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Thermal ground state for pure SU(2) Yang-Mills thermodynamics

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In this proceeding the emergence of a composite, adjoint-scalar field as an average over (trivial holonomy) calorons and anti-calorons is reviewed. This composite field acts as a background field to the dynamics of perturbative gluons, to which it is coupled via an effective, gauge invariant Lagrangian valid for temperatures above the deconfinement phase transition. Moreover a Higgs mechanism is induced by the composite field: two gluons acquire a quasi-particle thermal mass. On the phenomenological side the composite field acts as a bag pressure which shows a linear dependence on the temperature. As a result the linear rise with temperature of the trace anomaly is obtained and is compared to recent lattice studies.

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1. Introduction

The SU(2) Yang-Mills (YM) theory at nonzero temperature is subject both of theoretical and numerical on-going efforts. The aim is a deeper understanding of its non-perturbative properties, which makes this theory complex and rich. This is also a necessary step toward the understanding of the quark gluon plasma (QGP), see for instance [1] for a review. In this work, based on Refs. [2, 3, 4, 5, 6], the non-perturbative sector of SU(2) is described by a composite, (adjoint-)scalar field ϕ in the deconfined phase ($T > T_c$).

Topological objects named calorons (i.e. instantons at nonzero T) with zero and nonzero holonomy -the latter being a property of the fields at spatial infinity- have been described in [7]. A non-trivial holonomy caloron carries also monopole-antimonopole constituents. The 'composite' field ϕ, which we want to introduce, emerges as an 'average' over calorons and anticalorons with trivial holonomy, see [2, 3] for a microscopic derivation and [4] for a macroscopic one. It depends only on the temperature T and the YM-scale Λ. On a length scale $l > |\phi|^{-1}$ it is thermodynamically exhaustive to consider only the average field ϕ and neglect the (unsolvable) microscopic dynamics of all YM-field configurations, such as calorons and monopoles. One can then build up an effective theory for YM-thermodynamics valid for $T > T_c$, in which the scalar field ϕ acts as background field coupled to the residual, perturbative gluons. It also acts as an Higgs-field in the thermal medium, implying that two gluons (out of three) acquire a non-vanishing quasi-particle thermal mass. On a phenomenological level it contributes to the energy and pressure as a temperature-dependent bag constant. Here we shall focus on one particular implication of this effective description: the linear growth with T of the stress-energy tensor, which has been obtained in some analyses of lattice data.

2. Linear growth of $\theta = \rho - 3p$ and bag constant

Be ρ the energy density and p the pressure of a system at a given temperature T. The quantity $\theta = \rho - 3p$ is the trace of the stress-energy tensor and vanishes for a conformal theory (as, for instance, a gas of photons). In SU(2) this symmetry is broken by quantum effects (trace anomaly). In [8], based on the new lattice data of Ref. [9], it is found that θ growths linearly with T:

$$\theta = aT, \quad 2T_c \lesssim T \lesssim 5Tc, \; a \simeq 1.5 \text{ GeV}^3. \tag{2.1}$$

(We notice that in the analysis of the same lattice data a quadratic rise of θ, rather than a linear one, has been found [10].) We also refer to [11], where a simple linear fit $\theta = aT$ was found to reproduce old lattice results. Recently, in [12] the linear rise has been confirmed by studying the Lattice data of [13].

We now turn to a phenomenological description of a plasma of quasi-particles [14], where also a T-dependent bag is introduced to mimic a non-perturbative behavior. As an example let us consider only one scalar field with a T-dependent mass $m = m(T)$ and a bag $B = B(T)$. The energy density and the pressure read (see, for instance, [15]):

$$\rho = \rho_p + B(T), \; p = p_p - B(T), \tag{2.2}$$

$$\rho_p = \int_k \frac{\sqrt{k^2+m^2(T)}}{\exp\left[\frac{\sqrt{k^2+m^2(T)}}{T}\right]-1},\ p_p = -T\int_k \log\left[1-\exp\left[-\frac{\sqrt{k^2+m^2(T)}}{T}\right]\right]$$

where $\int_k = \int \frac{d^3k}{(2\pi)^3}$. Requiring the validity of the thermodynamical self-consistency [15, 16] $\rho = T(dp/dT) - p$ (which is a consequence of the first principle of thermodynamics), we obtain the equation

$$\frac{dB}{dT} = -D(m)\frac{dB}{dT},\ D(m) = \int_k \frac{m}{\sqrt{k^2+m^2}}\frac{1}{\exp\left[\frac{\sqrt{k^2+m^2(T)}}{T}\right]-1}. \tag{2.3}$$

Imposing that $B(T) = cT$ and following the analytical steps of Ref. [5], we obtain:

$$\theta = \rho - 3p = 4B + \rho_p - 3p_p \overset{T\text{ large}}{=} 6B(T) = 6cT. \tag{2.4}$$

Thus, also the quasi-particle excitation contributes *linearly* to θ at high T. The important result is that a linear rise of the bag constant implies also a linear rise of $\theta = \rho - 3p$. If the linear rise shall be confirmed on the lattice, it means that the SU(2), nonperturbative bag shall be a linear rising function with T.

3. A thermal ground state

The SU(2) (euclidean) YM Lagrangian reads $\mathscr{L}_{YM} = \frac{1}{2}Tr[G_{\mu\nu}G^{\mu\nu}]$, where $G_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - ig[A_\mu, A_\nu]$ and $A_\mu = A_\mu^a t^a$ (t^a are the SU(2) generators). g is the fundamental coupling constant, which, upon renormalization, is function of the renormalization scale μ. The non-abelian nature of $\mathscr{L}_{YM}$ is at the origin of its nonperturbative properties. A general field configuration can be decomposed as $A_\mu = A_\mu^{\text{top}} + a_\mu$, where A_μ^{top} refers to a topologically non-trivial function and a_μ to the quantum fluctuations. It is very hard, if not impossible, to take into account at a microscopic level all the topological objects such as calorons and monopoles. As described in Refs. [2, 4] we introduce an adjoint scalar gauge field $\phi = \phi^a t^a$ as

$$\phi = \text{'Spatial average over (anti-)calorons with trivial holonomy'}. \tag{3.1}$$

The field ϕ can be univocally determined by the three following conditions [4]: (i) Being a spatial average it depends (periodically) on τ only. It transforms as $\phi \to U\phi U^\dagger$ under (space-independent) gauge transformations. (ii) The corresponding Lagrangian ϕ reads $\mathscr{L}_\phi = Tr[(\partial_\tau\phi)^2 + V]$; BPS saturation of (anti-)calorons implies also BPS-saturation for ϕ: $\mathscr{H}_\phi = Tr[(\partial_\tau\phi)^2 - V] = 0$. (iii) ϕ represents (part of) the vacuum of the YM-system at nonzero T and a background field to the dynamics of the trivial quantum fluctuations a_μ. The gauge-invariant quantity $|\phi|$ acts as a 'condensate' (strictly related to the gluon condensate, see [6]) and does not correspond to any new particle. As a consequence $|\phi|$ shall *not* depend on τ.

The conditions (i), (ii) and (iii) imply that $V(\phi) \propto 1/|\phi|^2$ [4]. Introducing a scale Λ, which is the only free parameter of the theory and is naturally identified with the YM-scale, we obtain $V(\phi) = \frac{\Lambda^6}{|\phi|^2}$. Solving the equation of motion one obtains as a unique solution (up to a phase) $|\phi| = \sqrt{\Lambda^3/(2\pi T)}$. On the length scale $l > |\phi|^{-1}$ (part of) the field configurations A_μ^{top} are effectively described by ϕ.

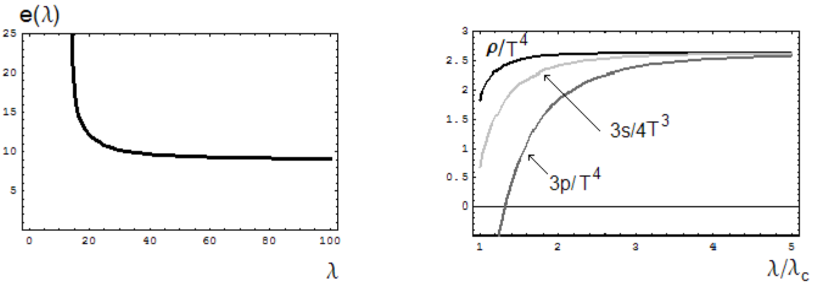

Figure 1: Left: effective coupling as function of $\lambda = 2\pi T/\Lambda$. Right: (scaled) thermodynamical relevant quantities as function of $\lambda/\lambda_c = T/T_c$.

As a last step we couple ϕ, which is a given background solution $|\phi| = \sqrt{\Lambda^3/(2\pi T)}$, to the quantum fluctuations a_μ in a gauge invariant manner and obtain the final form of the effective theory:

$$\mathscr{L}^{\text{eff}} = Tr\left[F_{\mu\nu}F^{\mu} + (D_\mu\phi)^2 + \frac{\Lambda^6}{|\phi|^2}\right], \tag{3.2}$$

with $F_{\mu\nu} = \partial_\mu a_\nu - \partial_\nu a_\mu - ie[a_\mu, a_\nu]$ and $D_\mu\phi = \partial_\mu\phi - ie[A_\mu, \phi]$. The new, T-dependent coupling constant $e = e(T)$ is univocally obtained by imposing thermodynamical self-consistency as described in Section 2. In Ref. [6] the fundamental coupling constant g is related to $e(T)$ and it is found that also g admits a Landau pole, which is just slightly shifted from the perturbative result. The effective coupling constant $e(\lambda = \frac{2\pi T}{\Lambda})$ together with some relevant thermodynamical quantities evaluated at tree-level in Fig. 1. Note that $e(\lambda)$ diverges at $\lambda_c = 13.86$, corresponding to a critical temperature of $T_c \sim 2\Lambda$. Thus the effective theory is valid for $T > T_c$. For $T >> T_c$ a plateaux is reached for $e(\lambda) \sim \sqrt{8}\pi$. Moreover, as evident from eq. (3.2) a Higgs mechanism takes place: two gluons acquire a quasi-particle mass $m(T) = 2e(T)|\phi|$ which diverges at the phase boundary.

Finally, we summarize the effective theory in Fig. 2. Its tree-level equations are similar to those presented in Section 2. Corrections beyond tree-level are small (1%) [17].

4. Result for θ and conclusions

The last term of eq. (3.2) acts as a T-dependent bag term, $B(T) \sim \frac{\Lambda^6}{|\phi|^2} = 2\pi T\Lambda^3$. The important point is that $B(T)$ is linear in T! It then implies, as seen in Section 2, that θ of SU(2) YM-theory grows linearly in T for high T. The precise result can be obtained analytically [5]:

$$\theta = \rho - 3p \overset{T>2T_c}{\sim} 24\pi\Lambda^3 T \simeq (1.7 \text{ GeV}^3)T. \tag{4.1}$$

Note that the coefficient 1.7 GeV^3 is similar to 1.5 GeV^3 found in Ref. [8] and is also not far from the theoretical result of Ref. [18]. We also note that the same approach can be applied to the $SU(3)$ case, where one also finds $\theta \overset{T>2T_c}{\sim} 24\pi\Lambda^3 T$.

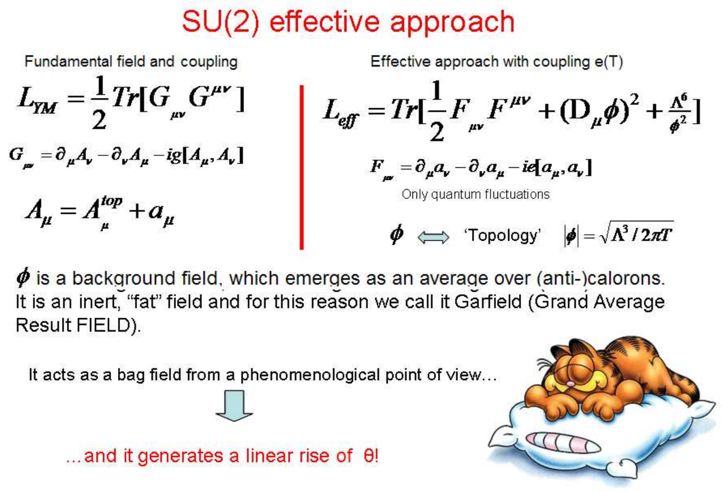

Figure 2: Summary of the effective theory: a transparency of the talk at Confinement8 in Mainz.

We conclude this brief report on an effective approach for the description of YM at nonzero T by summarizing its main idea: the introduction of a simple, average background field over calorons and anticalorons. The aim is a to avoid an impossible, microscopic evaluation of complicated field configurations, and to obtain a thermodynamically exhaustive description of a YM system. A simple consequence of this effective approach, namely the linear growth with T of the trace anomaly, is in agreement with recent lattice simulations. Some applications of this approach can be found in Refs. [19].

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