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Full Salpeter Equation with Confining Interactions: Analytic Stability Proof

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The most popular 3-dimensional reduction of the Bethe–Salpeter formalism for the description of bound states within quantum field theory is the Salpeter equation, found as the *instantaneous* limit of the Bethe–Salpeter framework if allowing, *in addition*, for *free* propagation of the bound-state constituents. Unfortunately, depending on the chosen Lorentz nature of the Bethe–Salpeter kernel, which encodes all interactions between the bound-state constituents, supposedly stable results of Salpeter's equation with *confining* interactions exhibit (un-)expected instabilities, probably related to Klein's paradox. Clearly, bound states may be regarded as stable if, for appropriate interactions, their energy eigenvalues or, in the center-of-momentum system, their mass eigenvalues belong to a *real* and *discrete* (part of the) *spectrum* that is *bounded from below*. Discreteness means either that eigenvalues and a possible continuous spectrum are *disjoint* or that the spectrum is *purely discrete*. Some general features of the eigenvalue spectra of any Salpeter equation, common to all solutions, are well-established: Since Salpeter's equation proves to be of the same algebraic structure as the random-phase-approximation equation familiar in nuclear physics all *bound-state masses squared* are real. Of course, this implies neither that the mass spectrum itself is necessarily real nor, even in those cases where it may be shown to be real, that it is also bounded from below. Direct inspection reveals that, for a large class of sufficiently reasonable Bethe–Salpeter kernels—which includes all interactions of phenomenological relevance in QCD—, the spectrum of mass eigenvalues consists, in the complex bound-state mass plane, at most of *real* pairs of opposite sign and *imaginary* points. In general, if the Lorentz structure of a confining kernel is a mixture of time-component vector and scalar stability is achieved only if the Lorentz structure is predominantly a time-component vector. For time-component vector *harmonic-oscillator* kernels, a straightforward stability proof is given.

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Building on experience gained in earlier investigations focused to the simpler *reduced* Salpeter equation [1, 2] and its improvement [3, 4] obtained by inclusion of full (dressed) propagators for the bound-state constituents [5, 6], a rigorous proof of the *full*-Salpeter solutions' stability for confining (such as harmonic-oscillator) interactions of *time-component Lorentz-vector* nature is constructed. For interactions described, in configuration space, by central potentials of harmonic-oscillator form, $V(\mathbf{x}) = a\,\mathbf{x}^2$, $a \neq 0$, the *integral* equation representing the Salpeter equation simplifies to a system of second-order homogeneous linear ordinary *differential* equations. Instabilities should appear first in *pseudoscalar* bound states, and this problem is most severe and restrictive for massless constituents. Such setting allows for a thorough spectral analysis of the problem: The interaction of the fermionic bound-state constituents is of time-component Lorentz-vector nature if both their couplings to some effective Lorentz-scalar potential are represented by the Dirac matrix γ^0. In this case, bound states exist for $a > 0$; the *squares* of their masses are determined by the eigenvalues of the product $O \equiv BA$ of two positive self-adjoint (Schrödinger) operators A and B on the Hilbert space $L^2(\mathbb{R}^3)$, defined by

$$A \equiv -a\Delta + 2|\mathbf{x}| = A^\dagger \geq 0 \ , \qquad B \equiv -a\Delta + 2|\mathbf{x}| + \frac{2a}{\mathbf{x}^2} = B^\dagger \geq 0 \ , \qquad \Delta \equiv \nabla \cdot \nabla \ .$$

Using the (unique) positive self-adjoint square root $A^{1/2} = (A^{1/2})^\dagger \geq 0$ of A, the eigenvalue problem for O is found to be equivalent to that for the *positive self-adjoint* operator $Q \equiv A^{1/2} B A^{1/2} = Q^\dagger \geq 0$ while the *operator inequality* $A \leq B$, clearly satisfied by A and B, translates into the relation $A^2 \leq Q$. Because its potential, $2|\mathbf{x}|/a$, is bounded from below and rises to infinity for $|\mathbf{x}| \to \infty$, the operator A and thus also the operator A^2 have *purely discrete* spectrum consisting of real eigenvalues which rise to infinity. Then, applying the minimum–maximum principle to $A^2 \leq Q$ reveals that our bound-state masses squared and, since $Q \geq 0$, also all the masses themselves are part of a *real discrete spectrum*.

References

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