Spontaneous CP violation in the NJL model at $\theta = \pi$

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As is well-known, spontaneous CP-violation in the strong interaction is possible at $\theta = \pi$, which is commonly referred to as Dashen's phenomenon. This phenomenon has been studied extensively using chiral Lagrangians. Here the two-flavor NJL model at $\theta = \pi$ is discussed. It turns out that the occurrence of spontaneous CP-violation depends on the strength of the 't Hooft determinant interaction, which describes the effect of instanton interactions. The dependence of the phase structure, and in particular of the CP-violating phase, on the quark masses, temperature, baryon and isospin chemical potential is examined in detail. The latter dependence shows a modification of the charged pion condensed phase first discussed by Son and Stephanov.

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1. Introduction

The study of spontaneous CP violation (SCPV) in the strong interaction at $\theta = \pi$ has a long history. Already in 1971 Dashen [1] pointed out its possibility and it has been studied extensively using chiral Lagrangians, see for example Refs. [2, 3, 4]. The angle θ stands for the vacuum angle that enters the QCD Lagrangian through the term $\mathscr{L}_\theta = \frac{\theta g^2}{32\pi^2} F\tilde{F}$ to which instantons contribute. This θ-term leads to explicit CP violation, except when $\theta = 0 \pmod{\pi}$. When $\theta = \pi$ it may happen that while the Lagrangian is invariant under CP, the ground state is not.

We will show that the actual occurrence of SCPV depends on the strength of the instanton interaction. This will be discussed for the two-flavor Nambu-Jona-Lasinio (NJL) model [5]. In models like the NJL-model and in low energy effective theories of the strong interaction in general, the effects of instantons are mimicked by an effective interaction, the 't Hooft determinant interaction [6]. We will investigate how SCPV depends on the strength of this interaction.

Other properties of the SCPV phase are studied by calculating its dependence on temperature and nonzero baryon and isospin chemical potential. This could be of relevance to heavy-ion collisions, despite the fact that in Nature $\theta < 10^{-10}$. Namely, it has been suggested that metastable CP-violating bubbles might be created in such collisions. These states would be characterized by an effective, possibly large θ, cf. Refs. [7]. Experimental signatures for these bubbles have been discussed in Refs. [8].

Finally, we discuss mixing of mesons with their parity partners, which arises whenever CP invariance is broken. This affects the charged pion condensed phase that arises at sufficiently large isospin chemical potential. The results presented here are based on Ref. [9] to which we refer for details.

2. The NJL model

The following form of the 2-flavor NJL-model is used

$$\mathscr{L} = \bar{\psi}\left(i\gamma^\mu\partial_\mu + \gamma_0\mu\right)\psi - \bar{\psi}M_0\psi + G_1\left[(\bar{\psi}\lambda_a\psi)^2 + (\bar{\psi}\lambda_a i\gamma_5\psi)^2\right] + $$
$$G_2\left[e^{i\theta}\det\left(\bar{\psi}_R\psi_L\right) + \text{h.c.}\right]. \quad (2.1)$$

The interaction with coupling constant G_1 is chirally symmetric. The interaction proportional to G_2 is the 't Hooft determinant interaction and represents the effects of instantons. As the model is non-renormalizable, a cut-off is needed. Here a three-dimensional cut-off is used, which is set by the value of the chiral condensate $\langle\bar{\psi}\psi\rangle$, via a gap equation. The coupling constants depend on this cut-off as $G_i \sim \mathcal{O}(1)/\Lambda^2$ for dimensional reasons. We restrict to two flavors, using λ_a with $a = 0, ..., 3$ as the generators of U(2).

We wish to investigate the dependence of SCPV at $\theta = \pi$ on the strength of the instanton interaction G_2, while keeping the physics at $\theta = 0$ unchanged. This means that the sum $G_1 + G_2$ has to be kept fixed. The parameters are chosen in such a way that at $\theta = 0$ reasonable values for the pion mass, chiral condensate and pion decay constant are obtained [10]. As we want to study the effects of instantons, the parameter $c = G_2/(G_1 + G_2)$ is varied. In order for the model to have a stable ground state, this parameter has to be between 0 and $1/2$. Often G_1 and G_2 are taken equal,
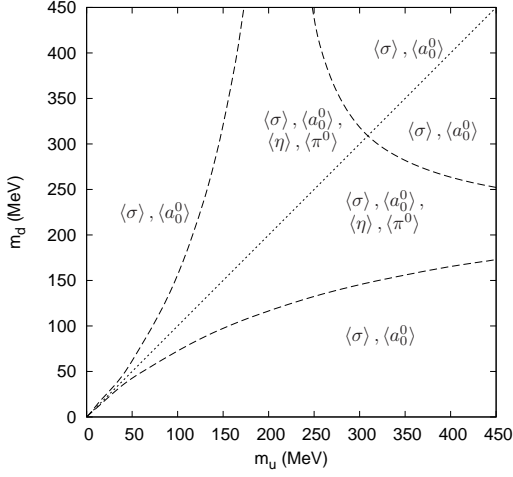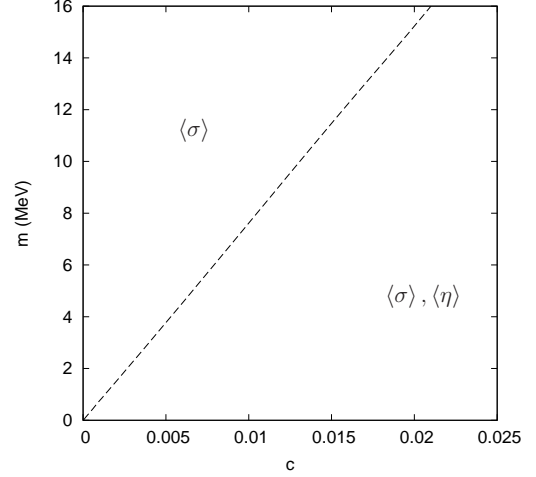
Figure 1: (m_d, m_u) phase diagram with $c = 0.4$.

Figure 2: (m, c) phase diagram.

i.e. $c = 1/2$, which means that at $\theta = 0$ the low energy spectrum consists of σ and $\boldsymbol{\pi}$ fields only. Using the strange quark condensate it was argued in Ref. [10] that a realistic value of c would be around 0.2.

The ground state is obtained by minimizing the effective potential, which is calculated in the mean-field approximation. As usual, the ground state is described by meson condensates, i.e.

$$\langle \sigma \rangle = \langle \bar{\psi} \lambda_0 \psi \rangle, \qquad\qquad \langle \boldsymbol{a}_0 \rangle = \langle \bar{\psi} \boldsymbol{\lambda} \psi \rangle,$$
$$\langle \eta \rangle = \langle \bar{\psi} \lambda_0 i \gamma_5 \psi \rangle, \qquad\qquad \langle \boldsymbol{\pi} \rangle = \langle \bar{\psi} \boldsymbol{\lambda} i \gamma_5 \psi \rangle. \tag{2.2}$$
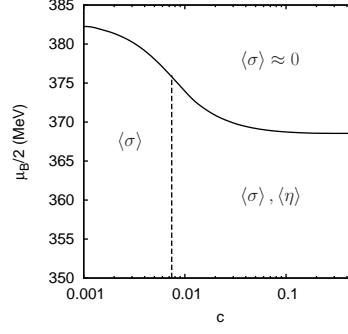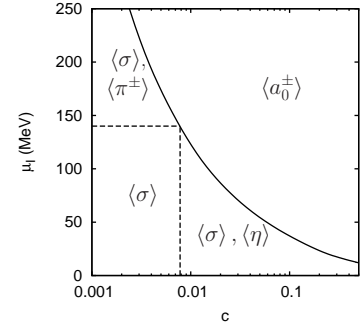
3. Results

All results presented here are for $\theta = \pi$. In the phase diagrams a solid line denotes a first-order phase transition, a dashed line a second-order, and a dotted line a cross-over.

We start with a discussion of the quark-mass dependence of the model. In Fig. 1 the phase diagram as a function of the up and down quark masses is shown for $c = 0.4$. The region where $\langle \eta \rangle$ and $\langle \pi^0 \rangle$ are non-zero corresponds to a phase which violates CP invariance. Nonzero $\langle \pi^0 \rangle$ only occurs for nondegenerate quark masses, as it arises purely in combination with explicit $SU(2)_V$ breaking. The asymptotes of the phase transition are proportional to c. Similar results have been obtained using a two-flavor chiral Lagrangian in Ref. [4]. However, in that calculation the phase transition at high $m_{u,d}$ (above the asymptotes) is absent. This is in contrast to the three-flavor chiral Lagrangian case studied by Creutz [3], which does exhibit this second order phase transition. However, in that case the asymptotes correspond to the mass of the strange quark.

To get more insight into the c dependence of the SCPV phase, we show in Fig. 2 the (m, c) phase diagram for degenerate quark masses ($m_u = m_d = m$). For every mass a critical c exists, above which SCPV occurs. From this, we can conclude that the instanton interaction has to be strong enough w.r.t. the quark masses in order to have SCPV.

In Figs. 3 and 4 the (T, c) and (μ_B, c) phase diagrams are shown, where T is the temperature and $\mu_B = \mu_u + \mu_d$ is the baryon chemical potential. When the temperature or the baryon chemical
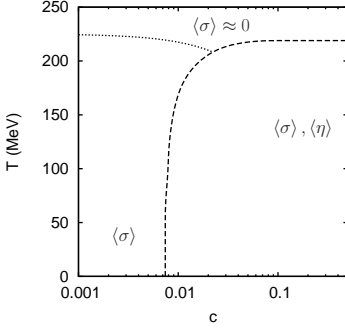
Figure 3: (T, c) phase diagram.   **Figure 4:** (μ_B, c) phase diagram.   **Figure 5:** (μ_I, c) phase diagram.

potential is increased, the phase which violates CP invariance disappears, which indicates that SCPV is a low energy phenomenon. It is therefore absent in the deconfined phase at $\theta = \pi$. The CP-restoring phase transition at high temperature is found to be of second order. This is in contrast to the results of Ref. [11], where a first-order phase transition was found using a linear sigma model coupled to quarks.

The final phase diagram which we discuss is the one as a function of c and the isospin chemical potential $\mu_I = \mu_u - \mu_d$, shown in Fig. 5. Son and Stephanov [12] have shown that a (second-order) phase transition to a charged pion condensed phase occurs at $\theta = 0$ when μ_I becomes larger than the vacuum pion mass. This condition still holds at $\theta = \pi$ as long as there is no SCPV. However, for even larger μ_I there is second phase transition to a novel phase of $\boldsymbol{a}_0^{\pm}$-condensation, which is characteristic for $\theta = \pi$. Moreover, this second phase transition is of first order. Above the critical c, the charged pion condensate does not occur for any value of μ_I, only the charged $\boldsymbol{a}_0$ condensate. In order to discuss the condition for $\boldsymbol{a}_0^{\pm}$-condensation, we have to calculate the mass eigenstates in the presence of SCPV. SCPV causes mixing between parity partners, i.e. mass eigenstates are no longer CP eigenstates. The mass eigenstates, denoted with a tilde, are defined as

$$|\tilde{\sigma}\rangle = \cos\theta_\eta \,|\sigma\rangle + \sin\theta_\eta \,|\eta\rangle\,, \qquad\qquad |\tilde{\eta}\rangle = \cos\theta_\eta \,|\eta\rangle - \sin\theta_\eta \,|\sigma\rangle\,,$$
$$|\tilde{a}_0\rangle = \cos\theta_{\boldsymbol{\pi}} \,|\boldsymbol{a}_0\rangle + \sin\theta_{\boldsymbol{\pi}} \,|\boldsymbol{\pi}\rangle\,, \qquad\qquad |\tilde{\boldsymbol{\pi}}\rangle = \cos\theta_{\boldsymbol{\pi}} \,|\boldsymbol{\pi}\rangle - \sin\theta_{\boldsymbol{\pi}} \,|\boldsymbol{a}_0\rangle\,, \qquad (3.1)$$

where θ_η and $\theta_{\boldsymbol{\pi}}$ are the mixing angles. The states on the right-hand side are the usual states of
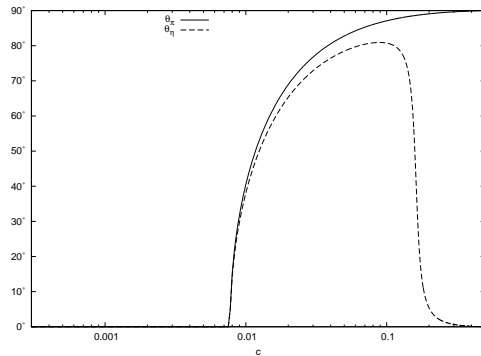
Figure 6: The c-dependence of the meson mixing angles at $\mu_I = 0$.
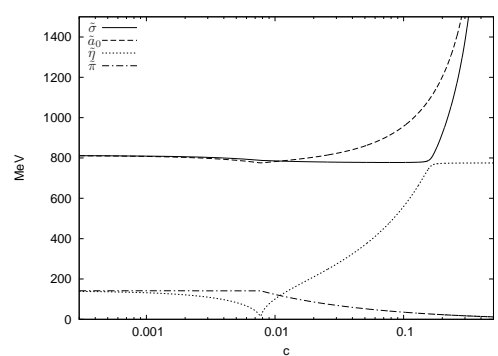
Figure 7: The c-dependence of the meson masses at $\mu_I = 0$.

definite parity.

The masses and mixing are calculated in the random phase approximation [13]. In Fig. 6 the c dependence of the mixing angles is shown. When the CP violating condensate turns on, the mixing angles become non-zero. In Fig. 7 the corresponding vacuum masses are plotted. Comparing these masses with the phase diagram we can conclude that the phase transition between the η and the $a_0^{\pm}$-condensate corresponds to the vacuum mass of $\tilde{\pi}$. Fig. 6 shows that for the special case of $c = 1/2$ ($G_1 = G_2$) the state $|\tilde{\boldsymbol{\pi}}\rangle$ is entirely $|\boldsymbol{a}_0\rangle$. Furthermore, one can observe that when c approaches $1/2$ the mass of the σ and $\boldsymbol{a}_0$ fields go to infinity which indicates that these fields decouple, as expected for $c = 1/2$.

In conclusion, the results presented here show that the phase structure of the strong interactions at $\theta = \pi$ is more diverse than at $\theta = 0$, thanks to spontaneous CP violation and the effects of instantons. We expect the presented two-flavor NJL-model results to remain valid in the case of three flavors and when going beyond the mean-field approximation, but this remains to be studied. Also, it would be interesting if the results could in the future be compared to lattice QCD results on the low-energy physics at $\theta = \pi$.

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