PROCEEDINGS OF SCIENCE

Poles in $K^-\pi^+$ Amplitude

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We present a simple chiral model for the $J = 0$, $I = 1/2$, elastic $K\pi$ amplitude which allows a transparent determination of its poles and preserve the essential physics. In the case of the K-matrix approximation, the model yields a quadratic equation in s. The solutions to this equation can then be well approximated by polynomials of masses and coupling constants. This analytic structure allows a clear understanding why, depending on the values of one of the coupling constants, one may have one or two physical poles. The model yields a pole, associated with the κ, at $\sqrt{s} = (0.75 - i0.24)$ GeV.

8th Conference Quark Confinement and the Hadron Spectrum
September 1-6 2008
Mainz, Germany

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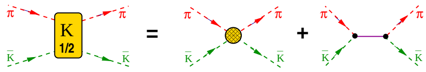
The $K^-\pi^+$ elastic scaterring amplitude for $(J,I)=(1/2,0)$ is discribed by the diagram

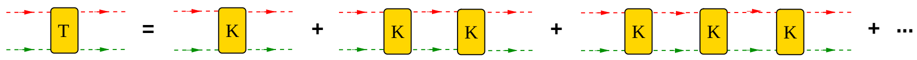


The contact[1] and resonant[2] term are derived from $SU(3)\times SU(3)$ chiral effective lagrangians

$$\mathscr{L}^{(2)} = \frac{F^2}{4}\,\langle \nabla_\mu U^\dagger\,\nabla^\mu U + \chi^\dagger U + \chi U^\dagger\rangle + c_d\,\langle S\,u_\mu\,u^\mu\rangle + c_m\,\langle S\,\chi_+\rangle\,. \tag{1}$$

U is the pseudoscalar field, S represent scalar resonaces and c_d and c_m are scalar-pseudoscalar coupling constants.

The $(J,I)=(0,1/2)$ amplitude is unitarized considering all $K\pi$ buble loop interactions[3]



and the amplitude is written as

$$T_{1/2}(s) = \gamma^2(s)/D(s)\,,$$
$$D(s) = [m_R^2 - s + \gamma^2(s)\,\bar{R}_{1/2}(s)] - i\left[\gamma^2(s)\,\frac{\rho(s)}{16\pi}\right], \tag{2}$$

where:
- s is the usual Mandelstam variable and $\rho(s)=\sqrt{1-2\,(M_K^2{+}M_\pi^2)/s+(M_K^2{-}M_\pi^2)^2/s^2}$;
- m_R is the parameter present in the chiral lagrangian, called *nominal* resonance mass;
- $\bar{R}_{1/2}(s)$ is the function describing off-shell effects in the two-meson propagator, given by

$$\begin{aligned}\bar{R}_{1/2}(s) &= -\Re\,[L(s)-L(m_R^2)]\,/16\pi^2\,,\\ \Re L(s) &= \rho(s)\log[(1-\sigma)/(1+\sigma)] - 2 + [(M_K^2-M_\pi^2)/s]\,\log(M_K/M_\pi)\,,\\ \sigma &= \sqrt{|s{-}(M_K{+}M_\pi)^2|/|s{-}(M_K{-}M_\pi)^2|}\,;\end{aligned} \tag{3}$$

- $\bar{R}_{1/2}(m_R^2)=0$ by construction and therefore the phase shift is $\pi/2$ at $s=m_R^2$;
- $\gamma^2(s)$ is the function which incorporates chiral dynamics, given by

$$\begin{aligned}\gamma^2(s) &= \{(1/F^2)\,[(1-3\,\rho^2(s)/8)\,s-(M_\pi^2+M_K^2)]\;(m_R^2-s)\}_L\\ &\quad+\{(3/F^4)\,[c_d\,(s{-}M_\pi^2{-}M_K^2)+c_m\,(4M_K^2{+}5M_\pi^2)\,/6]^2\}_R\,.\end{aligned} \tag{4}$$

Poles are zeros in $D(s)$ (2). In the results from numerical solution is dificult to identify dynamic. In other way, analitical solution are approximation but transparent physics. To find the anlitical equation we consider $m_\pi=0\;\Rightarrow\;SU(2)$ limit and K-matrix approximation $\rightarrow\;\bar{R}_{1/2}(s)=0$. Then $D(s)$ became a quartic function

$$\begin{aligned}&\left(\frac{5}{8}-\frac{3c_d^2}{8}\right)s^4+\left[-(5m_r^2+7m_K^2)/8+\frac{c_d}{F^2}(9c_d-4c_m)m_K^2\;+i16\pi F^2\right]s^3\\ &\quad+\left[(7m_r^2-m_K^2)\frac{m_K^2}{8}-(c_d-2c_m/3)(9c_d-2c_m)\,\frac{M_K^4}{F^2}\,m_r^2\;-i16\pi F^2\right]s^2\\ &+\left[(m_r^2\,+3m_K^2)/8+3(c_d-2c_m/3)^2\,\frac{m_K^2}{F^2}\right]M_K^4\,s\;-3m_r^2M_K^6/8=0\end{aligned} \tag{5}$$

Close to pole position $m_K^2/|s|\ll 1$ and $D(s)$ is reduced to a quadratic function

$$\begin{aligned}A\,s^2+B\,s+C=0\qquad &A=[5/8\;-\;3c_d^2/F^2]\,;\\ &B=[-\,(5m_R^2\;+\;7M_K^2)\,/8+c_d(9c_d-4c_m)\,\frac{M_K^2}{F^2}+i16\pi F^2]\,;\\ &C=[\,7M_K^2/8\;-\;i16\pi F^2]\,m_R^2\,.\end{aligned} \tag{6}$$

The coefficient $A = 5/8 - 3c_d^2/F^2$ is very important. $A = 0 \to c_d/F = \sqrt{5/24} = 0.047$ and the quadratic function have a single solution

$$s_-(0) = \frac{[7M_K^2/5 - i\,128\pi F^2/5]}{1 + \left[\frac{7M_K^2}{5} - 8c_d(9c_d - 4c_m)\frac{M_K^2}{F^2} - i\,128\pi\frac{F^2}{5}\right]/m_R^2}\,. \tag{7}$$

In $A = 5/8 \longrightarrow c_d = 0$ resonance R is decoupled bound state in the real axis,

$$s_+(5/8) = m_R^2 \quad \text{and} \quad s_-(5/8) = \left[7M_K^2/5 - i\,128\pi F^2/5\right]\,; \tag{8}$$

Analitical solution: (approximate),

$$s_+ = \frac{1}{A}\left\{\frac{5}{8}m_R^2 - \frac{c_d}{F}\left(\frac{24c_d}{5F} - \frac{4c_m}{F}\right)M_K^2 - \frac{3c_d^2}{m_R^2F^2}\left(1 - \frac{24c_d^2}{5F^2}\right)\left(\frac{128\pi F^2}{5}\right)^2\right.$$
$$\left. - i\,\frac{c_d}{F}\left[3\frac{c_d}{F} - \left(\frac{3c_d}{5F} - \frac{4c_m}{F}\right)\frac{M_K^2}{m_R^2} - \frac{3c_d}{F}\left(1 - \frac{24c_d^2}{5F^2}\right)\left(\frac{128\pi F^2}{5m_R^2}\right)^2\right]\frac{128\pi F^2}{5}\right\}, \tag{9}$$

$$s_- = \frac{7}{5}M_K^2 + \frac{24m_R^2c_d^2}{5F^2}\left(\frac{128\pi F^2}{5m_R^2}\right)^2 - i\left[1 - \frac{24c_d^2}{5F^2}\left(\frac{128\pi F^2}{5m_R^2}\right)^2\right]\frac{128\pi F^2}{5}\,. \tag{10}$$

The graphic 1, shows that the inclusion of the pion mass is not numerically important and off-shell

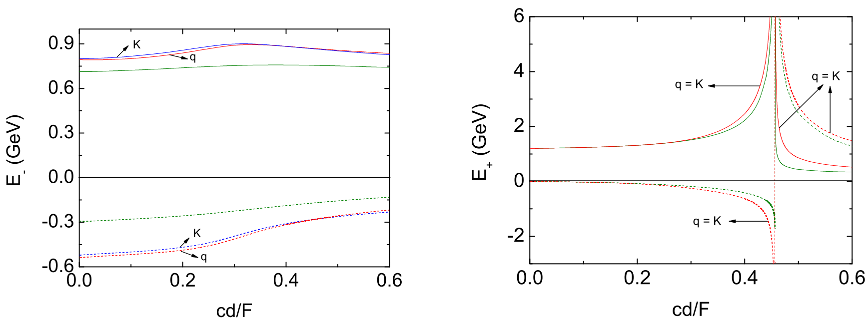


Figure 1: Real (full) and imaginary (dashed) components of respectively functions $E_- = \sqrt{s_-}$ and $E_+ = \sqrt{s_+}$. Green ≡numerical; K and q are K-matrix and quadratic approximation.

effects in the two-meson propagator do influence the positions of the poles. We recognize $\sqrt{s_+}$ as been $K_0^*(1430)$ and $\sqrt{s_-}$ as been κ.

The following scenario is supported by eqs.(9-10):

- if resonance R is absent we have only $\sqrt{s_-}$ originated in contact interaction;
- if resonance R is present and $c_d = c_m = 0$, we have a bound state $\sqrt{s_+}$ in the real axis at $s = m_R^2$;
- if $c_d \neq 0$ the mass and width of $\sqrt{s_+}$, eq.(9), *increase* monotonically, driven by the factor A in the denominator;
- the pole $\sqrt{s_+}$ blows up at the critical value $c_d/F = \sqrt{5/24} \;\to$ beyond this point, just $\sqrt{s_-}$ is present.

Prediction: $K_0^*(1430)$ pole at $[(1.414 \pm 0.006) - i\,(0.145 \pm 0.010)]$ GeV$\Rightarrow m_R = 1.1865 \pm 0.079$ GeV and $c_d = 0.02786 \pm 0.00078$ GeV

$\Rightarrow \kappa$ pole at $(0.7505 \pm 0.0010) - i\,(0.2363 \pm 0.0023)$ GeV.

References

[1] G. Colangelo, J. Gasser and H. Leutwyler, Nucl. Phys. B **603** (2001) 125.

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[3] J.A. Oller and E. Oset, Nucl. Phys. A **620** (1997) 438.