = \frac{z}{6} \frac{1}{2} \sum_n \delta\left(\frac{p_-}{z} - p_- - p_{n-}\right) \langle p, p_n | \bar{\psi} | 0 \rangle \gamma^+ \langle 0 | \psi | p, p_n \rangle.$$

This relation, which has been derived originally from the hadronic tensors for the inclusive and semi-inclusive processes[15], can be expressed as

$$D_q^h(z) = -\Theta(1-z) \frac{z}{6} F\left(x = \frac{1}{z}\right).$$

(If  $h$  is a boson, there is no minus sign in the above equation.) This indicates that  $f_q^h$  and  $D_q^h$  are essentially one and the same function, defined in different regions of the variable. This result would allow a straight forward method to extend our calculations of distribution functions to fragmentation functions. More detailed investigations[16], however, show that this formal connection has several severe limitations: (i) The regularization scheme used for the distribution functions cannot be extended in a straight forward way to the fragmentation functions. (ii) The  $Q^2$  evolution gives rise to a logarithmic singularity at  $x = 1$ , which is essentially an infrared singularity due to the vanishing gluon mass. (iii) Multiple fragmentation processes, as discussed in the framework of the jet model of Field and Feynman [17] are important so that the light cone momentum of the fragmenting quark is completely transferred to hadrons. The detailed analysis of these points, together with numerical results, will be presented in a future publication [16].

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