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Contributions to the QCD Pressure Beyond Perturbation Theory

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In this article we report on a new proposal to treat the infrared problems of thermal QCD by taking into account explicitly the confining influence of the Gribov horizon. In order to make clear the possible value of such an approach, we briefly review the most important arguments why a straightforward perturbative description of finite-temperature QCD is unlikely to be successful. From the infrared problems of thermal perturbation theory one can conclude that confinement effects and bound states probably play an important role also in the high-temperature phase.

To set the stage we recount the supposed role of the Gribov horizon for confinement, before we turn to the application to finite-temperature theory. In the current approach it has been found that the contributions to the free energy from the explicit inclusion of the horizon begin to set in at order g^6 – precisely where the infrared problems of thermal QCD lead to a breakdown of ordinary perturbation theory.

From the study of observables (free energy, anomaly, bulk viscosity) we also note that for thermodynamic observables the leading order term obtained by such an expansion in the coupling strongly deviates from the more complete numerical solution. This can be regarded as yet another sign for general problems of series expansions in thermal QCD.

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1. Infrared Problems in Thermal QCD

The properties of QCD at high temperatures ($\mathscr{O}(100\,\mathrm{MeV})$ and higher) are obviously important for an understanding of the early universe. While this domain is to some extent accessible experimentally (at RHIC and LHC), it remains a challenge for theory.

Initially one had expected to find a phase transition where at some temperature hadrons melt into a "quark-gluon plasma" [1]. In such a plasma, quarks and gluons were supposed to be almost free particles which could be reliably described by perturbation theory. Such a purely perturbative description was motivated by the fact that high temperatures imply high average momentum transfer μ and thus – due to asymptotic freedom – a small coupling $g(\mu)$.

However, this picture turned out to be too naive. In principle, this should have been clear at least since 1980, when it was shown [2, 3] that at order g^6 a natural barrier arises for any perturbative description (the *Linde problem*). Even earlier than that, the simple fact that the infinite-temperature limit of four-dimensional Yang-Mills theory is a three-dimensional *confining* Yang-Mills theory could and should have been regarded as a sign that any straightforward perturbative approach to high-temperature QCD was problematic.

Still it took more than 20 years until it began to be accepted that the high-temperature phase of QCD has little to do with a conventional plasma. The results of the RHIC experiments [4] clearly showed that also above the transition, bound state phenomena can not be neglected.

Since perturbation theory is limited to a finite order (and, in addition, for experimentally accessible temperatures the apparent convergence of the perturbation series is bad [5, 6]), other methods have been invoked to treat the properties of finite temperature QCD:

One popular proposal [7] is to explicitly separate the three relevant scales (hard – $2\pi T$, chromoelectric – gT and chromomagnetic – g^2T) by introducing effective theories at each scale. Two of these theories can be treated perturbatively, while one (which governs the magnetostatic sector) is genuinely nonperturbative.

Therefore nonperturbative methods are – either in the direct or in the effective theory approach – urgently needed. One possibility is given by functional methods, based for example on Dyson-Schwinger equations; for an application to finite temperature see e.g. [8, 9]. These methods provide valuable insights, but unfortunately up to now the pressure (and quantities derived from it) are difficult to access. Still functional methods provide further evidence for the picture of bound states playing an important role even at very high temperatures and parts of the gluon spectrum being confined at any temperature.

The nonperturbative methods most actively pursued for gauge theories are lattice-based, and indeed lattice data is available for finite temperatures up to $T \approx 5T_C$. In this region for pure SU(3) gauge theory the problem of determining the equation of state is regarded as solved [10]. Also the AdS/CFT duality [11] and the AdS/QCD duality [12] are employed to study finite temperature gauge theories.

In this article we report on a project which follows an alternative path: Since the footprints of confinement are visible also in the "deconfined" phase, the confining influence of the Gribov horizon is explicitly taken into account. The method has been proposed in [13, 14] and further developed in [15, 16]. To explain it, we will briefly summarize the importance of the Gribov horizon in section 2, while in section 3 we will discuss the application to finite-temperature theory.

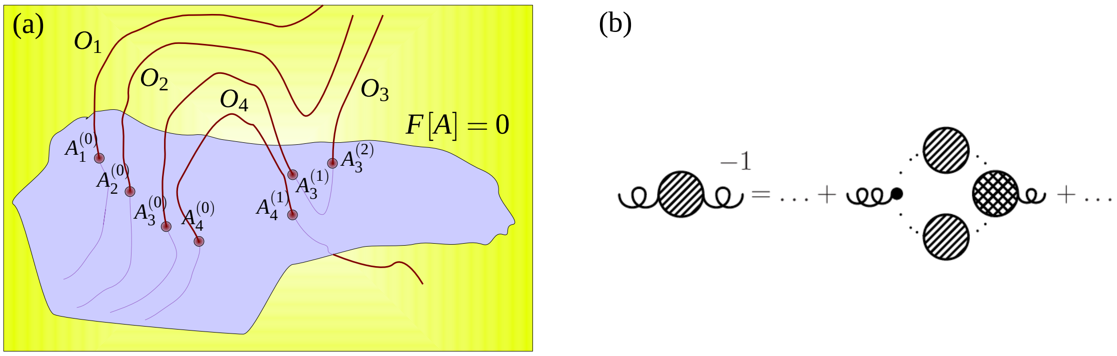

Figure 1: (a) In non-Abelian gauge theories there are typically configurations $A_i^{(k)}$, $k = 0,1,2,\ldots$ on the same gauge orbit O_i which fulfill the same gauge condition $F[A] = 0$. (b) The gluon Dyson-Schwinger equation (only the infrared-dominant part is displayed on the rhs).

2. The Gribov Horizon and Confinement

The initial goal of gauge-fixing was to single out precisely one configuration on each gauge orbit in order to embed the physical configuration space into the space of all field configurations. As it has been shown by Gribov [17] and further discussed by Singer [18], in non-Abelian gauge theories the usual gauge-fixing procedure does not yield uniqueness. So for any local gauge-fixing condition $F[A] = 0$ one typically finds, as sketched in figure 1a, several gauge-equivalent configurations $A^{(0)}$, $A^{(1)} = {}^{g_1}A^{(0)} := g_1^{-1}A^{(0)}g_1 + g_1^{-1}\partial g_1$, $A^{(2)} = {}^{g_2}A^{(0)}$, ... which all fulfill this condition.

This does not affect perturbation theory (since only the vicinity of $A = 0$ plays a role there), but in order to account for nonperturbative effects, one has to consider the effect of such *Gribov copies*. In particular the infrared properties are dramatically changed.

In order to eliminate Gribov copies one typically restricts the domain of integration to the *Gribov region*, i.e. the region where the Faddeev-Popov operator $\mathscr{M}(A)$ is positive semidefinite.[1] The boundary of the Gribov region (where $\mathscr{M}(A)$ has at least one zero eigenvalue) is called the *Gribov horizon*. The presence of the Gribov horizon (and the restriction of the functional integral to the Gribov region) leads to modifications of the Faddeev-Popov procedure [20]. The influence of the Gribov horizon can be incorporated in a local action [21, 22].

While the Gribov problem seems to be a nuisance at first glance, in fact a whole confinement scenario is based on it – proposed in [17] and further elaborated by one of the authors [21, 22]: Since the Gribov region is (on the lattice) a high- or (in the continuum) an infinite-dimensional region, due to geometrical reasons most of the volume is concentrated close to the boundary ("the entropy argument"). So most configurations which contribute to the path integral have an almost vanishing Faddeev-Popov operator and correspondingly an enhanced ghost propagator.

Such an enhanced ghost can be made responsible for the suppression of the gluon propagator in the infrared. This is most easily understood by analyzing the gluon Dyson-Schwinger equation (see figure 1b) [23, 24, 25]. The ghost loop on the right-hand side is infrared enhanced and thus the gluon propagator is accordingly suppressed – the gluon cannot propagate over long distances.

[1] However, as discussed in [19], in principle one has to restrict the integration even further – to the Fundamental Modular Region (FMR), which is unfortunately characterized by a global condition and thus difficult to handle.

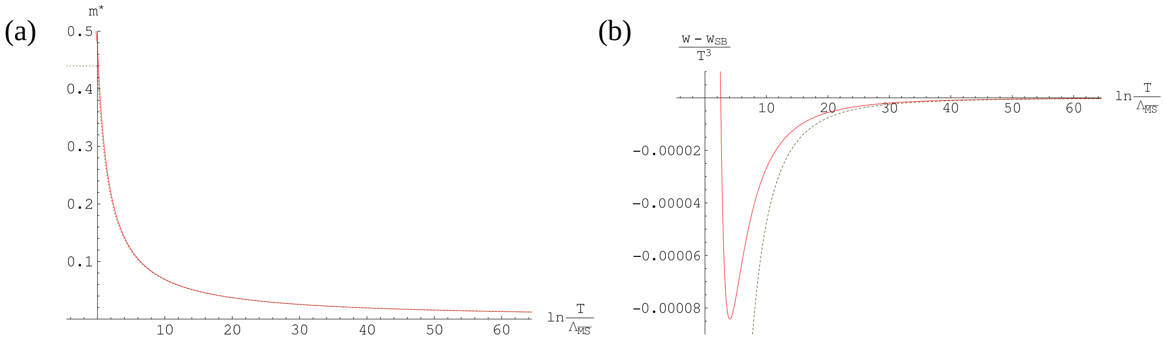

Figure 2: (a) the rescaled Gribov mass $m^* = \frac{m}{T}$, (b) the rescaled free energy $\frac{w-w_{\rm SB}}{T^3}$, where the Stefan-Boltzmann term $w_{\rm SB}$ has been subtracted. [solid – numerical solution, dotted – leading order expansion]

3. Application to Finite Temperature Theory

We now turn to the "marriage" of finite temperature theory and Gribov's confinement scenario. The basic physical idea is that the infrared divergences of finite-temperature perturbation theory (which are responsible for the Linde problem) do not arise when the domain of functional integration is cut off at the Gribov horizon. The cut-off is done in Coulomb gauge which is well adapted to finite-temperature calculations.

Technically the cut-off at the Gribov horizon is implemented by adding a "horizon function" to the action [21, 26]. The initially non-local term then gets replaced by a local, renormalizable term in the action by means of an integration over a multiplet of auxiliary ghost fields. The new term in the action depends on a mass parameter m; the functional cut-off at the Gribov horizon imposes the condition that the free energy W or quantum effective action Γ be stationary with respect to that mass, $\frac{\partial W}{\partial m} = -\frac{\partial \Gamma}{\partial m} = 0$.

This "horizon condition" has the form of a non-perturbative gap equation that determines the Gribov mass $m = m(T, \Lambda_{QCD})$ and thereby provides a new vacuum, around which a perturbative expansion is again possible. Lowest-order expansion gives the Gribov-type dispersion relation

$$E\left(\vec{k}^2\right) = \sqrt{\vec{k}^2 + \frac{m^4}{\vec{k}^2}}. \tag{3.1}$$

From knowledge of the Gribov mass one can deduce contributions to the free energy w, the energy e, the (rescaled) anomaly $A_{\rm r} = \frac{e-3p}{T^4}$ and (making use of a relation obtained in [27]) the bulk viscosity ζ. Results for m and w are displayed in figure 2.

An expansion of the results in powers of g is possible at least in leading order, but this leading order term strongly deviates from the full numerical result. This is probably not a fault of the method, but should be regarded as yet another sign that series expansions in thermal QCD have to be treated with great care.

Nevertheless as a matter of principle, it is a significant success that for thermodynamic observables this procedure gives finite results precisely at the order, g^6 at which ordinary perturbation theory diverges. It also confirms the picture of thermal QCD composed of two sectors with one being accessible perturbatively, while the other one is genuinely nonperturbative.

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