Decay Properties of $Q\bar{Q}$ Mesons in Potential Models and Effective Field Theories

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The decay rates of $Q\bar{Q}$ mesons ($Q\ \varepsilon\ c,b$) are studied in the NRQCD formalism in terms of their short distance and long distance coefficients. The long distance coefficients are obtained through phenomenological potential model description of the mesons. The mass spectrum of the $c\bar{c}$ and $b\bar{b}$ mesons are reviewed in non-relativistic phenomenological quark antiquark potential of the type $V(r) = -\frac{\alpha_c}{r} + Ar^{\nu}$, with ν varying from 0.5 to 2. The spin hyperfine and spin-orbit interactions are employed to obtain the masses of the pseudoscalar and vector mesons. The digamma and dileptonic decays of $c\bar{c}$, and $b\bar{b}$ mesons are investigated using some of the known potential models by incorporating radiative corrections up to the lowest order. The heavy quarkonium decays into two photons, and the vector state into lepton pairs are computed within the NRQCD formalism. Our theoretical predictions of the decays of the $c\bar{c}$ and $b\bar{b}$ mesons and the results obtained from some of the other potential schemes are compared with the experimental values.

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1. Introduction

The new results of many experimental facilities world wide CLEO, DELPHI, Belle, BaBar, LHCb *etc.*, have generated considerable interest in the hadron spectroscopy from light flavor to heavy flavor sector [1].The heavy flavour mesons are those in which at least one of the quark or antiquark or both the quark and antiquark belong to heavy flavour sector, particularly the charm or beauty. They are represented by $Q\bar{Q}$ mesonic systems which include the quarkonia ($c\bar{c}$ and $b\bar{b}$). The investigation of the properties of these mesons gives very important insight into heavy quark dynamics. For the mass predictions, the nonrelativistic potential models with Buchmüller and Tye [2], Martin [3], Log[4], Cornell [5] *etc.*, were successful at the heavy flavour sectors. Relativistic models were also being employed in the study of heavy flavour sectors [6], the non-relativistic potential model has been successful in many respects [7, 8, 9, 10, 11, 12]. A more comprehensive review of developments in heavy quarkonium physics is available in ref [13].

The new role of the heavy flavour studies as the testing ground for the non-perturbative aspects of QCD, demands extension of earlier phenomenological potential model studies on quarkonium masses to their predictions of decay widths incorporating the non-perturbative aspects. In this context, an elegant effort was provided by the NRQCD formalism [14]. It consists of a nonrelativistic Schrodinger field theory for the heavy quark and antiquark that is coupled to the usual relativistic field theory for light quarks and gluons. NRQCD not only organize calculation of all orders in α_s, but also elaborate systematically the relativistic corrections to the conventional formula. Next to leading order in v^2/c^2, the decay rates satisfy a more general factorization formula, which contain two additional independent nonperturbative matrix elements related to their radial wave functions. We study the di-gamma decay of pseudoscalar states and the leptonic decay width of the vector states of the $c\bar{c}$ and $b\bar{b}$ quarkonia in the frame work of the NRQCD formalism and compare with results obtanied using conventional Van Royen-Weisskopf formula with radiative correction to the decay widths.

2. Nonrelativistic Treatment for $Q\bar{Q}$ systems

For the study of heavy-heavy bound state systems such as $c\bar{c}$, $c\bar{b}$, $b\bar{b}$, we consider a nonrelativistic Hamiltonian given by [10, 11, 12]

$$H = M + \frac{p^2}{2m} + V(r) \tag{2.1}$$

where $M = m_Q + m_{\bar{Q}}$, $m = \frac{m_Q\, m_{\bar{Q}}}{m_Q + m_{\bar{Q}}}$, m_Q and $m_{\bar{Q}}$ are the mass parameters of quark and antiquark respectively, p is the relative momentum of each quark and $V(r)$ is the quark-antiquark potential. Though linear plus coulomb potential is a successful well studied nonrelativistic model for heavy flavour sector, their predictions for decay widths are not satisfactory owing to the improper value of the radial wave function at the origin compared to other models [11]. Recently, we have considered a general power potential with colour coulomb term of the form $V(r) = \frac{-\alpha_c}{r} + Ar^\nu$ as the static quark-antiquark interaction potential (CPP_ν) [15, 16]. Here, for the study of mesons, $\alpha_c = \frac{4}{3}\alpha_s$, α_s being the running strong coupling constant, A is the potential parameter and ν is a general power.

Table 1: Decay rates (in keV) of $0^{-+} \to \gamma\gamma$ and the relevant correction terms of η_c and η_b mesons.

	Models	Γ_0	Γ_R	Γ_{0+R}	Γ_{NRQCD}		$\Gamma_{NRQCD_{frs}}$	Γ_{Others}
					$O(v^2)$	$O(v^4)$		
	ERHM[9]	7.460	-2.855	4.605	4.005	4.225	–	
	BT[2]	10.870	-4.206	6.664	6.555	6.561	–	7.2±
	PL(Martin)[3]	13.406	-6.196	7.210	8.434	10.691	–	0.7±2.0[1]
η_c	Log [4]	10.937	-4.349	6.588	6.691	6.697		7.500[17]
	Cornell [5]	19.512	-6.581	12.931	13.779	17.447	–	9.02 ±
	CPP$_\nu$ =0.5	8.173	-2.635	5.538	2.511	6.078	2.992	0.8 [14]
	1.0	14.649	-4.724	9.925	5.552	10.077	5.549	
	1.5	19.971	-6.440	13.531	8.556	13.142	7.536	
	ERHM[9]	0.444	-0.114	0.326	0.315	0.317	–	
	BT[2]	0.574	-0.149	0.424	0.445	0.455	–	
	PL(Martin)[3]	0.406	-0.118	0.288	0.312	0.340	–	0.364 [14]
η_b	Log [4]	0.435	-0.115	0.320	0.337	0.345	–	
	Cornell [5]	1.244	-0.290	0.954	1.015	1.112	–	
	CPP$_\nu$ =0.5	0.345	-0.086	0.259	0.254	0.254	0.195	
	1.0	0.529	-0.132	0.397	0.390	0.391	0.353	
	1.5	0.683	-0.171	0.512	0.505	0.505	0.510	

We employ the exponential trial wave function of the hydrogenic type to generate the Schrödinger mass spectra. Within the Ritz variational scheme using the trial radial wave function we obtain the expectation values of the Hamiltonian ($\langle H\rangle = E(\mu,\nu)$) as

$$E(\mu,\nu) = M + \frac{\mu^2}{8m} + \frac{1}{2}\left(-\mu\alpha_c + A\,\frac{\Gamma(\nu+3)}{\mu^\nu}\right) \tag{2.2}$$

Eqn(2.2) gives the spin average mass of the ground state and μ is the variational parameter.

The spin average mass for the ground state is computed for the values of ν from 0.5 to 2. We have taken the quark mass parameters $m_b = 4.66\ GeV$ and $m_c = 1.31\ GeV$. The experimental spin average masses are computed from the experimental masses of the pseudoscalar and vector mesons using the relation, $M_{SA} = M_P + \frac{3}{4}(M_V - M_P)$ have been used to determine the potential strength A for each choice for ν.

3. Decay rates of quarkonia

Along with the mass spectrum, successful predictions of various decay widths of heavy flavoured systems have remained as testing ground for the success of phenomenological models. Experimentally, the excited states and the leptonic, di-gamma and other hadronic decay width, of the heavy flavour mesons have been reported. However, experimentally, the pseudoscalar $b\bar{b}$ bound state η_b is still elusive though a single experimental result has been reported recently [18].

As an attempt to improve the theoretical predictions involving the phenomenological description of the meson, using the radial wave functions and other model parameters of the different potential models, we study the decay of $^1S_0$ quarkonium into di-gamma and the decay of $^3S_1$ into lepton pairs using the NRQCD formalism [14]. It is expected that the NRQCD formalism has all the corrective contributions for the right predictions of the decay rates. For the present calculations of the decay rates of quarkonia, we employ the NRQCD factorization expressions given by [19].

Table 2: Decay rates (in keV) of $1^{--} \to l^+ \, l^-$ and the relevant correction terms of J/ψ and Υ mesons.

	Models	Γ_{VW}	Γ_{rad}	Γ_{VW+rad}	Γ_{NRQCD}		$\Gamma_{NRQCD_{frs}}$	Γ_{EXP}[1]
					$O(v^0)$	$O(v^4)$		
	ERHM[9]	5.595	-3.381	2.214	2.543	3.246	–	
J/ψ	BT[2]	8.152	-4.982	3.170	2.539	2.809	–	5.55±
	PL (Martin)[3]	10.055	-7.341	2.714	3.311	4.698	–	0.14
	Log[4]	8.203	-0.171	3.057	1.967	2.094	–	
	Cornell [5]	14.634	-7.701	6.933	7.920	10.294	–	
	CPP_ν= 0.5	6.130	-0.624	5.506	4.212	4.973	0.973	
	1.0	11.053	-1.165	9.888	9.353	12.398	2.822	
	1.5	15.165	-1.645	13.520	12.605	19.037	4.749	
	ERHM[9]	1.320	-0.540	1.303	1.221	1.228	–	
Υ	BT[2]	1.720	-0.076	1.644	1.249	1.267	–	
	PL(Martin)[3]	1.218	-0.761	0.457	0.693	0.774	–	1.340±
	Log[4]	1.305	-0.032	1.273	0.924	0.943	–	0.018
	Cornell [5]	3.733	-0.232	3.501	2.025	2.270	–	
	CPP_ν= 0.5	1.035	-0.010	1.025	0.935	0.938	0.710	
	1.0	1.587	-0.017	1.570	1.436	1.442	1.279	
	1.5	2.047	-0.022	2.025	1.857	1.865	1.844	

4. Conclusion and discussion

The potential model and the quark mass parameters obtained from the experimental masses of the respective quarkonium states and the resulting radial wave functions have been employed to study their decay properties in the frame work of NRQCD formalism (Γ_{NRQCD}) as well as using the conventional Van-Royen-Weisskopf nonrelativistic formula (Γ_{VW}). The theoretical (CPP_ν) predictions of the decay widths for $J/\psi \to l^+l^-$ and $\Upsilon \to l^+l^-$ listed in Table 2 are found to be in accordance with other potential model predictions with the radiative correction (Γ_{VW+rad}) as well as with the widths computed using NRQCD formalism. Though the radiative corrections are found to be important in most of the phenomenological models, the NRQCD predictions with their matrix elements computed at finite radial separation defined through the 'colour compton radius'[15] ($\Gamma_{NRQCD_{frs}}$) are found to be in better agreement with the experimental values for most of the cases.

The NRQCD width for $\eta_c \to \gamma\gamma$ predicted in the present study based on the potential model parameters of BT [2], Log [4], CPP$_\nu$ between 0.5 and 1 fall with in the range of the experimental results. However for the $\eta_b \to \gamma\gamma$ case, most of the model predictions based on NRQCD formalism are very close to similar theoretical predictions of [14]. The predictions based on V-W formula with radiative corrections (Γ_{0+R}) are also found to be in close agreement with the prediction of[14] and [17] respectively.

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