Universality in QCD and Halo Nuclei

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Effective Field Theory (EFT) provides a powerful framework to exploit a separation of scales in order to perform systematically improvable, model-independent calculations. We apply this method to strongly interacting quantum systems with short-range interactions and large scattering length. Such systems display remarkable universal properties which include a geometric spectrum of shallow three-body states called "Efimov states" and log-periodic dependence of scattering observables on the scattering length. We review the EFT for large scattering length and some of its applications in the physics of cold atoms and nuclear physics. In particular, we discuss the possibility of an infrared limit cycle in QCD and the extension of the EFT to halo nuclei.

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Figure 1: Integral equation for the particle-dimer scattering amplitude. A single (double) line indicates a particle (full dimer) propagator.

Introduction. The Effective Field Theory (EFT) approach provides a powerful framework that exploits the separation of scales in physical systems. Only low-energy (or long-range) degrees of freedom are included explicitly, while everything else is parametrized in terms of the most general local contact interactions. Thus, the EFT describes universal low-energy physics independent of detailed assumptions about the short-distance dynamics. Physical observables can be described in a controlled expansion in powers of kl, where k is the typical momentum and $l \sim r_e$ is the characteristic low-energy length scale of the system. We focus on applications of EFT to few-body systems with large S-wave scattering length $|a| \gg l$. For a generic system, the scattering length is of the same order of magnitude as the low-energy length scale l. Only a very specific choice of the parameters in the underlying theory (a so-called *fine tuning*) will generate a large scattering length a. The fine tuning can be accidental or it can be controlled by an external parameter. Examples with an accidental fine tuning are the S-wave scattering of nucleons or of ^{4}He atoms. The scattering length of alkali atoms close to a Feshbach resonance can be tuned experimentally by adjusting the external magnetic field. At very low energies these systems behave similarly and show universal properties associated with large a [1]. We start with a brief review of the EFT for few-body systems with large a and then discuss some applications in nuclear and atomic physics.

Three-body system with large scattering length. We consider a two-body system of bosonic particles with large S-wave scattering length a and mass m. The generalization to fermions is straightforward. For momenta k of the order of the inverse scattering length $1/|a|$, the problem is nonperturbative in ka. The exact two-particle scattering amplitude can be obtained analytically by summing the so-called *bubble diagrams* with a 2-body contact interaction. The resulting amplitude reproduces the leading order of the effective range expansion for the particle-particle scattering amplitude: $f_{AA}(k) = (-1/a - ik)^{-1}$, where the total energy is $E = k^2/m$. If $a > 0$, f_{AA} has a pole at $k = i/a$ corresponding to a shallow dimer with binding energy $B_2 = 1/(ma^2)$. Higher-order derivative interactions are perturbative and generate corrections suppressed by powers of $\ell/|a|$. We now turn to the 3-body system. At leading order, the particle-dimer scattering amplitude is given by the integral equation shown in Fig. 1. The inhomogeneous term consists of the one-particle exchange and the 3-body contact interaction. The integral equation simply sums these diagrams to all orders. An ultraviolet cutoff Λ must be introduced in order to regulate the loop integrals involved. This cutoff guarantees that the integral equation has a unique solution. All physical observables, however, must be independent of Λ, which determines the behavior of the 3-body contact interaction H as a function of Λ [2]:

$$H(\Lambda) = \frac{\cos[s_0 \ln(\Lambda/\Lambda_*) + \arctan s_0]}{\cos[s_0 \ln(\Lambda/\Lambda_*) - \arctan s_0]}, \tag{1}$$

where $s_0 = 1.00624$ is a transcendental number and Λ_* is a 3-body parameter introduced by dimensional transmutation. It cannot be predicted by the EFT and must be determined from a 3-body
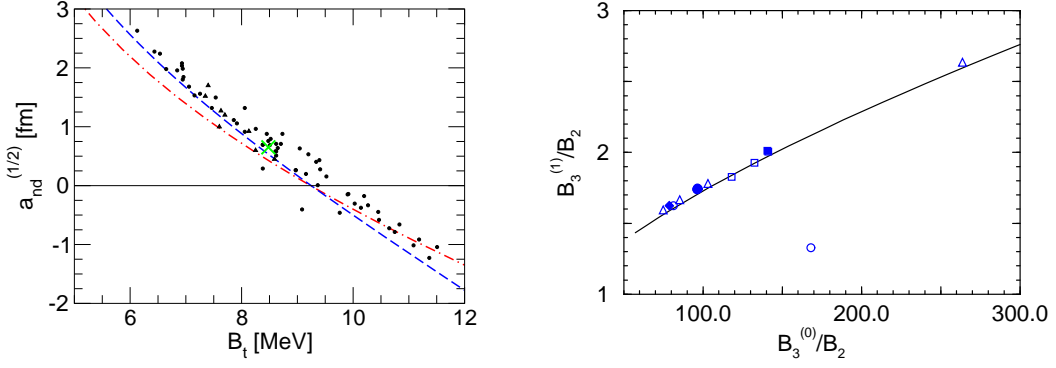
Figure 2: Universal correlations between the triton binding energy and the spin-doublet neutron-deuteron scattering length (left panel) and between ^{4}He trimer ground and excited state energies $B_3^{(0)}$ and $B_3^{(1)}$ (right panel).

observable. Note also that $H(\Lambda)$ is periodic and runs on a limit cycle. When Λ is increased by a factor of $\exp(\pi/s_0) \approx 22.7$, $H(\Lambda)$ returns to its original value. In summary, two parameters are required to specify a 3-body system at leading order in $l/|a|$: they may be chosen as the scattering length a (or equivalently B_2 if $a > 0$) and the 3-body parameter Λ_* [2]. This universal EFT confirms and extends the universal predictions for the 3-body system derived by Efimov including the *Efimov effect*, the accumulation of infinitely many 3-body bound states at threshold as $a \to \pm\infty$ [3].

Universal correlations. Since up to corrections of order $l/|a|$, low-energy 3-body observables depend on a and Λ_* only, they obey non-trivial scaling relations. If dimensionless combinations of such observables are plotted against each other, they must fall close to a line parametrized by Λ_* [1, 2, 3]. These correlations are purely driven by the large scattering length and are independent of the mechanism responsible for it. In Fig. 2, we show two examples of such universal correlations [1]. In the left panel, we show the Phillips line, a correlation between the triton binding energy and the doublet neutron-deuteron scattering length. In the right panel, we show the correlation between the ^{4}He trimer ground and excited state energies $B_3^{(0)}$ and $B_3^{(1)}$. The data points show calculations using various interaction potentials. Since these potentials have approximately the same scattering length but include different short-distance physics, the points on this line correspond to different values of Λ_*. The small deviations of the potential model calculations are mainly due to effective range effects. They are suppressed by $r_e/|a|$ and can be calculated at next-to-leading order. The extension of this EFT to the 4-body system requires no new 4-body parameter at leading order in $l/|a|$ [4]. Consequently, the universal correlations persist in the 4-body system.

An infrared renormalization group limit cycle in QCD. Nuclear phenomena can be described within a chiral EFT which has the explicit dependence on the quark masses [5]. It has been used to study the quark mass dependence of nuclear forces [6, 7]. The extrapolation of the S-wave nucleon-nucleon scattering lengths a_t (spin triplet) and a_t (spin singlet) to larger values of M_π predicts that a_t diverges and the deuteron becomes unbound at a critical value in the range $170\,\mathrm{MeV} < M_\pi < 210$ MeV. It is also predicted that a_s is likely to diverge and the spin-singlet deuteron becomes bound at some critical value of M_π not much larger than 150 MeV. Based on this behavior it was conjectured that one should be able to reach the critical point by varying the up- and down-quark masses m_u and m_d independently because the spin-triplet and spin-singlet channels have different isospin [8].
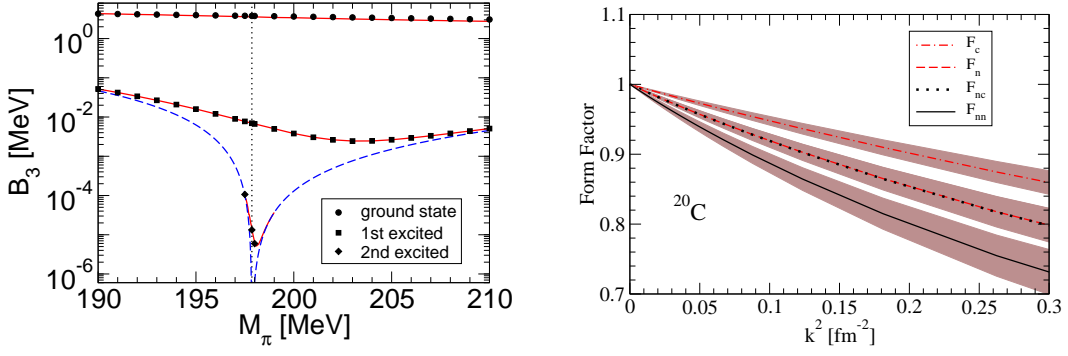
Figure 3: Left panel: Binding energies of the triton ground and first two excited states in the critical region as a function of M_π. Right panel: The one- and two-body matter density form factors F_c, F_n, F_{nc}, and F_{nn} with leading order error bands for the ground state of ^{20}C as a function of the momentum transfer k^2.

In this case, the triton would display the Efimov effect which corresponds to the occurence of an infrared limit cycle in QCD. In Refs. [9, 10], the properties of the triton around the critical pion mass were studied for one particular solution with a critical pion mass $M_\pi^{crit} = 197.8577$ MeV. The binding energies of the triton and the first two excited states in the vicinity of the limit cycle were calculated for this scenario in chiral EFT. They are given in the left panel of Fig. 3 by the circles, squares, and diamonds. The dashed lines indicate the neutron-deuteron ($M_\pi \leq M_\pi^{crit}$) and neutron-spin-singlet-deuteron ($M_\pi \geq M_\pi^{crit}$) thresholds where the 3-body states become unstable. Directly at the critical mass, these thresholds coincide with the 3-body threshold and the triton has infinitely many excited states. The solid lines are leading order calculations in the universal EFT using the pion mass dependence of the nucleon-nucleon scattering lengths and one triton state from chiral EFT as input. Both calculations agree very well and the universal EFT has also been used to calculate scattering [10]. The binding energy of the triton ground state varies only weakly over the whole range of pion masses. The excited states, however, are more influenced by the thresholds and vary strongly. Whether the limit cycle can be realized in QCD can only be answered definitely by directly solving QCD. In particular, one would like to know whether this can be achieved by approriately tuning the quark masses in a Lattice QCD simulation [11].

Halo Nuclei. Halo nuclei can be described by extensions of the universal EFT. One can assume the core to be structureless and treats the nucleus as a few-body system of the core and the valence nucleons. Corrections from the structure of the core appear in higher orders and can be included in perturbation theory. A new facet is the appearance of resonances as in the neutron-alpha system [12]. The first application of effective field theory methods to halo nuclei was carried out in Refs. [12, 13], where the $n\alpha$ system ("^{5}He") was considered. It was found that for resonant P-wave interactions both the scattering length and effective range have to be resummed at leading order. At threshold, however, only one combination of coupling constants is fine-tuned and the EFT becomes perturbative. More recent studies have focused on the consistent inclusion of the Coulomb interation in two-body halo nuclei such as the $p\alpha$ and $\alpha\alpha$ systems [14, 15].

Three-body halo nuclei composed of a core and two valence neutrons have the possibility to exhibit excited states due to the Efimov effect [16]. A comprehensive study of S-wave halo nuclei in EFT including structure calculations with error estimates was recently carried out in Ref. [17].

Currently, the only possible candidate for an excited Efimov state is ^{20}C, which consists of a core nucleus with spin and parity quantum numbers $J^P = 0^+$ and two valence neutrons. The value of the ^{19}C energy, however, is not known well enough to make a definite statement about the appearance of an excited state in ^{20}C. The structure of halo nuclei can also be calculated in the halo EFT. As an example, we show the various one- and two-body matter density form factors F_c, F_n, F_{nc}, and F_{nn} (for a definition, see [17]) with leading order error bands for the ground state of ^{20}C as a function the momentum transfer k^2 in the right panel of Fig. 3. The theory breaks down for momentum transfers of the order of the pion-mass squared ($k^2 \approx 0.5$ fm^{-2}). From the slope of the form factors one can extract the radii: $F(k^2) = 1 - \frac{1}{6}k^2 \langle r^2 \rangle + \dots$. Experimental information on these radii is available for some halo nuclei. For the neutron-neutron radius of the Borromean halo nucleus ^{14}Be for example, the leading order halo EFT result is $\sqrt{\langle r_{nn}^2 \rangle} = 4.1 \pm 0.5$ fm. The value $\sqrt{\langle r_{nn}^2 \rangle_{exp}} = 5.4 \pm 1.0$ fm was obtained from 3-body correlations in the dissociation of ^{14}Be using intensity interferometry and Dalitz plots [18]. Within the errors there is good agreement between both values. Results for further halo nuclei are given in Ref. [17]. With upcoming experiments much more knowledge can be obtained about the structure of these intriguing systems as well as discovering whether they show universal behavior and excited Efimov states.

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