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Determination of the relativistic corrections to the static inter-quark potential from lattice QCD

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We investigate the relativistic corrections to the static potential, i.e. the $O(1/m)$ potential and the $O(1/m^2)$ spin-dependent potentials, in SU(3) lattice gauge theory. Utilizing the multilevel algorithm, we obtain a very clean signal for these potentials in the region 0.25 fm $\lesssim r \lesssim$ 1.2 fm. We confirm long-range nonperturbative contributions in the $O(1/m)$ potential and $O(1/m^2)$ spin-orbit potentials.

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1. Introduction

A possible strategy of studying heavy quarkonium in QCD is to employ potential nonrelativistic QCD (pNRQCD) [1, 2], which is obtained by integrating out the scale above the heavy quark mass $m \gg \Lambda_{\rm QCD}$ and the scale mv, where v is quark velocity. The effective hamiltonian of pNRQCD up to $O(1/m^2)$ [2] is then given by

$$H = \frac{\vec{p}_1^{\,2}}{2m_1} + \frac{\vec{p}_2^{\,2}}{2m_2} + V^{(0)}(r) + \frac{1}{m_1}V^{(1,0)}(r) + \frac{1}{m_2}V^{(0,1)}(r) + \frac{1}{m_1^2}V^{(2,0)}(r) + \frac{1}{m_2^2}V^{(0,2)}(r) + \frac{1}{m_1 m_2}V^{(1,1)}(r) + O(1/m^3)\,, \tag{1.1}$$

where m_1 and m_2 denote the masses of quark and antiquark, placed at $\vec{r}_1$ and $\vec{r}_2$, respectively. The static inter-quark potential $V^{(0)}(r \equiv |\vec{r}_1 - \vec{r}_2|)$ emerges, accompanied by relativistic corrections classified in powers of $1/m$. The potentials $V^{(1,0)}(r) = V^{(0,1)}(r)$ $(\equiv V^{(1)}(r))$ are the corrections at $O(1/m)$. The potentials $V^{(2,0)}(r)$, $V^{(0,2)}(r)$, and $V^{(1,1)}(r)$ are the corrections at $O(1/m^2)$, which contain the leading order velocity-dependent potentials [3, 4] and spin-dependent potentials [5, 6]. The spin-dependent potentials are conventionally parametrized as

$$V_{\rm SD}(r) = \left(\frac{\vec{s}_1\cdot\vec{l}_1}{2m_1^2} - \frac{\vec{s}_2\cdot\vec{l}_2}{2m_2^2}\right)\left(\frac{V^{(0)\prime}(r)}{r} + 2\frac{V_1'(r)}{r}\right) + \left(\frac{\vec{s}_2\cdot\vec{l}_1}{2m_1m_2} - \frac{\vec{s}_1\cdot\vec{l}_2}{2m_1m_2}\right)\frac{V_2'(r)}{r} + \frac{1}{m_1m_2}\left(\frac{(\vec{s}_1\cdot\vec{r})(\vec{s}_2\cdot\vec{r})}{r^2} - \frac{\vec{s}_1\cdot\vec{s}_2}{3}\right)V_3(r) + \frac{\vec{s}_1\cdot\vec{s}_2}{3m_1m_2}V_4(r), \tag{1.2}$$

where $\vec{s}_1$ and $\vec{s}_2$ denote the spins, and $\vec{l}_1 = -\vec{l}_2 = \vec{l}$ the orbital angular momenta.

Once these potentials are determined from QCD, various properties of heavy quarkonium can be investigated systematically by solving the Schrödinger equation. Since the binding energy of a quark-antiquark system is typically of the scale mv^2, which can be of the same order as $\Lambda_{\rm QCD}$ due to the nonrelativistic nature of the system, $v \ll 1$, as well as the fact that perturbation theory cannot incorporate quark confinement, it is essential to determine the potentials nonperturbatively.

Monte Carlo simulations of lattice QCD offer a powerful tool for the nonperturbative determination of the potentials. Recently, we investigated the $O(1/m)$ potential [7, 8], and the $O(1/m^2)$ spin-dependent [9, 10] and momentum-dependent potentials [8] on a lattice utilizing the multilevel algorithm [11], and obtained remarkably clean signals up to distances of around 0.6 fm. In certaincases we observed deviations from the perturbative potentials.

In this report we present updated results of the $O(1/m)$ potential and $O(1/m^2)$ spin-dependent potentials. In particular, we aim to clarify the long-distance behavior of these corrections.

2. Formulation and Numerical Procedures

According to pNRQCD, the $O(1/m)$ and $O(1/m^2)$ potentials can generally be expressed by the matrix elements and the energy gaps which appear in the spectral representation of the color-electric and color-magnetic field strength correlators (FSCs) on the quark-antiquark source [1, 2], where we adopt the Polyakov loop correlation function (PLCF) as a quark-antiquark source. By utilizing the multilevel algorithm [11], we measure these FSCs of various geometries on a $V = L^3T$

Table 1: Simulation parameters used in this study. $N_{\rm tsl}$ is the number of time slices in a sublattice and $N_{\rm iupd}$ the number of internal update within a sublattice, both are parameters for the multilevel algorithm. The lattice spacing a is set from the Sommer scale $r_0 = 0.5$ fm.

$\beta = 6/g^2$	a [fm]	$N_{\rm tsl}$	Spin-dependent potential			$O(1/m)$ potential		
			$(L/a)^3(T/a)$	$N_{\rm iupd}$	$N_{\rm conf}$	$(L/a)^3(T/a)$	$N_{\rm iupd}$	$N_{\rm conf}$
5.85	0.123	3	24^4	50000	77	24^4	50000	124
6.00	0.093	4	$20^3 40$	7000	33	$24^3 32$	50000	100
6.20	0.063	5	———			$30^3 40$	50000	39
6.30	0.059	6	24^4	6000	39	———		

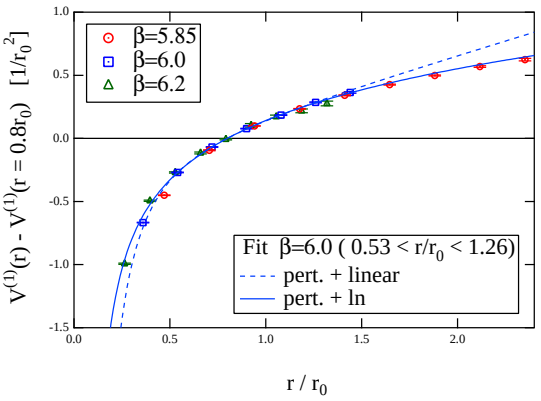

Figure 1: The $O(1/m)$ potential $V^{(1)}(r)$. Two lines are the fit results obtained from the data at $\beta = 6.0$ and $0.53 < r/r_0 < 1.26$ (5 points) with the two fit functions, $V_{\rm linear}(r) = -A_1/r^2 + B_1 r + C_1$ and $V_{\ln}(r) = -A_2/r^2 + B_2 \ln r + C_2$. Both functional forms can describe the data behavior within the fit window, however, $V_{\ln}(r)$ seems to be favored for the long-range data around $r \sim 2.0 r_0$.

lattice with lattice spacing a. Exploiting the spectral representation, we can extract the matrix elements and the energy gaps from the measured FSCs. Definitions and all technical details are described in Refs.[7, 8, 10].

3. Numerical results

We carry out simulations using the standard Wilson gauge action in SU(3) lattice gauge theory. We summarize our simulation parameters in Table 1. In Fig. 2, we show the $O(1/m)$ potential, $V^{(1)}(r)$, normalized at $r = 0.8 r_0$ together with the two fit curves $V_{\rm linear}(r) = -A_1/r^2 + B_1 r + C_1$ and $V_{\ln}(r) = -A_2/r^2 + B_2 \ln r + C_2$. Perturbation theory at $O(\alpha_s^2)$ provides $V_{\rm pert}(r) = -\frac{C_F C_A \alpha_s^2}{4r^2}$, where C_F and C_A are the Casimir charges of the fundamental and the adjoint representations, respectively, and $\alpha_s = g^2/(4\pi)$ the strong coupling [1]. Owing to using larger spatial volumes compared to [7] and a re-tuning of the algorithm, we have now succeeded in determining $V^{(1)}(r)$ up to $r = 2.35 r_0 \sim 1.2$ fm. Results for $V^{(1)}(r)$ at different β show a reasonable scaling behavior, except for the data at $r/a = 2$, which are likely to suffer from discretization errors. We confirm that the $O(1/m)$ potential contains nonperturbative contribution. The new data set at $r \gtrsim 1.5 r_0$ suggest that the nonperturbative contribution may be described by a logarithmic function, as proposed in [12]. It is quite interesting to examine the effect of the $O(1/m)$ potential on the spectrum as this is the leading-order relativistic correction in the $1/m$ expansion.

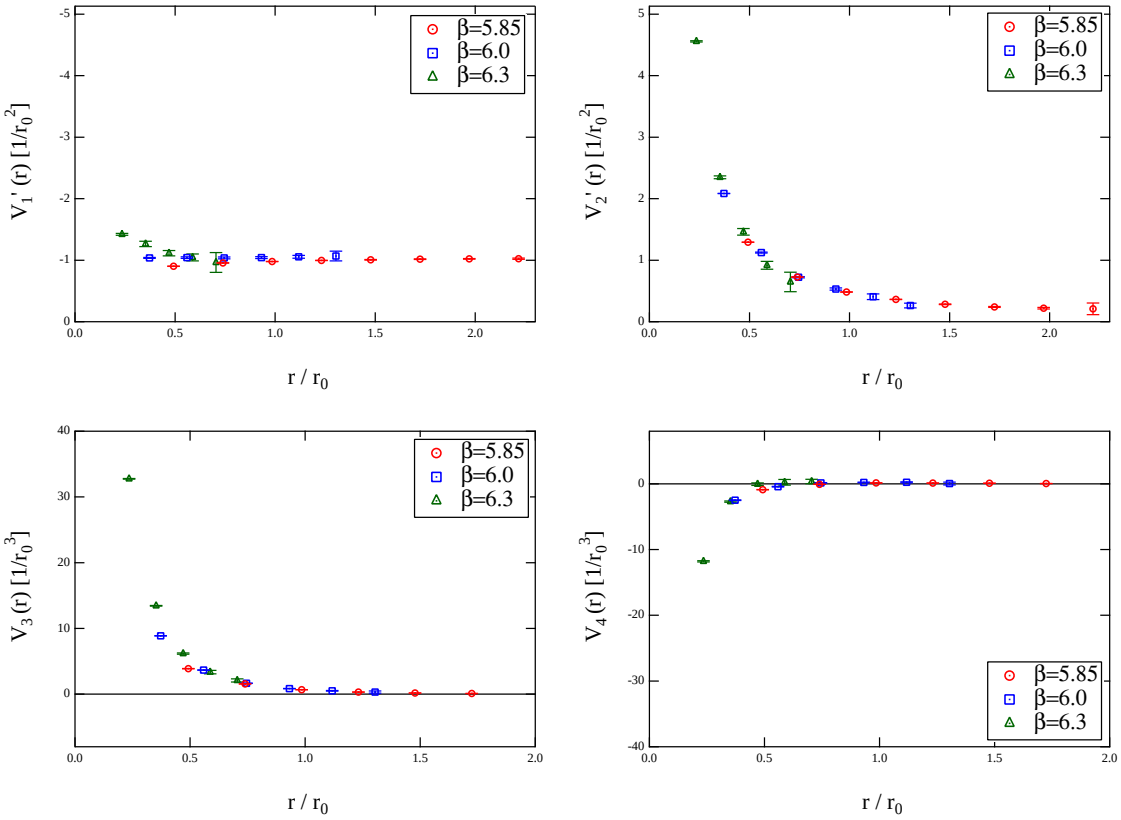

Figure 2: The spin-dependent potentials $V_1'(r), V_2'(r), V_3(r)$, and $V_4(r)$.

In Fig. 2, we present the spin-dependent potentials, $V_1'(r), V_2'(r), V_3(r)$, and $V_4(r)$[1]. The $O(1/m^2)$ spin-orbit potentials, $V_1'(r)$ and $V_2'(r)$, are non-vanishing up to $r = 2.23r_0$. The finite tail of $V_2'(r)$ is an example of the observed deviation from perturbative potentials [10]. The Gromes relation, which is an important analytic relation derived from the Lorentz invariance, $V^{(0)\prime}(r) = V_2'(r) - V_1'(r)$ [6], is approximately satisfied as shown in Fig. 3, where the deviation from the relation $1 - (V_2' - V_1')/V^{(0)\prime}$ is plotted. The deviation is 10 to 12 % at $a = 0.123$ fm, while 4 to 10 % at $a = 0.093$ fm. It will be interesting to study whether the deviation vanishes after taking the continuum limit. For the spin-spin potentials, $V_3(r)$ and $V_4(r)$, we confirm that they have no long-range contribution. In order to investigate their functional form, we definitely need data at $r \lesssim 0.5r_0$.

4. Summary

We have investigated the relativistic corrections to the static potential, the $O(1/m)$ potential and the $O(1/m^2)$ spin-dependent potentials, in SU(3) lattice gauge theory. They are important ingredients of the pNRQCD hamiltonian for heavy quarkonium. By evaluating the color-electric and color-magnetic FSCs on the PLCF with the multilevel algorithm, and exploiting the spectral

[1]The data at $\beta = 6.0$ and 6.3 are already published in Ref. [10].

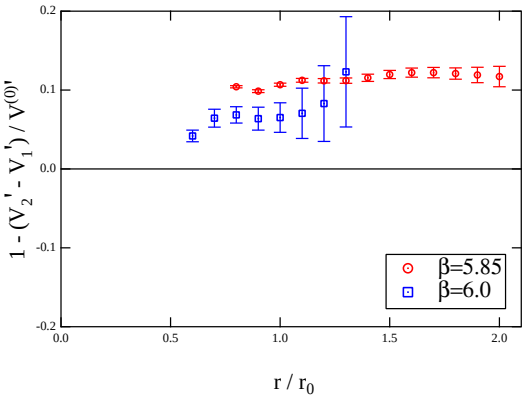

Figure 3: The quantity $1-(V_2'-V_1')/V^{(0)\prime}$ is plotted as a function of r. If the Gromes relation is exactly satisfied, this quantity should be zero at all r.

representation of the correlator, we have obtained a very clean signal for these potentials in the region $0.5r_0 \lesssim r \lesssim 2.4r_0$.

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