PROCEEDINGS OF SCIENCE

Extra Dimensions and their Ultraviolet Completion

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Large extra dimensions are one of the constructions addressing the hierarchy problem of the Standard Model. Their main theoretical and phenomenological challenge is that already predicting LHC effects requires an ultraviolet completion of TeV-scale gravity. In these lecture notes we first give a basic introduction into TeV-scale gravity models and their collider effects and then discuss possible ultraviolet completions, like string theory and fixed-point gravity.

Workshop on Continuum and Lattice Approaches to Quantum Gravity
September 17-19 2008
Brighton, University of Sussex, United Kingdom

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PoS(CLAQG08)009

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Many years of experimental tests have lead us to accept the Standard Model as the effective theory valid at and below the weak energy scale. This includes its range in terms of direct particle searches as well as high-precision tests of quantum effects, both of them related by the renormalizability of gauge theories. However, the crucial ingredient of electroweak symmetry breaking is not yet understood, *i.e.* we still have not seen a fundamental scalar Higgs boson or any kind of indication of strong interactions breaking the electroweak $SU(2)_L \times U(1)_Y$ symmetry to the observed electromagnetic $U(1)_Q$. While the search for some variation of a simple fundamental Higgs scalar has been the major motivation in experimental searches as well as theory model building for at least the past 25 years there is a little more to it. To date we see four serious problems with our Standard Model

1. experimentally, it does not include dark matter, even though generically dark matter could be explained by a stable weak-scale particle with typical weak-scale Standard Model couplings [1].

2. theoretically, a truly fundamental quantum theory including masses for the W and Z gauge bosons should either include a Higgs boson or an additional strong interaction with its appropriate resonances. All we know to date is that a light Higgs scalar is consistent with electroweak precision data [2].

3. if the Higgs boson is a fundamental scalar, its mass has to be protected. Otherwise, quantum corrections would betray the underlying principle of fundamental gauge theories and force us to order by order fine tune a counter term to stabilize the fragile Higgs mass — the hierarchy problem [3]. Decoupling a corresponding new-physics sector from the electroweak precision data mentioned in point (2) can be achieved with a discrete symmetry which in passing introduces a stable dark matter particle as required by point (1).

4. gravity is not included in this picture of particle physics, even though we know that it includes the remaining fourth fundamental force between particles.

Note that this is of course not a complete list of problems in fundamental physics, which would have to include the cosmological constant, the baryon asymmetry of the Universe, or the absence of gravitational waves. This list simply includes issues which might well be solved by TeV-scale new physics.

On the other hand, this list makes it obvious that Higgs searches, or searches for the mechanism of electroweak symmetry breaking, cannot be separated from searches for TeV-scale new physics. Both are different sides of the same medal. Proof that all four problems can indeed be linked together is given by supersymmetry: by roughly doubling the Standard Model's particle spectrum above the TeV scale it provides a dark matter candidate, radiatively breaks electroweak symmetry, stabilizes the Higgs mass, allows for a perturbative extrapolation to high energies (including a grand unified theory) and links the Standard Model to a local theory of gravity. The problem is that even the minimal supersymmetric Standard Model can be viewed as more on the elaborate than on the minimal side. Instead, we can ask the question: how far can we get in solving as many of the above issues with as little extra input as possible?

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Tackling the last of the problems listed above we run into a fundamental problem of field theory — we know from the classical theory that the gravitational coupling carries a mass dimension, which means we cannot quantize it in a perturbatively renormalizable manner. What we can do is explicitly exclude the possibly dangerous high-energy regime and treat gravity as an effective field theory, *i.e.* a theory with a built-in cutoff scale which should nevertheless describe low-energy observables well. In Sections 1 and 2 we will construct two such effective theories of extra-dimensional gravity valid up to LHC energies and show the limitations of this approach. In Section 3 we will compute the same observables based on two ultraviolet completions of gravity which cure the poor ultraviolet behavior of the effective theory of gravity.

1. Flat extra dimensions

One answer to this question is given by large extra dimensions [4, 5]. This model has the most important feature that it does not introduce any additional states, which in turn means that we would have to invoke some other mechanism to explain dark matter. But as we will see below, it does successfully tackle the three other problems and might even offer an explanation for the small cosmological constant.

Initially, large extra dimensions were suggested as an explanation for the observed hierarchy between the electroweak and Planck [4] or GUT [6] scales while allowing the Higgs mass to remain comfortably at a mass around 100 GeV. Such models with large (compared to the Planck length) and flat extra dimensions are referred to as ADD models. Their basis is a low fundamental Planck scale ($M_\star \sim$ TeV) which also locates the onset of quantum gravitational effects. This new scale serves as the ultraviolet cutoff in the loop contributions to the renormalized Higgs mass, which limits the size of quadratic quantum corrections. This construct appears to be in clear contradiction to all 4-dimensional data which determines the Planck mass from Newton's constant $G_N \sim 1/M^2_{\text{Planck}}$, describing the force on an object in a gravitational field. The ADD model solves this apparent contradiction by deriving the observed value of M_{Planck} from the fundamental Planck mass $M_\star$ and a a particular geometry of space-time.

At the classical level we can see how this occurs in a universe with extra spatial dimensions. The Einstein-Hilbert action in any number of $(4+n)$ dimensions is given as

$$S_{\text{bulk}} = -\frac{1}{2}\int d^{4+n}x\,\sqrt{-g^{(4+n)}}\,M_\star^{n+2}\,R^{(4+n)}. \tag{1.1}$$

We denote 4-dimensional space-time coordinates with Greek indices $\mu,\nu,\alpha = 0,1,2,3$ and extra dimensional coordinates with lower case Roman letters $a,b,c = 5,6,7,\cdots n$. These are unified to capital Roman letters $M,N,L = 0,1,2,3,5,\cdots n$. The 4-dimensional coordinates we write as x_μ, while the $(4+n)$-dimensional coordinates are y_a, such that $z_M = x_\mu + y_a$.

The Einstein Hilbert action has a number of interesting features: first of all, the propagating degrees of freedom are carried exclusively by the the metric g_{MN}. To act as a metric in the conventional sense (most notably connecting vectors to form inner products) it is symmetric, and to produce the correct mass dimensions, g_{MN} must be dimensionless. The mass dimension of the Ricci scalar is independent of the underlying space-time dimension. This is merely the statement that the dimensionality of R is completely determined by derivatives and not fields. The action eq.(1.1) enjoys full $(4+n)$-dimensional invariance under general coordinate transformation with an arbitrary parameter $\xi_M(z)$

$$z_M \to z_M + \xi_M(z), \tag{1.2}$$

where the induced variation in the metric is

$$\delta g_{AB} = \partial_A \xi^M g_{MB} + \partial_B \xi^M g_{MA} + \xi^M \partial_M g_{AB}. \tag{1.3}$$

The ADD model breaks this symmetry explicitly by treating the 4-dimensional space x_μ and the n-dimensional space y_i differently. Coordinate transformation can no longer mix these components. The requirements on the space described by metric g_{ij} are

- *spatial*: the signature for the n extra dimensions is $(-1,-1,\cdots)$.
- *separable*: the extra dimensions must be orthogonal to the brane so that the measure $d^{4+n}z$ is well defined. In other words, the metric decomposes as a product space $g^{(4+n)} = g^{(4)} \otimes g^{(n)}$.
- *flat*: the dimensions must be flat so that they can be integrated out explicitly in the action. In standard gravity the same is true unless sources induce $T_{ij} \neq 0$. We therefore restrict matter to the $y_i = 0$ brane:

$$T_{AB}(x;y) = \eta_A^\mu \, \eta_B^\nu \, T_{\mu\nu}(x) \, \delta^{(n)}(y) = \begin{pmatrix} T_{\mu\nu}(x)\,\delta^{(n)}(y) & 0 \\ 0 & 0 \end{pmatrix}. \tag{1.4}$$

  The assumption of an infinitely thin brane for our 4-dimensional world might have to be weakened to generate realistic higher-dimensional operators for flavor physics or proton decay [7, 8]. Einstein's equation purely in the extra dimensions

$$R_{jk} - \frac{1}{n+2} g_{jk} R = 0. \tag{1.5}$$

  contracted with g^{jk} requires $R = 0$. The full Ricci scalar is then

$$R = g_{MN}R^{MN} = g_{\mu\nu}R^{\mu\nu} + g_{ij}R^{ij} + g_{i\mu}R^{i\mu} = g_{\mu\nu}R^{\mu\nu} = R^{(4)} \ , \tag{1.6}$$

  using $g^{jk}R_{jk} = 0$ along with the fact that $g_{\mu i}$ no longer transforms under general coordinate transformations.
- *compact/periodic*: the simplest compact space is a torus with periodic boundary conditions and a radius r of the compactified dimension $y_i = y_i + 2\pi r$.

In addition to the Ricci scalar, the Einstein–Hilbert action contains explicit dependency on the determinant of the metric $\sqrt{-g^{(4+n)}}$. Since the extra dimensions are flat and spatial, the contribution to $\det(g_{MN}) \equiv g$ is at most a sign. We assume that $\sqrt{-g}$ is synonymous with $\sqrt{|g|}$. Now, it is straightforward to simplify the higher–dimensional bulk action

$$\begin{aligned} S_{\text{bulk}} &= -\frac{1}{2} M_\star^{n+2} \int d^{4+n}z \, \sqrt{-g^{(4+n)}} \, R^{(4+n)} \\ &= -\frac{1}{2} M_\star^{n+2} \, (2\pi r)^n \int d^4x \, \sqrt{-g^{(4)}} \, R^{(4)} \\ &\equiv -\frac{1}{2} M_{\text{Planck}}^2 \int d^4x \sqrt{-g^{(4)}} \, R^{(4)}. \end{aligned} \tag{1.7}$$

In the last line we have matched the two theories, *i.e.* we have assumed that from a 4-dimensional point of view the actions have to be identical, as long as we do not probe high enough energy scales to observe quantum gravity effects.

This leads us to the basis of extra dimensions as a solution to the hierarchy problem: our 4-dimensional Planck scale $M_{\text{Planck}} \sim 10^{19}$ GeV is not the fundamental scale of gravity. It is merely a derived parameter which depends on the fundamental $(4+n)$–dimensional Planck scale and the geometry of the extra dimensions, *e.g.* the compactification radius of the n–dimensional torus. Matching the two theories translates into

$$M_{\text{Planck}} = M_\star \,(2\pi r M_\star)^{n/2} \tag{1.8}$$

If the proportionality factor $(2\pi r M_\star)^n$ is large we can postulate that the fundamental Planck scale $M_\star$ be not much larger than 1 TeV. In that case the UV cutoff of our field theory is of the same order as the Higgs mass and there is no problem with the stability of the two scales.

Assuming $M_\star = 1$ TeV we can solve the equation above for the compactification radius r — transferring the hierarchy problem into space-time geometry:

n	r
1	10^{12} m
2	10^{-3} m
3	10^{-8} m
...	...
6	10^{-11} m

At least in the simplest model $\delta = 1$ is ruled out by classical bounds on gravity as well as astrophysical data. A possible exception is if there is a non trivial mass gap between massless and massive excitations [9]. For larger values of n we need to test Newtonian gravity at small distances [10]. Note that the analysis in this section is purely classical, and it is obvious that its physical degrees of freedom do not survive compactification. For this we resort to the original ideas of Kaluza and Klein and decompose the higher-dimensional gravitational theory as an effective 4-D theory with residual gauge symmetries [11].

1.1 Gravitons in extra dimensions

The first step towards a viable description of extra dimensional effects in experiment is deriving the properties of spin-2 gravitons in these extra dimensions [12, 13]. Generically, a massless graviton in higher dimensions can be described by an effective theory of massive gravitons and gauge fields in four dimensions. The inclusion of massive spin two fields is particularly interesting from a theoretical point of view since the Pauli–Fierz mass term [14] and the coupling to matter fields is highly restricted. In particular, it is inconsistent to introduce massive spin-2 fields not originating from some type of Kaluza–Klein decomposition [15].

We start with the $(4+n)$-dimensional Einstein equation

$$R_{AB} - \frac{1}{2} g_{AB} R \;=\; \frac{T_{AB}}{M_\star^{2+n}} \tag{1.9}$$

and rewrite the metric in terms of our flat background metric η_{AB} and a fluctuating spin-2 field h_{AB}

$$g_{AB} = \eta_{AB} + 2\frac{h_{AB}}{M_\star^{1+n/2}} . \tag{1.10}$$

The prefactor ensures that h (and with it the kinetic term in the Lagrangian) has the appropriate mass dimension for a propagating bosonic field $[h] = m^{(2+n)/2}$. In terms of h, Einstein's equations to linear order give

$$\begin{aligned} M_\star^{1+n/2}\left(R_{AB} - \frac{1}{2}g_{AB}R\right) &= \Box h_{AB} - \partial_A\partial^C h_{CB} - \partial_B\partial^C h_{CA} + \partial_A\partial_B h_C^C - \eta_{AB}\Box h_C^C + \eta_{AB}\partial^C\partial^D h_{CD} \\ &= -\frac{T_{AB}}{M_\star^{1+n/2}} . \end{aligned} \tag{1.11}$$

The equation of motion follows from the bilinear action which we refer to as the linearized Einstein–Hilbert action:

$$\mathscr{L} = -\frac{1}{2}h^{MN}\Box h_{MN} + \frac{1}{2}h\Box h - h^{MN}\partial_M\partial_N h + h^{MN}\partial_M\partial_L h_L^N - M_\star^{-(1+n/2)}h^{MN}T_{MN}, \tag{1.12}$$

The slightly circumvent logic (Einstein–Hilbert action→ Einstein's equation → linearized Einstein's equation → linearized Einstein–Hilbert action) leading us to eq.(1.12) is necessary because the energy momentum tensor is generated through $T^{\mu\nu} = 2/\sqrt{-g}\;\delta S/\delta g_{\mu\nu}$ when computing the equations of motion. Had we inserted the graviton decomposition into the Einstein–Hilbert action directly, the resulting linearized Einstein equations would describe a freely propagating field. The linearized variation analogous to eq.(1.3) is

$$\delta h_{AB} = \partial_A\xi^M + \partial_B\xi^M , \tag{1.13}$$

leaving the linearized action invariant up to terms $\mathscr{O}(\xi^2)$ with h and ξ treated as the same order.

The ADD model breaks this full symmetry by compactifying the extra dimensions. Periodic boundary conditions allow us to Fourier decompose the y component of the graviton field

$$\begin{aligned} h_{AB}(z) &= \sum_{m_1=-\infty}^{\infty}\cdots\sum_{m_n=-\infty}^{\infty}\frac{h_{AB}^{(\vec{n})}(x)}{\sqrt{(2\pi r)^n}}e^{in_jy_j/r} \\ &= h_{AB}^{(0)}(x) + \sum_{n_1=1}^{\infty}\cdots\sum_{n_n=1}^{\infty}\frac{1}{\sqrt{(2\pi r)^n}}\left[h_{AB}^{(\vec{n})}(x)e^{in_jy_j/r} + h_{AB}^{\dagger(\vec{n})}(x)e^{-in_jy_j/r}\right] \end{aligned} \tag{1.14}$$

where $h_{AB}^{(\vec{n})}(x)$ is a four dimensional bosonic field with mass dimension one. The second step is possible because $h_{AB}(z)$ is real.

To avoid confusion we emphasize that $h_{AB}^{\dagger(\vec{n})}(x)$ does not constitute an additional degree of freedom in the theory. The internal index $\vec{n}$ can be thought of as a discretized momentum index, such that $h_{AB}^{(\vec{n})}(x)$ and $h_{AB}^{\dagger(\vec{n})}(x)$ differ only by the sign of the extra-dimensional momentum $h_{AB}^{\dagger(\vec{n})}(x) = h_{AB}^{(-\vec{n})}(x)$. This is also obvious from the fact that $h_{AB}^{(\vec{n})}(x)$ and $h_{AB}^{(\vec{n}')}(x)$ are not distinct field excitations. It is now simple to work out the form of Einstein's equations in terms of the field $h_{AB}^{(\vec{n})}(x)$. For example, the first term in eq.(1.11) decomposes as

$$\begin{aligned}\Box^{(4+n)} h_{AB}(z) &= \sum_{n_j} \frac{1}{(2\pi r)^{n/2}} \partial_C \partial^C \left[h_{AB}^{(\vec{n})}(x)\, e^{i(n\cdot y)/r} \right] \\ &= \sum_{m_j} \frac{1}{(2\pi r)^{n/2}} \partial_C \left[\left(\delta_\mu^C \partial^\mu h_{AB}^{(\vec{n})}(x) + \delta_j^C h_{AB}^{(\vec{n})}(x) \frac{i n_j}{r} \right) e^{i(n\cdot y)/r} \right] \\ &= \sum_{m_j} \frac{1}{(2\pi r)^{n/2}} \left[\Box^{(4)} - \frac{n^j n_j}{r^2} \right] h_{AB}^{(\vec{n})}(x)\, e^{i(n\cdot y)/r}\end{aligned} \tag{1.15}$$

Multiplying by $e^{-i(n\cdot y)/r}$ and using the energy momentum tensor from eq.(1.4) eliminates the exponential on the right-hand side of eq.(1.11). An independent check on the consistency of this method is that the 4-dimensional massive graviton field $h_{\mu\nu}^{(\vec{n})}(x)$ only has a Pauli–Fierz mass term, $\propto \left[h_{\mu\nu} - \eta_{\mu\nu} h \right]$ and no mass terms originating from mixed index derivatives. This is required for a consistent spin-2 field [14].

From the Einstein equations we can brute force derive the action bilinear in the fields $h_{AB}^{(\vec{n})}(x)$. This field does not transform irreducibly under the Lorentz group in four dimensions. As an ansatz we introduce a field decomposition [12] which forms irreducible representations. Using the convenient definitions $\hat{n} \equiv \vec{n}/r$ and $\kappa \equiv \sqrt{3(n-1)/(n+2)}$, the action in terms of these new fields manifestly carries the correct degrees of freedom:

$$\begin{aligned} G_{\mu\nu}^{(\vec{n})} &= h_{\mu\nu} + \frac{\kappa}{3} \left(\eta_{\mu\nu} + \frac{\partial_\mu \partial_\nu}{\hat{n}^2} \right) H^{(\vec{n})} - \partial_\mu \partial_\nu P + \partial_\mu Q_\nu + \partial_\nu Q_\mu \\ V_{\mu j}^{(\vec{n})} &= \frac{1}{\sqrt{2}} \left(i h_{\mu j} - \partial_\mu P_j - \hat{n}_j Q_\mu \right) \\ S_{jk}^{(\vec{n})} &= h_{jk} - \left(\eta_{jk} + \frac{\hat{n}_j \hat{n}_k}{\hat{n}^2} \right) \frac{H^{(\vec{n})} \kappa}{n-1} + \hat{n}_j P_k + \hat{n}_k P_j - \hat{n}_j \hat{n}_k P \\ H^{(\vec{n})} &= \frac{1}{\kappa} \left[h_j^j + \hat{n}^2 P \right] \\ Q_\mu^{(\vec{n})} &= -i \frac{\hat{n}_j}{\hat{n}^2} h_\mu^j \\ P_j^{(\vec{n})} &= \frac{\hat{n}_k}{\hat{n}^2} h_j^k + \hat{n}_j P \\ P^{(\vec{n})} &= \frac{\hat{n}_k \hat{n}_j}{\hat{n}^4} h_{jk} \end{aligned} \tag{1.16}$$

The fields Q_μ, P_j and P are not invariant under general coordinate transformations eq.(1.13) and cannot appear independently in the effective 4-dimensional action. In this sense they are gauge degrees of freedom and setting $Q_\mu = P_j = P = 0$ corresponds to a unitary gauge with no propagating ghosts.

The decompositions is similar in spirit to the well-known Kaluza–Klein [11] decomposition where a 5-dimensional metric g_{AB} is decomposed into a 4-dimensional metric $g_{\mu\nu}$, a vector A_μ and a scalar ϕ. At the massless level these fields decouple and the five degrees of freedom for a 5-dimensional graviton decompose appropriately as $2+2+1$. Including masses the vector and scalar fields are eaten by the graviton to build a massive 4-dimensional graviton with five degrees of freedom.

Similarly, starting from eq.(1.16) our unitary gauge choice allows $G^{\vec{n}}_{\mu\nu}$ to become massive by eating P and Q_μ. However, as opposed to the 5-dimensional case this does not exhaust the degrees of freedom; there is an additional massive $(n-1)$-multiplet of vectors $V_{\mu j}$ which eat P_j to obtain their longitudinal polarization. Finally, there are $(n^2-n-2)/2$ scalars in the symmetric tensor S_{jk} as well as the singlet scalar H. The total number of degrees of freedom is

$$1\cdot 5 \,+\, (n-1)\cdot 3 \,+\, \frac{n^2-n-2}{2}\cdot 1 \,+\, 1\cdot 1 = \frac{(n+4)(n+1)}{2}\,. \tag{1.17}$$

A similar analysis of $(4+n)$-dimensional gravity gives an identical counting of degrees of freedom: the only physical field is a symmetric tensor h_{AB} with $(4+n)(5+n)/2$ components. These are reduced by fixing the gauge; typically, the harmonic condition $\partial_A h^A_B = \partial h^A_A/2$ amounts to $4+n$ constraints. Furthermore, we are free to add terms to the variation parameter in eq.(1.13) with $\Box\xi_M$ leaving the action invariant. Altogether there are $2(4+n)$ constraints, and counting of degrees of freedom is identical to eq.(1.17).

In terms of the new physical fields Einstein's equations simplify to

$$\begin{aligned}
(\Box+\hat{n}^2)G^{(\vec{n})}_{\mu\nu} &= \frac{1}{M_{\text{Planck}}}\left[-T_{\mu\nu}+\left(\frac{\partial_\mu\partial_\nu}{\hat{m}^2}+\eta_{\mu\nu}\right)\frac{T^\lambda_\lambda}{3}\right]\\
(\Box+\hat{n}^2)V^{(\vec{n})}_{\mu j} &= 0\\
(\Box+\hat{n}^2)S^{(\vec{n})}_{jk} &= 0\\
(\Box+\hat{n}^2)H^{(\vec{n})} &= \frac{\kappa}{3M_{\text{Planck}}}T^\mu_\mu
\end{aligned} \tag{1.18}$$

so that the linearized Lagrangian eq.(1.12) in terms of the fields eq.(1.16) reads (omitting the sum over the index $\vec{n}$ for all fields)

$$\begin{aligned}
\mathscr{L} \sim &-\frac{1}{2}G^{\dagger\mu\nu}(\Box+m^2_{\text{KK}})G_{\mu\nu}-\frac{1}{2}G^{\dagger\mu}_{\mu}(\Box+m^2_{\text{KK}})G^\nu_\nu-G^{\dagger\mu\nu}\partial_\mu\partial_\nu G^\lambda_\lambda\\
&+G^{\dagger\mu\nu}\partial_\mu\partial_\lambda G^\lambda_\mu-\frac{1}{2}H^\dagger(\Box+m^2_{\text{KK}})H\\
&-\frac{1}{M_{\text{Planck}}}\left[G^{\mu\nu}-\frac{\kappa}{3}\eta^{\mu\nu}H\right]T_{\mu\nu}+\cdots
\end{aligned} \tag{1.19}$$

where the ellipses stand for free field kinetic terms. Here and henceforth we define $m^2_{\text{KK}} \equiv \hat{n}^2 = \vec{n}^2/r^2$. The structure of Einstein's equations eq.(1.18) reveals a few particularities: the fields $V_{\mu j}$ and S_{jk} do not couple to the energy momentum tensor, *i.e.* to the Standard Model. The massive gravitons $G_{\mu\nu}$ do couple to the Standard Model. Their Fourier coordinate only appears as a mass-squared $\hat{n}^2$ and in the coupling to the trace of the energy-momentum tensor. This means their couplings are level–degenerate and their masses and couplings depend only on the length, but not on the orientation of the vector $\hat{n}$.

We focus on the properties of conformally invariant theories, where $T^\mu_\mu = 0$, because this is a good approximation of all relevant particle masses as compared to the LHC energy. For such massless theories

$$(\Box + m_{\rm KK}^2)\, G_{\mu\nu}^{(m)} = -\frac{T^{\mu\nu}}{M_{\rm Planck}} \tag{1.20}$$

describes physical gravitons produced by quark or gluon interactions and either vanishing or decaying to leptons. The scalar mode H plays a special role. Its massless radion mode corresponds to a fluctuation of the volume of the compactified extra dimension. We assume that the compactification radius r is stabilized in some way [16], giving mass to the radion [17]. More importantly, the radion only couples to a massive theory, so it is not surprising that as a scalar with no Standard Model charge it will mix with a Higgs boson without too drastic effects.

Before deriving Feynman rules we will briefly outline the significance of Kaluza–Klein towers of massive gravitons: first of all, the basic relation derived at the classical level can be rewritten as $M_{\rm Planck}^2 \equiv M_\star^2 N$ where $N \sim (2\pi r M_\star)^n$ is the number of Kaluza–Klein species existing below the scale $M_\star$. A heuristic argument for this relies on the spacing between consecutive KK modes $\delta m_{\rm KK} \sim 1/r$, so that $rM_\star = M_\star/\delta m_{\rm KK}$ gives the number of KK modes with the vector $\vec{n}$ occupied only in one direction *i.e.* $\vec{n} = (j,0,0,\cdots)$ with j being some integer. A generic vector has n such directions, so in general there are $(rM_\star)^n$ possibilities. Realistic numbers for $M_\star \sim$ TeV give $N \sim 10^{32}$. A similar result is achieved by considering black hole evaporation [18]. This multiplicity of states is what determines the visible effects of ADD models at the LHC.

As a side remark, it is not altogether mysterious that we are summing over a very large number of KK states. In $(4+n)$-dimensional language the graviton propagator is simply $1/(p^A p_A)$. The momentum p^A obeys momentum conservation at each vertex to two Standard Model particles. This way 4-dimensional external lines fix the momentum in four directions, leaving an integration over p_j

$$\int d^n p_j \frac{1}{p^A p_A} = \int d^n p_j \frac{1}{p^\mu p_\mu - p^j p_j} \sim \sum_{m_{\rm KK}} \frac{(\delta m_{\rm KK})^n}{p^\mu p_\mu - m_{\rm KK}^2} \; . \tag{1.21}$$

For KK modes as intermediate states, proper treatment of the KK tower implies a closed integral, similar to an additional n-dimensional loop integral $\int \mathscr{M}$. A similar argument reveals that the additional momentum directions available for final state KK particles amounts to a modified phase space integral $\int |\mathscr{M}|^2$.

To summarize our main results relevant for ADD phenomenology; the spin-2 graviton field couples to the energy momentum tensor universally for massless states, suppressed by the 4-dimensional Planck scale $M_{\rm Planck}$. This predicts the production of massive gravitons at the LHC from gluon as well as quark initial states. The index structure of the massive graviton propagator and vertices are a mess, but theoretically well defined in our effective field theory. There are a large number of gravitons organized by a KK tower which again couple universally to Standard Model fields. The mass splitting between the KK states inside the tower is given by $1/r$ which translates into ($M_\star = 1$ TeV as before):

$$\delta m_{\rm KK} \sim \frac{1}{r} = 2\pi M_\star \left(\frac{M_\star}{M_{\rm Planck}}\right)^{2/n} = \begin{cases} 0.003 \text{ eV} & (n=2) \\ 0.1 \text{ MeV} & (n=4) \\ 0.05 \text{ GeV} & (n=6) \end{cases} \tag{1.22}$$

On the scale of high-energy experiments or the weak scale ($m_Z \sim 91$ GeV), this mass splitting is tiny. For example the LHC will be unable to resolve such mass differences, which allows us to generally replace the sum over graviton modes (either as intermediate states or as final states) by an integration over a continuous variable. We will show this conversion to an integral in the next section. What we can also see from this mass splitting is that gravitons in this model might well be stable, just because they are too light to decay to two Standard Model particles even via a gravitational interaction. A KK tower of gravitons appears as missing energy at the LHC.

1.2 Feynman rules

To lowest order in the graviton field the coupling to massless matter is given by eq.(1.19). We illustrate the extraction of the vertices from the energy-momentum tensor in a manifestly symmetric way using the QED Lagrangian

$$\mathscr{L}_{\rm QED} = \frac{\sqrt{-g}}{M_P} \left(i\bar{\psi}\gamma^a \mathscr{D}_a \psi - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} \right) \tag{1.23}$$

where the covariant derivative contains a gauge and coordinate connection. Taking the variation in the metric for only the gauge field and noting $\delta\sqrt{-g} = -\sqrt{-g}\, g^{\mu\nu}\ \delta g_{\mu\nu}/2$ we find

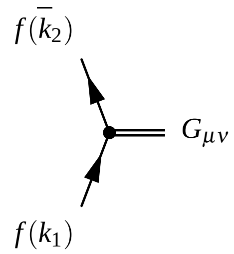
$$\begin{aligned} T^{\mu\nu}_{\rm gauge} &= \frac{2}{\sqrt{-g}} \frac{\delta}{\delta g_{\mu\nu}} \sqrt{-g} \left(-\frac{1}{4} F_{\rho\sigma} F_{\alpha\beta}\, g^{\rho\alpha} g^{\sigma\beta} \right) \\ &= \frac{\eta^{\mu\nu}}{4} F_{\alpha\beta} F^{\alpha\beta} + F^{\mu}_{\alpha} F^{\alpha\nu} \end{aligned} \tag{1.24}$$

and for the the purely fermionic contribution

$$T^{\mu\nu}_{\rm fermion} = \frac{i}{4} \bar{\psi} \left(\partial^\mu \gamma^\nu + \partial^\nu \gamma^\mu \right) \psi - \frac{i}{4} \left(\partial^\mu \bar{\psi} \gamma^\nu + \partial^\nu \bar{\psi} \gamma^\mu \right) \psi \tag{1.25}$$

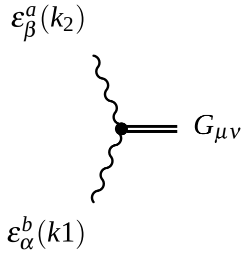
All momenta are incoming to the vertex. To derive the Feynman rules we need to symmetrize the graviton and gauge boson indices separately.

In the following, we will quote the Feynman rules relevant to our LHC analysis. Because gravity couples to every particle in and beyond the Standard Model there are in fact many other graviton vertices [12, 13]. The fermion–graviton and gluon–graviton vertices are



$$= -\frac{i}{4M_{\rm Planck}} \left[W_{\mu\nu} + W_{\nu\mu} \right]$$

and



$$= -\frac{i\delta_{ab}}{M_{\rm Planck}} \left[W_{\mu\nu\alpha\beta} + W_{\nu\mu\alpha\beta} \right]$$

with

$$
\begin{aligned}
W_{\mu\nu} =& (k_1 - k_2)_\mu \gamma_\nu \\
W_{\mu\nu\alpha\beta} =& \frac{1}{2}\eta_{\mu\nu}\left(k_{1\beta}k_{2\alpha} - k_1 \cdot k_2 \eta\alpha\beta\right) + \eta_{\alpha\beta}k_{1\mu}k_{2\nu} + \eta_{\mu\alpha}\left(k_1 \cdot k_2 \eta_{\nu\beta} - k_{1\beta}k_{2\nu}\right) - \eta\mu\beta k_{1\nu}k_{2\alpha}
\end{aligned}
\tag{1.26}
$$

The non-abelian part of the field strength does not contribute, and the color factor δ_{ab} reflects the fact that the graviton is a gauge singlet. The final ingredient needed is the graviton propagator, which is the momentum-space inverse two-point function from the bilinear terms in eq.(1.19). This Lagrangian describes the physical fields and — as opposed to the massless graviton — no additional gauge fixing is required.

$$
G_{\mu\nu}(k) \;=\!=\!=\!=\; G^\dagger_{\alpha\beta}(k) \; = \frac{iP_{\mu\nu\alpha\beta}}{k^2 - m^2}
$$

with

$$
\begin{aligned}
P_{\mu\nu\alpha\beta} = & \frac{1}{2}(\eta_{\mu\alpha}\eta_{\nu\beta} + \eta_{\mu\beta}\eta_{\nu\alpha} - \eta_{\mu\nu}\eta_{\alpha\beta}) \\
& - \frac{1}{2m^2_{\text{KK}}}(\eta_{\mu\alpha}k_\nu k_\beta + \eta_{\nu\beta}k_\nu k_\alpha + \eta_{\mu\beta}k_\nu k_\alpha + \eta_{\nu\alpha}k_\mu k_\beta) \\
& + \frac{1}{6}\left(\eta_{\mu\nu} + \frac{2}{m^2_{\text{KK}}}k_\mu k_\nu\right)\left(\eta_{\alpha\beta} + \frac{2}{m^2_{\text{KK}}}k_\alpha k_\beta\right).
\end{aligned}
\tag{1.27}
$$

It is easy to recognize the first line in eq.(1.27) as (ignoring overall normalization) the massless graviton in the De Donder gauge. A good exposition on different forms (different weak field expansions) of the massive and massless propagator is given in Ref.[19].

The amplitude for a generic s-channel process mediated by virtual gravitons will then look like

$$
\begin{aligned}
\mathscr{A} \sim & \frac{1}{M^2_{\text{Planck}}}\sum T_{\mu\nu}\,\frac{P_{\mu\nu\alpha\beta}}{s - m^2_{\text{KK}}}\,T_{\alpha\beta} \\
= & \frac{1}{M^2_{\text{Planck}}}\sum T_{\mu\nu}\,\frac{\eta_{\mu\alpha}\eta_{\nu\beta} + \eta_{\mu\beta}\eta_{\nu\alpha} - \eta_{\mu\nu}\eta_{\alpha\beta} + \eta_{\mu\nu}\eta_{\alpha\beta}/3}{2(s - m^2_{\text{KK}})}\,T_{\alpha\beta} \\
= & \frac{1}{M^2_{\text{Planck}}}\sum \frac{1}{s - m^2_{\text{KK}}}\,T_{\mu\nu}T^{\mu\nu} \\
\equiv & \mathscr{S}(s)\,\mathscr{T}\,.
\end{aligned}
\tag{1.28}
$$

On the way we use the conservation and tracelessness of the energy momentum tensor *i.e.* $T^\mu_\mu = k_\mu T^{\mu\nu} = 0$. This form is useful because the field content $\mathscr{T} \equiv T_{\mu\nu}T^{\mu\nu}$ and an appropriate coefficient form a general dimension-8 operator $\mathscr{S}\mathscr{T}$.

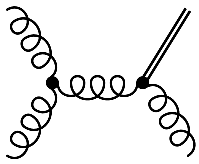
In addition, a loop-induced dimension-6 operator will be generated by diagrams of the form

The resulting four-fermion interactions couples two axial-vector currents

$$c_6 \sum \left(\bar{\psi}\gamma_5\gamma_\mu\psi\right)^2 \tag{1.29}$$

where the sum is over all fermions in the theory. The coefficient c_6 can be estimated by naive dimensional analysis [20]. Such KK graviton contributions can be compared to effects from a modified theory with non-symmetric connection [21]. Generically, all these loop effects will affect electroweak precision observables.

Our final consideration is real graviton emission off any kind of Standard Model process, preferably a so-called standard candle [22] which we expect to understand well from a theory as well as an experimental perspective. Calculating amplitudes corresponding to diagrams such as



requires polarization tensors for external gravitons. We can for example construct five tensors $\varepsilon^s_{\mu\nu}$ where $s = (1,2,\cdots 5)$ by taking outer products of the three massive gauge boson polarization vectors. A convenient parameterization is given in Ref. [13]. Most importantly, the $\varepsilon^s_{\mu\nu}$ obtained this way satisfy

$$\sum_s \varepsilon^s_{\mu\nu}(k)\varepsilon^s_{\alpha\beta}(k) = P_{\mu\nu\alpha\beta}(k) \tag{1.30}$$

when summed over all polarization states. This brief review of calculational details now puts us into a position to discuss LHC processes.

1.3 Collider observables

In this section we summarize possible direct and indirect signatures for massive Kaluza–Klein gravitons at colliders [12, 23, 24, 25, 26]. There is also a large amount of phenomenological work confronting electroweak precision data [27] or astrophysical data [28] and large extra dimensions, in part orthogonal to their collider effects [9], which we will not have space to cover here. Current limits strongly constrain ADD models with few extra dimensions favoring $n > 2$. As we will see in the following sections, such a scenario is also the most conceptually interesting. For two to seven extra dimensions, strong direct constraints on $M_\star$ come from recent Tevatron data [29, 30].

Of the two classes of collider observables we first consider the real emission of Kaluza–Klein gravitons at the LHC [12, 31]. The outgoing gravitons cannot be detected in our detectors — similar to neutrinos or possible dark matter agents — so they appear as missing transverse momentum or missing transverse energy $\not{E}_T$. One process to radiate gravitons off is single jet production [24]. The Feynman rules discussed above allow us to compute squared-averaged amplitudes for partonic sub-processes such as $qg \to qG$, $q\bar{q} \to gG$ and $gg \to gG$ all of which lead to the same final state: one hard QCD jet and missing transverse energy $\not{E}_T$. Due to the strong QCD coupling this is the most likely real emission search channel.

For the graviton–jet final state there is an obvious irreducible background coming from $q\bar{q} \to Zg$ where the gluon is emitted from an initial-state quark and the Z decays into neutrinos. This background is known to next-to-leading order [32], but at large partonic event energies the theoretical rate prediction becomes increasingly hard, due to large logarithms. Extracting new physics from pure QCD signatures at the LHC will therefore always be tough and somewhat dangerous (as we have seen in the past at the Tevatron, where many signals for new physics have come and gone over the years).

Due to the structure of the parton densities of quarks and gluons inside a proton, Tevatron searches for large extra dimensions concentrate on $\gamma \not{E}_T$ final states. Similarly, at the LHC a one photon final state could be resolved in the detectors optimized for Higgs searches in the $H \to \gamma\gamma$ decay channel. Hard single photon events would constitute a revealing signature for physics beyond the Standard Model.

Similarly, the Drell–Yan process $q\bar{q} \to \gamma^*, Z \to \ell^+\ell^-$ with two leptons (electrons or muons) in the final state is the arguably best known hadron collider process [33]. A large amount of missing energy in this channel would be a particularly clean signal for physics beyond the Standard Model at the LHC [34]. Depending on the detailed analysis, both of these electroweak signatures do have smaller rates than a jet+graviton final state, but the lack of QCD backgrounds and QCD-sized experimental and theory uncertainties result in discovery regions of similar size [12, 31].

Going back to the theoretical basis, the partonic cross section for the emission of one graviton is not the appropriate observable. What we are interested in is the entire KK tower contributing to the missing energy signature

$$d\sigma^{\text{tower}} = \sum_{\vec{n}} d\sigma^{\text{graviton}} = \int dN \, d\sigma^{\text{graviton}} \tag{1.31}$$

where $\int dN$ is an integration over an n-dimensional sphere in KK density space

$$\int dN \equiv S_{n-1} \, |\vec{n}|^{n-1} \, d|\vec{n}| \qquad\qquad S_{n-1} = \frac{2\pi^{n/2}}{\Gamma(n/2)} \tag{1.32}$$

In the ultraviolet the sum over $\vec{n}$ is truncated to those states which satisfy kinematic constraints. In particular, the KK mass satisfies $m_{\text{KK}} = |\vec{n}|/r < \sqrt{s}$ where $\sqrt{s}$ is the partonic center of mass energy (related to the proton center of mass energy via $s = (14 \text{ TeV})^2 \, x_1 x_2$).

The KK state density we can rewrite into a mass density kernel using $dm_{\text{KK}}/d|\vec{n}| = 1/r$

$$dN = S_{n-1} \, r^n \, m_{\text{KK}}^{n-1} \, dm_{\text{KK}} = \frac{S_{n-1}}{(2\pi M_\star)^n} \left(\frac{M_{\text{Planck}}}{M_\star} \right)^2 m_{\text{KK}}^{n-1} \, dm_{\text{KK}} \, . \tag{1.33}$$

This implies for the production of a Kaluza–Klein tower

$$d\sigma^{\text{tower}} = d\sigma^{\text{graviton}} \, \frac{S_{n-1} \, m_{\text{KK}}^{n-1} \, dm_{\text{KK}}}{(2\pi M_\star)^n} \, \left(\frac{M_{\text{Planck}}}{M_\star} \right)^2 \, . \tag{1.34}$$

The key aspects of this formula are:

– The factor M_{Planck}^2 from the KK tower summation can be absorbed into the one-graviton matrix element squared. The effective coupling of the entire tower at the LHC energy scale E is then $E/M_\star \gtrsim 1/10$ instead of E/M_{Planck}, *i.e.* roughly of the same size as the Standard Model gauge couplings.

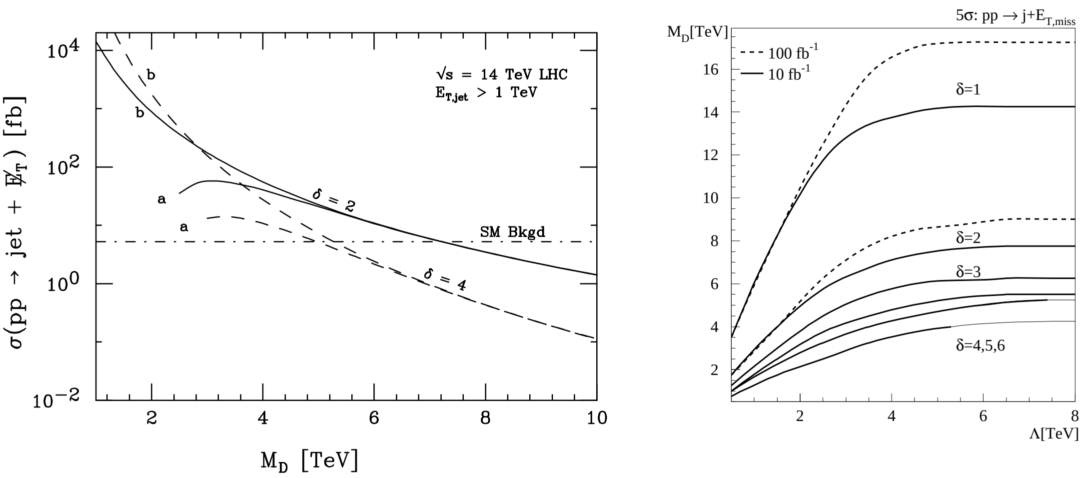


Figure 1: Left: production rates for graviton–jet production at the LHC. The cut on the transverse mass of the jet indirectly acts as a cut on the transverse momentum from the graviton tower, without the additional experimental smearing from measuring missing transverse energy. For the curve (a) a cutoff procedure sets $\sigma(s) = 0$ whenever $\sqrt{s} > M_\star$. For curve (b) the $m_{\rm KK}$ integration in eq.(1.34) includes the region $\sqrt{s} > M_\star$. Figure from Ref. [12]. Right: 5σ discovery contours for real emission of KK gravitons in the plane of $M_\star$ and the UV cutoff on $\sigma(s)$. The transition to thin lines indicates a cutoff $\Lambda_{\rm cutoff}$ above $M_\star$. Figure from Ref. [9]. Note that M_D in both figures corresponds to $M_\star$ in the text.

– In particular for larger n the integral is infrared finite with the largest contributions arising from higher mass modes. This is the effect that KK modes are more tightly spaced as we move to higher masses and even more so for a increasing number of extra dimensions n.

– Although $m_{\rm KK}$ appears explicitly in the polarization sum and thus is naively present in the amplitude, it does not appear once we square the amplitude due to the arguments following eq.(1.28). The $m_{\rm KK}$ integration at least on the partonic level — *i.e.* without the parton densities — can be done without specifying the process.

$$\int dN = \int_0^{\Lambda_{\rm cutoff}} \frac{S_{n-1}}{(2\pi M_\star)^n} \left(\frac{M_{\rm Planck}}{M_\star}\right)^2 m_{\rm KK}^{n-1}\, dm_{\rm KK} = \frac{S_{n-1}}{(2\pi M_\star)^n} \left(\frac{M_{\rm Planck}}{M_\star}\right)^2 \frac{\Lambda^n_{\rm cutoff}}{n} \quad (1.35)$$

In this form we indeed see that our effective theory of KK gravity requires a cutoff to regularize an ultraviolet divergence, simply reflecting the fact that gravity is not perturbatively renormalizable. The crucial question becomes if the prediction of LHC observables is sensitive to $\Lambda_{\rm cutoff}$.

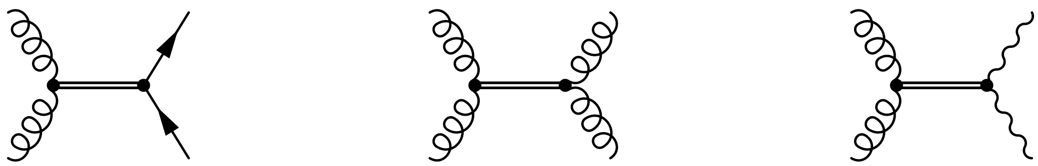
– For real graviton production the kinematic constraint $\mathscr{M} = 0$ for $m_{\rm KK} > \sqrt{s}$ provides a natural ultraviolet cutoff on the n-sphere integration. Therefore the result is insensitive to physics far above the LHC energy scale, which might or might not cover the fundamental Planck scale.

In the left panel of Fig.1 we see that the jet + $\not{E}_T$ cross section becomes seriously dependent on physics above $M_\star$ the moment the Planck scale enters the range of available energies at the LHC $\sqrt{s} \lesssim 3$ TeV. Above this threshold the difference between curves (a) and (b) is small.

In the region where the curves differ significantly, UV effects of our modelling of the KK spectrum become dominant and any analysis based on the Kaluza–Klein effective theory will fail. Luckily, the parton distributions, in particular the gluon density, drop rapidly towards larger parton energies. This effects effectively constrains the impact of the UV region which includes $\sqrt{s} > M_\star$.

In the right panel of Fig.1 we see the 5σ discovery reach at the LHC with a variable ultraviolet cutoff $\Lambda_{\rm cutoff}$ on the partonic collider energy. For each of the lines there are two distinct regimes: for $\Lambda_{\rm cutoff} < M_\star$ the reach in $M_\star$ increases with the cutoff. Once the cutoff crosses a universal threshold around 4 TeV the discovery contours reach a plateau and become cutoff independent. This universal feature demonstrates and quantifies the effect of the rapidly falling parton densities. The fact that the signal decreases with increasing dimension shows that the additional volume element from the n-sphere integration is less than the $1/M_\star$ suppression from each additional dimension.

Virtual gravitons at the LHC demand a markedly different analysis since by definition these signals do not produce gravitational missing energy. The dimension-8 operator $\mathscr{S}(s)\mathscr{T}$ as shown in eq.(1.28) is induced by integrating out a whole graviton tower exchange in s-channel processes at the LHC such as



In the Standard Model some of these final states, leptons and weak gauge bosons, can only be produced by a $q\bar{q}$ initial state. Because at LHC energies the protons mostly consist of gluons, such indirect graviton signatures get a head start. The Tevatron mostly looks for in two-photon or two-electron final states [30]. At the LHC the cleanest signal taking into account backgrounds as well as experimental complications is a pair of muons [23]. In Higgs physics the corresponding channel $H \to ZZ \to 4\mu$ is referred to as the 'golden channel', because it is so easy to extract.

In the Standard Model the Drell–Yan process mediates muon pair production via the s-channel exchange of on-shell and off-shell γ and Z bosons. Aside from the squared amplitude for graviton production, these Standard Model amplitudes interfere with the graviton amplitude, affecting the total rate as well as kinematic distributions. This mix of squared amplitudes and interference effects make it hard to apply any kind of golden cut to cleanly separate signal and background. One useful property of the s-channel process is that the final state particles decay from a pure d-wave (spin-2) state. This results in a distinctive angular separation $\Delta\phi$ of the final state muons [35].

What we are most interested in, though, is the theoretical basis of the dimension-8 operator, *i.e.* its derivation from the KK effective theory. Its dimension $1/m^4$ coefficient arises partly from the coupling and partly from the propagator structure

$$\mathscr{S}(s) = \frac{1}{M_{\rm Planck}^2} \sum \frac{1}{s - m_{\rm KK}^2} \; . \tag{1.36}$$

It exhibits a sum over the KK tower with its typical small mass spacing. To replace it by an integral we need a quantity acting as $\Delta m_{\rm KK}$ which when sent to zero provides the Riemannian measure. In a similar vein to the real mission case we can replace M_{Planck} with its definition in terms of $M_\star$. The factor of r^n appearing in the denominator is precisely $(\Delta m_{\text{KK}})^2$. This gives us an integral over KK masses. Next, recalling that $m_{\text{KK}} \sim |\vec{n}|/r$ we realize that the only relevant coordinate in the KK state space is the radial distance. Therefore, it is possible to perform the angular integration explicitly:

$$\begin{aligned}\mathscr{S}(s) &= \frac{1}{M_\star^{2+n}} \int d^n m_{\text{KK}} \frac{1}{s - m_{\text{KK}}^2} . \\ &= \frac{S_{n-1}}{M_\star^{2+n}} \int d m_{\text{KK}} \frac{m_{\text{KK}}^{n-1}}{s - m_{\text{KK}}^2}\end{aligned} \tag{1.37}$$

The crucial point is that similar to eq.(1.35) this integral is divergent for $n > 1$, *i.e.* for most phenomenologically relevant scenarios. This divergence of the integrals in eqs.(1.35,1.37) is the unique feature of ADD models which are based on an effective field theory of gravity. It can only be cured by going a step beyond the effective theory and employ some kind of UV completion of extra-dimensional gravity.

Previously, we mentioned that summing over virtual graviton states is similar to performing a loop-type integral. However, unlike for renormalizable gauge theories our effective theory of gravity has no such thing as a well-defined counter term to absorb the UV divergence. The remedy we use in this first discussion is to cut off the integral explicitly at the limiting scale of our effective theory. In the spirit of an effective theory we study the leading terms in $s/\Lambda_{\text{cutoff}}^2$.

$$\begin{aligned}\mathscr{S}(s) &= \frac{S_{n-1}}{M_\star^{2+n}} \int_0^{\Lambda_{\text{cutoff}}} d m_{\text{KK}} \frac{m_{\text{KK}}^{n-1}}{s - m_{\text{KK}}^2} \\ &= \frac{S_{n-1}}{M_\star^4 (n-2)} \left(\frac{\Lambda_{\text{cutoff}}}{M_\star} \right)^{n-2} \left[1 + \mathscr{O}\left(\frac{s}{\Lambda_{\text{cutoff}}^2} \right) \right] \approx \frac{S_{n-1}}{n-2} \frac{1}{M_\star^4}\end{aligned} \tag{1.38}$$

where in the final line we identify $M_\star \equiv \Lambda_{\text{cutoff}}$, lacking other reasonable options. Obviously, this relation should be considered an order-of-magnitude estimate rather than an exact relation valid to factors of two. For simplicity this term is further approximated in terms of a generic mass scale $\mathscr{S} = 4\pi/M^4$ in the literature [12, 20]. A number of interesting properties of virtual graviton exchange diagrams we can summarize.

– The identification in eq.(1.38) only parameterizes the effects of the transition scale between the well known linearized theory of gravitons and whatever new physics occurs above the scale $M_\star$, assuming the effective theory captures the dominant effects.

– The generic $2 \to 2$ s-channel amplitude $4\pi/M^4 \cdot \mathscr{T}$ requires powers of the process energy in the numerator. Unitarity is violated at some s signifying the breakdown of our effective theory. For first collider predictions it suffices to make a hard cut on events with $\sqrt{s} > \Lambda_{\text{cutoff}}$. This as it will turn out poor approximation will be addressed later.

– The function $\mathscr{S}(s)$ including Λ_{cutoff} can be integrated analytically for a Wick rotated graviton propagator $1/(s + m_{\text{KK}}^2)$, but the interpretation of particles in an effective field theory will be lost beyond a leading approximation $s \gg m_{\text{KK}}^2$ or $s \ll m_{\text{KK}}^2$.

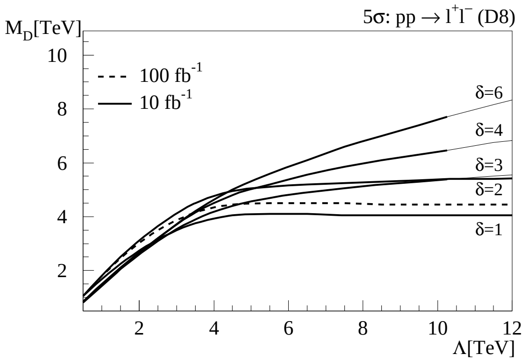


Figure 2: 5σ discovery contours for discovery of extra dimensions via virtual graviton contributions to the Drell–Yan process. Figure from Ref.[9].

Moving on to some results, the LHC reach in M_D is given in Fig.2, again in the presence of a variable cutoff Λ_{cutoff}. As opposed to the real emission case shown in Fig. 1, the result is clearly sensitive to Λ_{cutoff}. Only for $n = 1$ this is not true, reflecting that $\int_0^\infty dm\, m_{\text{KK}}^{-2}$ in fact converges. Along a similar thread of thought, the sensitivity becomes more pronounced with increasing n as the integral is more divergent. The value of Λ_{cutoff} depends on the details of the transition to UV behavior. Phrased differently, the LHC production cross section involving virtual graviton exchange is seriously sensitive to the UV physics of quantum gravity. This cutoff dependence and the non-unitarity of gravitational scattering amplitudes will be our main focus in the final section of these notes.

2. Warped extra dimensions

Our main focus thus far have been large flat dimensions as suggested by the ADD model. However, there is the alternative Randall–Sundrum model [36, 37] with a wealth of phenomenological applications [38, 39]. It also solves the hierarchy problem using an extra space dimension and claiming that the fundamental Planck scale resides around the TeV scale. The mechanism which generates the large hierarchy between $M_\star$ and M_{Planck} utilizes a spatially warped extra dimension.

For completeness, we also mention a third class of phenomenologically relevant extra dimensional models, universal extra dimensions UED [40]. In this model all Standard Model particles exist in the higher dimensional space, but the geometry is such that there is an experimentally constrained mass gap between the ground state and the first excited state. At the LHC, we would for example expect to see KK resonances of the gluon, *i.e.* a massive color octet vector particle. Since UED models do not provide a straightforward link to quantum gravity effects we will not consider them in detail.

For the Randall–Sundrum model, we compactify our 5th dimension y on a S^1/Z_2 orbifold. S^1 simply means a circle, equivalent to periodic boundary conditions. S^1/Z_2 means we map one half of this circle on the other, so we really only look at half a circle with no periodic boundary conditions, but two different branes at $y = 0$ and $y = b$. The key observation now is that nobody can stop us from postulating a 5-dimensional metric of the kind

$$ds^2 = e^{-2k|y|}\eta_{\mu\nu}dx^\mu dx^\nu - dy^2 \qquad \Leftrightarrow \qquad g_{AB} = \begin{pmatrix} e^{-2k|y|}\eta_{\mu\nu} & 0 \\ 0 & \eta_{jk} \end{pmatrix} \tag{2.1}$$

The metric in the four orthogonal directions to y depends on $|y|$. The absolute value appearing in $|y|$ corresponds to the Z_2 (orbifolding) as S^1/Z_2. When looking at our (3+1)-dimensional brane we can take into account the warp factor $e^{-2k|y|}$ in two ways (with some caveats):

1. Use $g_{\mu\nu} = \eta_{\mu\nu}e^{-2k|y|}$ everywhere, which is a pain but possible.
2. Replace x^μ in five dimensions by effective coordinates $e^{-k|y|}d\tilde{x}^\mu$ and $g_{\mu\nu}$ by $\tilde{g}_{\mu\nu} = \eta_{\mu\nu}$ where the tilde indicates 4-dimensional variables.

The second version means we shrink our effective 4-dimensional metric along y and forget about the curved space, because the warp factor does not depend on x^μ. The general–relativity action for Newtonian gravity we can write in terms of the 5-dimensional fundamental Planck scale M_{RS}. In our hand–waving argument we have to transform the 5-dimensional Ricci scalar. Just looking at the mass dimensions we see that R has mass dimension two (or by looking at the definition of R we find space dimension minus two). This suggests that the 4-dimensional Ricci scalar $\tilde{R}$ should roughly scale like $x^{-2} \sim \tilde{x}^{-2}\exp(+2k|y|)$, leading us to guess $R \sim \tilde{R}\exp(+2k|y|)$. The Einstein–Hilbert action with separated $\tilde{x}$ and y integrals reads

$$\begin{aligned} S &= -\frac{1}{2}\int_0^b dy \int d^4\tilde{x}\, e^{-4k|y|}\, R M_{\text{RS}}^3 \\ &\sim -\frac{M_{\text{RS}}^3}{2}\int_0^b dy\, e^{-2k|y|}\int d^4\tilde{x}\,\tilde{R} \\ &= -\frac{M_{\text{RS}}^3}{4k}\left(1-e^{-2kb}\right)\int d^4\tilde{x}\,\tilde{R} \\ &\sim -\frac{M_{\text{RS}}^3}{4k}\int d^4\tilde{x}\tilde{R} \qquad\qquad \text{assuming } kb \gg 1 \\ &\equiv -\frac{M_{\text{Planck}}^2}{2}\int d^4\tilde{x}\,\tilde{R} \qquad\qquad \text{with} \qquad M_{\text{Planck}}^2 \sim \frac{M_{\text{RS}}^3}{2k}\,. \end{aligned} \tag{2.2}$$

In the last step we have applied the usual matching with 4-dimensional Newtonian gravity. Note that this does yet not solve the hierarchy problem because $M_{\text{RS}} \sim k \sim M_{\text{Planck}} \sim 10^{19}$ GeV looks like the most reasonable solution to the matching condition.

Fortunately, this is not the whole story. Consider now the Standard Model Lagrangian on the TeV brane ($y = b$) in the $\tilde{x}^\mu$ coordinates, *i.e.* including the warp factor. To solve the hierarchy problem, the scalar Higgs Lagrangian is obviously crucial

$$S_{\text{SM}} = \int d^4\tilde{x}\, e^{-4kb}\left[(D_\mu H)^\dagger(D^\mu H) - \lambda(H^\dagger H - v^2)^2 + \ldots\right] \tag{2.3}$$

From the Higgs mass term we see that we can rescale all Standard Model fields and mass parameters — in this case H as well as v — by the warp factor on the TeV brane $\exp(-kb)$. The same we have to do for the space coordinate, as described above and for gauge fields appearing in the covariant derivative. To get rid of the entire pre-factor from the warped metric we need to absorb four powers of $\exp(-kb)$ in each term contributing to the Standard Model Lagrangian.

Four is a magic number in Lagrangians of renormalizable gauge theories: it fixes the mass dimension of the Lagrangian. This means that if we only consider contributions to $\mathscr{L}_{\rm SM}$ of mass dimension four, we can simply rescale all Standard Model fields according to their mass dimension:

$$\begin{aligned} \tilde{H} &= e^{-kb} H && \text{scalars} \\ \tilde{A}_\mu &= e^{-kb} A_\mu && \text{or } \tilde{D}_\mu = e^{-kb} D_\mu \\ \tilde{\Psi} &= e^{-3kb/2} \Psi && \text{fermions} \end{aligned} \tag{2.4}$$

which also means for all masses

$$\begin{aligned} \tilde{m} &= e^{-kb} m \\ \tilde{v} &= e^{-kb} v \end{aligned} \tag{2.5}$$

Yukawa couplings as dimensionless parameters are not affected. If we now assume $kb \sim 35$ we do solve the hierarchy problem:

$$\tilde{v} \sim 0.1\, e^{-kb}\, M_{\rm Planck} \sim 0.1 \text{ TeV} \tag{2.6}$$

The fundamental Higgs mass and the fundamental Planck mass are indeed of the same order, only the 4-dimensional Higgs mass (like all mass scales on the TeV brane) appears smaller, because of the warped geometry in the 5th dimension. In contrast, on the Planck brane with its warp factor $\exp(-k|y|) = 1$ nothing has happened.

Before we introduce gravitons as metric fluctuations into our RS model, it turns out to be useful to rewrite the metric by rescaling the 5th dimension $y \to z$ with

$$ds^2 = e^{-A(z)} \left(g_{\mu\nu} dx^\mu dx^\nu - dz^2 \right) \tag{2.7}$$

To simplify things we assume for the following brief discussion $y > 0$. This is obviously justified, as long as we limit our interest to the TeV brane. First, we define $A(z) = 2ky$ and rewrite the metric

$$e^{-2ky} = e^{-A(z)} = \frac{1}{(1+kz)^2} \qquad \Rightarrow \qquad dy = e^{-ky}\, dz = e^{-A(z)/2}\, dz \tag{2.8}$$

The Planck brane at $y = 0$ sits at $z = 0$. Assuming $k > 0$ we find that $y > 0$ corresponds to $z > 0$. The derivative indeed produces the correct pre-factor of dz^2.

To introduce tensor gravitons we expand the 4-dimensional part of the metric:

$$ds^2 = e^{-A(z)} \left(\eta_{\mu\nu} + h_{\mu\nu}(x,z)\, dx^\mu dx^\nu - dz^2 \right) \tag{2.9}$$

Einstein's equation without sources but in the presence of $A(z)$ includes a linear term which does not look at all like an equation of motion and which we therefore do not like. We can get rid of it rescaling (as usual) $h_{\mu\nu} = e^{(2+n)kb/4} \widetilde{h}_{\mu\nu}$, according to its bosonic mass dimension $[h] = m^{(2+n)/2}$. This gives

$$-\frac{1}{2} \partial_C \partial^C h_{\mu\nu} + \frac{2+n}{4}\, \partial^C A\, \partial_C h_{\mu\nu} = -\frac{1}{2} \partial^C \partial_C \widetilde{h}_{\mu\nu} + \left(\frac{9}{32} A'^2 - \frac{3}{8} A'' \right) \widetilde{h}_{\mu\nu} = 0 \tag{2.10}$$

as the equation of the motion for the rescaled graviton field $\widetilde{h}_{\mu\nu}$. We can solve this equation of the motion by separating variables $\widetilde{h}_{\mu\nu}(x,z) = \widehat{h}_{\mu\nu}(x)\,\Phi(z)$ and by giving mass to the tensor graviton solving $\partial_\mu \partial^\mu \widehat{h}_{\mu\nu} = m^2 \widehat{h}_{\mu\nu}$. The equation of motion

$$-\partial_z^2 \Phi + \left(\frac{9}{16} A'^2 - \frac{3}{4} A'' \right) \Phi = m^2 \Phi \tag{2.11}$$

is a Schrödinger-type equation for Φ with a potential term

$$V(z) = \frac{9}{16} \frac{4k^2}{(k|z|+1)^2} + \frac{3}{4} \frac{2k^2}{(k|z|+1)^2} = \frac{15}{4} \frac{k^2}{(k|z|+1)^2} \tag{2.12}$$

This equation is first of all solved by the zero mode

$$h_{\mu\nu}^{(0)} = e^{+3A/4} \widetilde{h}_{\mu\nu}^{(0)} = e^{+3A/4} \hat{h}_{\mu\nu}^{(0)}(x)\, \Phi^{(0)}(z) \equiv \hat{h}_{\mu\nu}^{(0)}(x) \tag{2.13}$$

which in terms of the 5th coordinate y means $\Phi^{(0)}(y) = e^{-3k|y|/4} = e^{-3kb/4}$ on our TeV brane. So indeed, gravity on the TeV brane is weak because of the exponentially suppressed wave-function overlap.

Again, using the Schrödinger-type equation with $V(z)$ as given in eq.(2.12) we can compute the KK graviton masses in our 4-dimensional effective theory The boundary conditions $\partial_z h_{\mu\nu} = 0$ on the branes are given by the orbifold identification $y \to -y$ and assuming $z > 0$. On the two different branes we find

$$\partial_z^2 \Phi = -\frac{3}{2} k \Phi \Big|_{\text{Planck}} \qquad \partial_z^2 \Phi = -\frac{3}{2} \frac{k}{kz+1} \Phi \Big|_{\text{TeV}} \tag{2.14}$$

The solution of the equation of motion can now be expressed in terms of Bessel functions, which are numbered by an index which corresponds to the mass introduced above:

$$\Phi_m(z) = \frac{1}{\sqrt{kz+1}} \left[a_m Y_2 \left(m \left(z + \frac{1}{k} \right) \right) + b_m J_2 \left(m \left(z + \frac{1}{k} \right) \right) \right] \tag{2.15}$$

The masses of these modes are given in terms of the roots of the Bessel function $J_1(x_j) = 0$ for $j = 1, 2, 3, 4 \cdots$

$$m_j = x_j\, k\, e^{-kb} \sim x_j\, M_{\text{Planck}}\, e^{-kb} \sim x_j \,\text{TeV} \qquad \text{with} \quad x_j = 3.8, 7.0, 10.2, 16.5 \cdots \tag{2.16}$$

This means that the KK excitations in the Randall–Sundrum model with one warped extra dimensions are almost, but not quite equally spaced. If we remember that we can choose $kb \sim 35$ to solve the hierarchy problem they are in the TeV range, *i.e.* in contrast to the ADD model not only resolvable by the LHC experiments put most likely out of reach beyond $j = 1$.

In the last step we need to compute the coupling strength of these heavy KK gravitons to matter, like quarks or gluons. Remember that in the ADD case we find tiny Planck-suppressed couplings for each individual KK graviton, which corresponds to an inverse-TeV-scale coupling once we integrate over the KK tower. For the warped model the relative coupling strengths on the Planck brane and on the TeV brane are approximately given by the ratio of the wave function overlaps. While the zero-mode graviton has to be strongly localized on the Planck brane, to explain the weakness of Newtonian graviton the TeV brane, the KK gravitons do not have strongly peaked wave functions in the additional dimension. From eq.(2.15) we can read off the ratio of wave functions — assuming that the Bessel functions with their normalized arguments will not make a big difference

$$\frac{\Phi(z)\big|_{\text{TeV}}}{\Phi(z)\big|_{\text{Planck}}} \sim \frac{\sqrt{kz+1}\big|_{\text{Planck}}}{\sqrt{kz+1}\big|_{\text{TeV}}} \sim \frac{1}{e^{kb/2}} \sim \frac{1\text{TeV}}{M_{\text{Planck}}} \tag{2.17}$$

The coupling of the KK states is given by the left-hand side of Einstein's equations which enters the Lagrangian just as for the large extra dimensions. We have to distinguish between the flat zero mode with un-suppressed wave function overlap and the KK modes with this ratio of wave functions

$$\mathscr{L} \sim \frac{1}{M_{\text{Planck}}} T^{\mu\nu} h^{(0)}_{\mu\nu} + \frac{1}{M_{\text{Planck}} e^{-kb}} T^{\mu\nu} \sum h^{(m)}_{\mu\nu} \tag{2.18}$$

We see that the heavy KK gravitons indeed couple with TeV-scale gravitational strength and can be produced at colliders in sufficient numbers, provided they are not too heavy. Similarly to the flat extra dimensions, the couplings of the different KK excitations are (approximately) universal. Remembering the way the effective theory of gravity breaks down in ADD models we see that integrating over an ultraviolet regime of a KK tower of gravitons is not a problem in RS models. However, in the large-energy limit we do find that first of all scattering amplitudes computed in the effective RS model violate unitarity, and that secondly in the ultraviolet regime the graviton widths become large, which means the effective KK picture becomes inconsistent. Both of these reasons again ultimately require an ultraviolet completion of gravity.

Obviously, this is phenomenologically very different from the flat (ADD) extra dimensions. For warped extra dimensions we will not produce a tightly spaced KK tower, but for example distinct heavy s-channel excitations. One advantage of such a scenario is of course that we can measure things like the KK masses and spins at colliders directly [35]. The disadvantage for phenomenology is that such resonance searches are boring and can be mapped on Z' searches [41, 42] one-to-one.

3. Ultraviolet completions

As explicitly seen in the last two sections, the effective field theory description of extra-dimensional gravity breaks down once the LHC energies approach the range of the fundamental Planck scale. This feature is expected — if a coupling constant has a negative mass dimension the relevant scale in the denominator has to be cancelled by an energy in the numerator. Once this ratio of the typical energy over the Planck scale becomes large gravity appears to become strongly interacting and will eventually encounter ultraviolet poles. These poles cannot be absorbed by our usual perturbative renormalization, which means we cannot meaningfully quantize gravity without an additional modification of this ultraviolet behavior.

We know several possible modifications of this dangerous ultraviolet behavior [43]: most well known, string theory includes its own fundamental scale M_S which is related to a finite size of its underlying objects. Such a minimum length acts as a an ultraviolet cutoff in the energy, which regularizes all observables described by the gravitational interaction [44, 45]. An alternative approach which avoids any ad-hoc introduction of radically different physics above some energy scale is based on the ultraviolet behavior of gravity itself [46]: the asymptotic safety scenario is based on the observation that the gravitational coupling develops an ultraviolet fixed point which avoids the ultraviolet divergences naively derived from power counting [47, 48, 49, 50, 51].

Going back to large flat extra dimensions and the ADD model the divergence of the integral representing the sum over KK states (for example shown in eq.(1.37)) is a major phenomenological problem. The unphysical cutoff dependence seriously weakens our ability to make precise LHC predictions or interpret possible LHC results. Furthermore, at a conceptual level this break-down of the KK effective theory already at LHC energies insinuates the immediate need for a more complete description of gravity [52, 53]. There are a number of effective-theory proposals which side-step this complication by defining the integral in a cutoff independent scheme. One such treatment relates to the eikonal approximation to the $2 \to 2$ process [54], another involves introducing a finite brane thickness [55]. In this example, the gravitational coupling is exponentially suppressed above $M_\star$ by a brane rebound effect. This is applied to the case of high energy cosmic rays interacting via KK graviton exchange [56]. However, none of these models offer a compelling UV completion for the KK effective theory.

The fundamental deficiency in the description of extra-dimensional gravity we discuss based on two approaches: in the context of string theory, one initial motivation for the ADD model [57], the expectation is for string Regge resonances to appear above the string scale. On the other hand, following our original motivation for the ADD model — its minimal structure with no additional states and a very straightforward geometry — we will focus on a UV completion based on the observation of asymptotic safety [46, 58]. This idea can be applied to LHC phenomenology in ADD [52] or RS [53] models.

3.1 String theory

One possible ultraviolet completion of gravity could be string theory with its finite minimum length scale regularizing the ultraviolet behavior of transition amplitudes. For example, we can compute the scattering $q\bar{q} \to \mu^+\mu^-$ using open string perturbation theory. Without tagging a certain vacuum with the Standard Model as its low-energy limit, we can nevertheless construct realistic string amplitudes for generic gauge and fermion fields. The first step is to restrict the (massless) fields to a D3 brane. Gauge bosons are included by adding Chan-Paton factors λ^a_{ij} at the string endpoints. For i,j running from 1 to N this implies N^2 additional degrees of freedom, identical to the generators for $U(N)$. The Standard Model subgroups $SU(2)$ and $U(1)$ are thus easily embedded. The helicity amplitudes for $2 \to 2$ scattering are simply analytic functions in s, t and u together with the common Veneziano amplitude [44]

$$\mathscr{S}(s,t) = \frac{\Gamma(1-\alpha' s)\Gamma(1-\alpha' t)}{\Gamma(1-\alpha' s-\alpha' t)}. \tag{3.1}$$

in terms of the inverse string scale $\alpha' = 1/M_S^2$. While we do not exactly know the size of this scale it should lie between the weak scale $v = 246$ GeV and the fundamental Planck scale $M_\star$. For our purposes it suffices to consider three distinct limits:

– In the hard scattering limit $s \to \infty$ and for a fixed scattering angle (or equivalently fixed Mandelstam ratio of variables t/s) the amplitude behaves as

$$\mathscr{S}(s,t) \sim e^{-\alpha'(s\log s + t\log t)} \tag{3.2}$$

This can be seen by applying Stirling's approximation. The physics is immediately apparent: due to the finite and dimensionful string scale $\alpha' = 1/M_S^2$ all scattering amplitudes becomes weak in the UV. Unfortunately, this particular limit is not very useful for LHC phenomenology.

– The Regge limit for small angle high energy scattering in terms of Mandelstam variables means $s \to \infty$ with t fixed. In this limit the poles in the Γ functions determine the structure: for $\sqrt{s} > M_S$ there appear single poles at negative integer arguments $1 - s/M_S^2 = -(n+1)$ where $n = 1, 2, \ldots$. These poles lie at $s = nM_S^2$, which tells us that string resonances appear as a tower of resonances in the s channel. Starting from the energies around M_S this UV completion consists of a string of real particles with masses $\sqrt{n}M_S$.

– The leading corrections in α' valid for energies $\sqrt{s}$ below the string scale is

$$\frac{\Gamma(1-s/M_S^2)\,\Gamma(1-t/M_S^2)}{\Gamma(1-(s+t)/M_S^2)} = 1 - \frac{\pi^2}{6}\,\frac{st}{M_S^4} + \mathscr{O}\left(\frac{1}{M_S^6}\right). \tag{3.3}$$

This form of the string corrections corresponds to our KK effective field theory, modulo a normalization factor which relates the two mass scales M_S and M_{Planck}. Hence, this series in M_S is not what we are interested as the UV completion of our theory.

– The physical behavior for scattering amplitudes above the string scale is a combination of Regge and hard scattering behavior. In other words, equally spaced string resonances together alongside exponential suppression, but at colliders the resonances should be the most visible effects.

So far, we have only considered the exchange of string resonances of Standard Model gauge bosons, not graviton exchange. The string theory equivalent of our process generating the effective dimension-8 operator is the scattering of four open strings via the exchange of a closed string. This amplitude is insignificant compared to the string excitations in the vicinity of the string scale, where eq.(3.3) is valid [45]. Most notably, the KK mass integration is finite for all n due to an exponential suppression of similar origin as the hard scattering behavior noted above. For fields confined to a D3 brane this integral is

$$\mathscr{S}(s) \sim \int d^6m\,\frac{e^{\alpha'(s-m^2)/2}}{s-m^2}\;. \tag{3.4}$$

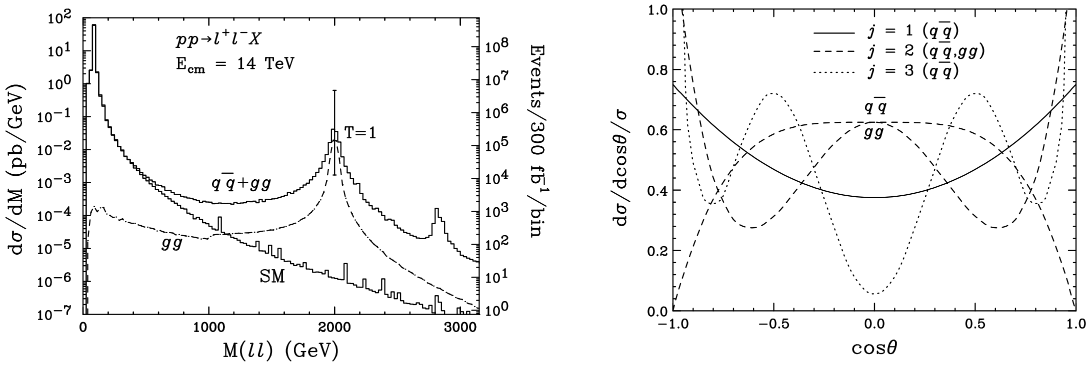


Figure 3: Left: invariant mass distribution for LHC dilepton production. The parameter $T = 1$ is the Chan-Paton number. Figure from Ref.[45]. Right: normalized angular distributions for $J = 1,2,3$ resonances. Figure from Ref. [45].

An explicit exponential factor regularizing the m integral also appears in the modification using a finite brane thickness [55]

$$\mathcal{S}(s) \sim \int d^6m \, \frac{e^{-\alpha' m^2}}{s-m^2} \; . \tag{3.5}$$

These results are thus approximately equivalent in the low $\sqrt{s}$ region for the m integration. The finite brane thickness approach still violates unitarity for large s.

Matching the string theory result with the effective theory amounts to a matching between the string scale and the cutoff scale appearing in the dimension-8 operator written as in eq.(1.38)

$$\frac{1}{M^4} \sim \frac{\pi^2}{32} \frac{g^4}{M_S^4} \tag{3.6}$$

This is the basis on which to argue that Regge string excitations are the dominant process for high energy particle scattering. Discovering string degrees of freedom at the LHC is primarily concerned with observing these resonances. An example for a possible distribution plot is given in the left panel of Fig.3. In addition, the partial wave decomposition of a string Regge amplitude shows superposition of different angular momentum state. Each resonance is degenerate with respect to the angular momentum number j. This degeneracy manifests itself in the angular correlations between final states in s-channel processes (right panel of Fig.3).

As mentioned above, the issue with the naive phenomenology of RS gravitons as well as string excitations is no experimental or phenomenological challenge to the Tevatron or LHC communities. For example the Tevatron experiments have been searching for (and ruling out) heavy gauge bosons, for a long time. So while a discovery of a Z' resonance at the LHC would trigger a great discussion of its origin, including KK gravitons, string resonances, KK gauge bosons, or simple heavy Z' gauge bosons from an additional gauged $U(1)$ symmetry [42], there is little to learn from such scenarios at this stage. Most of the papers you will find on the topic have not much to say about the generic structure and challenges of such signatures at hadron colliders.

3.2 Fixed-point gravity

If all hints concerning the asymptotic safety of gravity should hold there is no need at all to alter the structure of gravity at high energies — gravity will simply be its own ultraviolet completion [46, 47, 48] For our phenomenological discussion we will only sketch some qualitative features of asymptotic safety [59, 49, 50, 51]. Although most in this field is developed in four dimensions, the results generalize in a straightforward manner [58]. This allows us to splice together results from two seemingly disjoint fields and consider asymptotic safety in extra dimensional models. The major principles for asymptotic safety in gravity are

– The metric carries the relevant degrees of freedom in both the classical and quantum regime.

– IR and UV physics lie on a single trajectory and are connected by the renormalization group flow.

– Relevant degrees of freedom are anti-screening.

– the UV behavior is determined by an interacting (non-gaussian) fixed point of the gravitational coupling.

– Residual interactions appear 2-dimensional

Evidence for asymptotic safety comes in many forms: the concept of asymptotic safety or non-perturbative renormalizability was proposed originally in 1980 [46]. The first hints that gravity might have a UV fixed point were uncovered using a $2+\varepsilon$ expansion for the space-time dimensionality [60]. Further evidence was collected in the $1/N$ expansion where N is the number of matter fields coupled to gravity [47]. More modern results use exact renormalization group methods [48]. There are a number of reviews of the subject [59], and on the necessary non-perturbative techniques, namely the exact flow equation for the effective average action or Wetterich equation [61]. Gravitational invariants including R^8 and minimal coupling to matter are consistently included in flow equations without destroying the fixed point [59]. More recently, it has been shown that including invariants proportional to divergences in perturbation theory do not give divergent results non-perturbatively [62]. Furthermore, there is independent evidence for asymptotic safety coming from recent lattice simulations, causal dynamical triangulations [63]. Universal quantities, like *e.g.* critical exponents, computed using this method agree non-trivially with results derived using renormalization group methods.

The key point for our application is that in the UV the coupling exhibits a finite fixed point behavior. This ensures that the UV behavior of the complete theory is dominated by fixed point scaling, rendering all our computed transition rates weakly interacting at all energy scales.

It is useful to start again from the Einstein–Hilbert action to calculate the scaling behavior for the gravitational coupling $G_N \sim 1/M_{\text{Planck}}^{2+n}$

$$\Gamma = \int d^d x \, \sqrt{g} \, \left\{ \frac{1}{16\pi G_N} (-R + 2\Lambda_{\text{cc}}) + \mathscr{O}(R^2) + \mathscr{L}_{\text{matter}} + \mathscr{L}_{\text{gauge fixing}} + \mathscr{L}_{\text{ghost}} \right\} . \tag{3.7}$$

The gauge-fixing and ghost terms in the Lagrangian we will ignore in the following. The effective action eq.(3.7) we truncate to only the cosmological constant and Ricci scalar terms. It is necessary to include the cosmological constant because the flow in G_N is correlated with a flow in Λ_{cc}, so even if Λ_{cc} is set to zero at some point, quantum effects will generate a non-zero value.

The dependence of dimensionless couplings on the energy scale is relevant to understanding quantum effects and thus we define a dimensionless Newton constant [48, 50, 51, 49]

$$G_N \to \frac{G_N}{Z(\mu)} \equiv G_N(\mu) \equiv \frac{g(\mu)}{\mu^{2+n}} \tag{3.8}$$

not to be confused with the determinant of the metric $\sqrt{g}$, which will not appear anymore in the following. The Callan-Symanzik equation for $g(\mu)$ can be derived in the standard way: first, we note that as we vary the energy scale Newton's constant undergoes multiplicative renormalization. In addition, we define $\eta = -\mu\, d\log Z(\mu)/d\mu$ as its anomalous dimension. This anomalous dimension encodes how quantum effects affect the scaling behavior of our theory. Applying the differential operator $\mu\, d/d\mu = d/\log\mu$ to the definition of the dimensionless coupling we see that

$$\beta_g = \frac{d}{d\log\mu}\, g(\mu) = [2+n+\eta]\; g(\mu)\; . \tag{3.9}$$

This is the exact beta function of Newton's constant. Although it looks innocuous enough at first glance, the parameter $\eta(g)$ will in general contain contributions from all couplings in the Lagrangian, not only the dimensionless Newton's constant. However, one important property is immediately apparent: for $g = 0$ we have a perturbative gaussian fixed point, *i.e.* an IR fixed point which corresponds to classical general relativity where we have not observed a running gravitational coupling. Secondly, depending on the functional form of $\eta(g)$ the prefactor $2+n+\eta(g)$ can vanish, giving rise to a a non-gaussian fixed point $g_\star \neq 0$. The anomalous dimension at this ultraviolet fixed point will take only integer values

$$\eta(g_\star) = -2-n. \tag{3.10}$$

For two space-time dimensions the anomalous dimension vanishes in which case the fixed point is at zero coupling and becomes a gaussian fixed point, another manifestation of the perturbative renormalizability of two-dimensional gravity. It is tempting to think that $G_N(\mu)$ will vanish at the UV fixed point so gravity is really asymptotically free. However, Newton's dimensionful constant does not have a physical meaning itself, and only when divided by an area does it acquire significance. The corresponding dimensionless coupling does not vanish, which means the correct statement is that in $4+n$ dimensions the theory with $g_\star \neq 0$ is still coupled, just not ultraviolet divergent [64, 58]. Using the exact renormalization group flow equation we can compute the anomalous dimensions [51], which depends on the shape of the UV regulator. The beta function of the gravitational coupling becomes — neglecting the cosmological constant, which does not alter the qualitative behavior of the system

$$\beta_g(g) = \frac{(1-4(4+n)g)(2+n)g}{1-(4+2n)g} \qquad\qquad \eta(g) = \frac{2(2+n)(6+n)\,g}{2(2+n)\,g-1} \tag{3.11}$$

Indeed, we observe the two fixed points: the IR fixed point $\beta_g = 0$ appears at zero coupling $g = 0$ and the UV fixed point $g_* = 1/4/(4+n)$ for an anomalous dimension of $\eta(g_*) = -2-n$.

One way of interpreting the physical effects of the gravitational UV fixed point is to modify the original calculations by defining a running Newton's coupling and evaluate it at the energy scale given by the respective process. This approach is in complete accordance with the usual QCD calculations for high-energy colliders, based on a running strong coupling. To derive a renormalization group equation for the gravitational coupling we can integrate this form with respect to a reference value $g_0 = g(\mu_0)$ [52]

$$\log \frac{g(\mu)}{g_0} - \frac{6+n}{2(4+n)} \log \frac{g(\mu)-g_*}{g_0-g_*} = (2+n)\, \log \frac{\mu}{\mu_0} \; . \tag{3.12}$$

To motivate one method of including the ultraviolet fixed point in a cross section calculation the renormalization group equation eq.(3.12) can be cast into the form [65, 51, 53]

$$\frac{g(\mu)}{g_0} \left(\frac{g_0-g_*}{g(\mu)-g_*} \right)^{\omega g_*} = \left(\frac{\mu}{\mu_0} \right)^{2+n} \qquad \text{with} \quad \omega = \frac{6+n}{2(4+n)g_*} \tag{3.13}$$

For $\omega \sim g_*$ which happens to be a reasonable approximation this becomes simply

$$g(\mu) = \frac{g_0 \left(\dfrac{\mu}{\mu_0} \right)^{2+n}}{1 - \dfrac{g_0}{g_*} + \dfrac{g_0}{g_*} \left(\dfrac{\mu}{\mu_0} \right)^{2+n}} \tag{3.14}$$

As a very rough check this formula reproduces the non-gaussian fixed point values $g(\mu) = g_*$ for $\mu \to \infty$ as well as the gaussian fixed point $g = 0$ for $\mu \to 0$. The dimensionful coupling $G_N(\mu)$ which becomes Newton's constant $G_N(\mu_0 = 0) \equiv G_N$ in the far infrared is accordingly given by

$$G_N(\mu) = \frac{G_N(\mu_0)}{1 + \dfrac{G_N(\mu_0)\mu^{2+n}}{g_*} - \dfrac{G_N(\mu_0)\mu_0^{2+n}}{g_*}} = \left[\frac{1}{G_N} + \frac{1}{g_*} \mu^{n+2} \right]^{-1} \; . \tag{3.15}$$

The leading effects from the renormalization group running of the gravitational coupling we can now include into a correction (form factor) to the coupling which appears in the Lagrangian $G_N = (\sqrt{8\pi} M_\star)^{-2}$, *e.g.* in eq.(1.12)

$$\frac{1}{M_\star^{2+n}}\, h^{MN} T_{MN} \to \frac{1}{M_\star^{2+n}} \left[1 + \frac{1}{8\pi g_*} \left(\frac{\mu^2}{M_\star^2} \right)^{1+n/2} \right]^{-1} h^{MN} T_{MN} \equiv \frac{F(\mu^2)}{M_\star^{2+n}}\, h^{MN} T_{MN} \tag{3.16}$$

which carries through in the decomposition of h_{MN} to the 4-dimensional field $G^n_{\mu\nu}$. At high energies $\mu \gg M_\star$ the form factor scales like $F(\mu^2) \propto (M_\star/\mu)^{2+n}$. The factor $1/(8\pi g_*)$ is an $\mathcal{O}(1)$ parameter controlling the transition to fixed point scaling.

As for any renormalization scale choice, there is an inherent ambiguity where to choose the scale μ. In QCD calculations this scale dependence vanishes once we include arbitrarily high orders in perturbation theory, which in our case will not help. For a collider process, the simplest

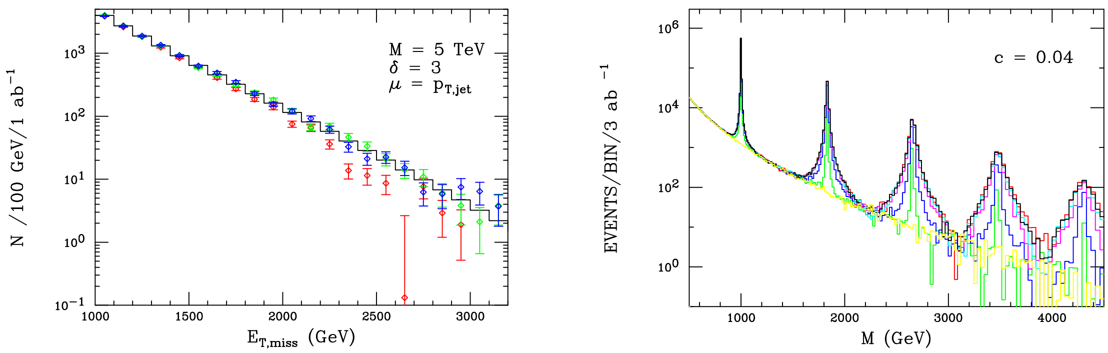


Figure 4: Left: Missing transverse energy distribution for single jet + $\not{E}_T$ signal. The black histogram represents standard ADD result for 1000 fb^{-1} of LHC integrated luminosity. Colored data points are for different parameterizations of the fixed point cross-over. Right: Resonance RS graviton production at the LHC for a lightest KK graviton of 1TeV Again, colored lines are for different parameterizations of the cross-over region. Both figures from Ref.[53].

choice is $\mu = \sqrt{s}$, in which case the form factor only has a noticeable effect for center of mass energies close to $M_\star$, as one might expect.

With this form factor the KK mass kernel in our virtual graviton exchange amplitude can be written as

$$\mathscr{S}(s) = \frac{1}{M_\star^{2+n}} \int d^n m_{\rm KK} \frac{1}{s - m_{\rm KK}^2} \quad \to \quad \left[1 + \left(\frac{s}{M_\star^2} \right)^{1+n/2} \right]^{-1} \frac{1}{M_\star^{2+n}} \int d^n m_{\rm KK} \frac{1}{s - m_{\rm KK}^2} . \qquad (3.17)$$

Note that if we treat $m_{\rm KK}$ and $\sqrt{s}$ as separate scales and evaluate the form factor in terms of $\sqrt{s}$, the $m_{\rm KK}$ integration still requires a cutoff. On the other hand, as far as the s integration is concerned, the form factor solves the unitarity problem associated with graviton scattering amplitudes at the LHC. This can be seen by power counting: the amplitude for graviton production is proportional to $1/M_{\rm Planck}^2$. Summing over the KK tower replaces this factor with the fundamental Planck scale $1/M_\star^2$. In addition, the geometry factor from the integration adds a factor $1/M_\star^n$, which together gives the $1/M_\star^{2+n}$ we observe for example in eq.(1.34). The form factor compensates this precisely with its UV scaling $F(s) \propto (M_\star/\sqrt{s})^{2+n}$. The only thing we have to ensure is that the numerical factor $1/(8\pi g_*)$ does not spoil this counter-play [53].

Since $F(s)$ modifies the Planck scale or the gravitational coupling in general, we can apply it to virtual as well as real graviton emission, in the ADD model as well as in Randall–Sundrum models. One example is the production of the first KK graviton excitations in warped extra dimensions. As discussed before, those are single particles produced for example in gluon fusion or quark-antiquark scattering and decaying to jets or leptons. The obvious effect of the form factor is to reduce the number of gravitons produced at high energies $\sqrt{\hat{s}}$. For the Randall–Sundrum model we can see this in the left panel of Fig.4.

Virtual graviton processes in models with warped extra dimensions can also be modified by fixed point effects. The collider signal is dominated by resonant graviton production, as opposed

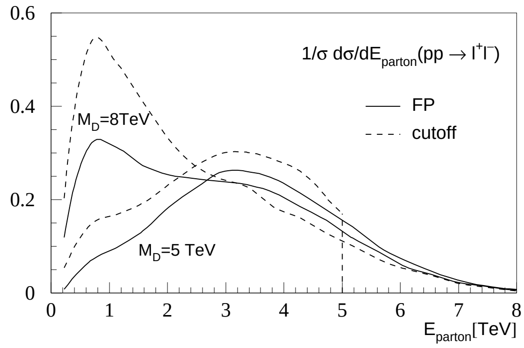


Figure 5: Normalized distribution of the partonic energy (or $m_{\ell\ell}$) in the Drell–Yan channel for $n = 3$. The non-trivial shape difference between the $M_\star = 5$ TeV and $M_\star = 8$ TeV is the result of interference effects. Figure from Ref. [52]

to the unspecific KK tower in ADD models. The form factor modifies the coupling used when computing the width Γ_j for the j-th RS graviton. For the production of one heavy state we run into the convenient fact that there is only one scale in the process $\mu = \sqrt{\hat{s}} = m_j$. The form factor becomes

$$F^{-1} = 1 + \left(\frac{m_j}{Me^{-\pi kr}} \right)^3 . \tag{3.18}$$

Without this form factor the width behaves like $\Gamma \sim m_j^3/M_{\rm RS}^2$ and the resonance interpretation becomes less and less valid at high energies (see Fig.4). Field theoretically this is inconvenient since once the width becomes of the same order as the mass spacing between modes the naive Breit-Wigner formalism breaks down [66]. Including the form factor the signal if formed by well defined resonances for higher masses, as shown in Fig.4.

An alternative (improved) method which is better suited for the case of virtual gravitons is based directly on the form of the anomalous dimension of the graviton eq.(3.9). It is motivated by renormalization group techniques from condensed matter physics or QCD [68]. In this picture, as usual there is a phase transition occuring at a critical point with anomalous dimension $\eta_\star = -2 - n$ for some energy at or beyond the Planck scale. At the critical point, correlation functions are expected to scale by a function of the anomalous dimension. The momentum-space two point function generically has the form $\Delta(p) \sim 1/(p^2)^{1-\eta/2}$, which reproduces the classical result for small η. In the vicinity of the non-gaussian fixed point, this becomes $1/(p^2)^{-(4+n)/2}$ [67]. The massive graviton propagator in the fixed point region is then modified as

$$\frac{1}{s - m_{\rm KK}^2} \to \frac{\Lambda_T^{2+n}}{(s - m_{\rm KK}^2)^{(4+n)/2}} \tag{3.19}$$

where the transition scale $\Lambda_T \sim M_\star$ in the numerator maintains the canonical dimensions for the propagator. The graviton kernel $\mathscr{S}(s)$ integrated over the entire $m_{\rm KK}$ range becomes

$$\mathscr{S}(s) \to \frac{1}{M_\star^{2+n}} \int_0^{\Lambda_T} d^n m_{\rm KK} \frac{1}{s - m_{\rm KK}^2} + \frac{1}{M_\star^{2+n}} \int_{\Lambda_T}^{\infty} d^n m_{\rm KK} \frac{\Lambda_T^{2+n}}{(s - m_{\rm KK}^2)^{(4+n)/2}} . \tag{3.20}$$

This integral is finite to for all n and the transition scale Λ_T parameterizes the crossover to fixed point scaling. The low-energy and high-energy contributions $\mathscr{S}_{\rm IR/UV}(s)$ can be easily calculated to leading order in s/Λ_T^2

$$\mathscr{S}_{\rm UV} = \frac{S_{n-1}}{4M_\star^4}\left(\frac{\Lambda_T}{M_\star}\right)^{n-2}\left[1+\mathscr{O}\left(\frac{s}{\Lambda_T^2}\right)\right].$$
$$\mathscr{S}_{\rm IR} = \frac{S_{n-1}}{(n-2)M_\star^4}\left(\frac{\Lambda_T}{M_\star}\right)^{n-2}\left[1+\mathscr{O}\left(\frac{s}{\Lambda_T^2}\right)\right]. \tag{3.21}$$

The transition from the IR to UV scaling we for now treat as a θ-function. The renormalization group predicts a smooth transition, which can be modelled using a $\tanh x$ function. The following approximations though will not be sensitive to the abruptness of the transitions, so we forgo implementing the smooth transition for the remainder of this work.

The combined IR and UV integral is given to leading order as

$$\mathscr{S} = \frac{S_{n-1}}{(n-2)M_\star^4}\left(\frac{\Lambda_T}{M_\star}\right)^{n-2}\left(1+\frac{n-2}{4}\right) \tag{3.22}$$

This result has several basic features:

- We do not need any artificial cutoff scale.
- The result from the $m_{\rm KK}$ integral has a small sensitivity to the precise value of the transition scale $\Lambda_T \sim M_\star$, but including a more elaborate description of the transition region will remove this.
- For hadronic cross sections there is an additional integral over s coming from the convolution with the parton distribution functions. Only in our leading-order approximation $\mathscr{S}$ is independent of s.
- For the full $\mathscr{S}_{\rm UV}(s)$ perturbative unitarity is maintained by the large-s behavior $\mathscr{S}_{\rm UV}(s) \sim s^{-2}$ given by dimensional analysis. $\mathscr{S}_{\rm UV}(s)$ and $\mathscr{S}_{\rm IR}(s)$ do not naively match perfectly at the boundary $\sqrt{s} = \Lambda_T$, which requires a more careful treatment of this matching for the final numerical results.
- Phenomenologically, we do not expect resonance peaks or clearly distinctive features in the UV regime of graviton production. This feature is clearly different from the string theory completion.

The anomalous dimension shift may also be implemented by evaluating the Euclidean propagator

$$\mathscr{S}_{\rm FP}(s) = \frac{1}{M_\star^{2+n}}\int_0^{\Lambda_T} d^n m_{\rm KK}\frac{1}{s+m_{\rm KK}^2} + \frac{1}{M_\star^{n-2}}\int_{\Lambda_T}^\infty d^n m_{\rm KK}\frac{\Lambda_T^{2+n}}{(s+m_{\rm KK}^2)^{(4+n)/2}} \tag{3.23}$$

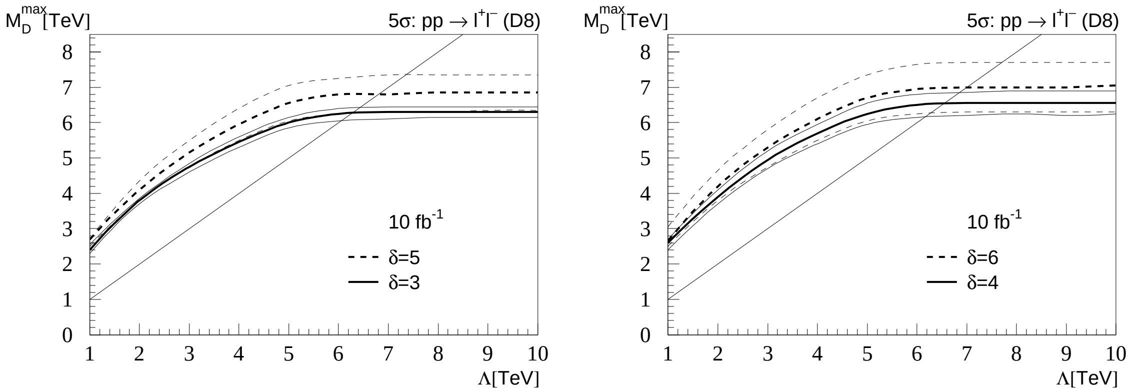


Figure 6: 5-σ discovery contours at the LHC. The solid diagonal line is for $M_\star = \Lambda_{\text{cutoff}}$. The plateau in the discovery contours is the result of the UV fixed point (compare to Fig.2).

As we will see below, this agrees with the form factor provided that we evaluate the coupling at a scale with $\mu = m$, reminiscent of the RS form factor used to regulate the graviton width — except that it is expected to hold in general off-shell. For large KK masses the form factor behaves as

$$\frac{1}{\overline{M}_\star^{2+n}} d^n m \left[1 + \frac{\omega}{8\pi} \left(\frac{m_{\text{KK}}^2}{\overline{M}_\star^2} \right)^{1+n/2} \right]^{-1} \frac{1}{s + m_{\text{KK}}^2} \approx S_{n-1} \left[\frac{8\pi}{\omega} \right] \frac{dm}{m_{\text{KK}}^5} . \tag{3.24}$$

This is in agreement with the anomalous dimension shift which gives

$$\left(\frac{\Lambda_T}{M_\star} \right)^{2+n} \frac{d^n m}{(s + m_{\text{KK}}^2)^{n/2+2}} \approx S_{n-1} \left(\frac{\Lambda_T}{M_\star} \right)^{2+n} \frac{dm}{m_{\text{KK}}^5} . \tag{3.25}$$

For the s integration there is no clear agreement between the two approaches. The form factor in s falls of much quicker than the $1/s^2$ and gives a reduced cross section compared to our estimate. This is expected since the second term in the expansion is of the opposite sign. Note that by construction this Eucledian argument avoids real particle poles in the graviton propagator and is hence limited when comparing to an effective field theory.

The search for extra dimensions at the LHC has two purposes. The first is to pin down the exact geometry of the non-visible space. The other and more interesting possibility is to ascertain a viable theory of quantum gravity by probing energies beyond $M_\star$. The UV completion to the KK integral can help us in both regards. First of all, the signal is enhanced by including the UV portion of the integral. In some cases this increase can be significant. For Drell–Yan production leading to final state muons the cross sections are given in the following table. Note that the LHC is expected to provide ~ 100 fb^{-1} per year at full luminosity, which leads to a non-trivial event number for the following scenarios.

σ[fb]	$n=3$			$n=6$		
$M_\star$	2 TeV	5 TeV	8 TeV	2 TeV	5 TeV	8 TeV
$\mathscr{S}_{IR}$	173	0.72	0.0204	66	0.28	0.008
$\mathscr{S}_{IR} + \mathscr{S}_{UV}$	408	1.24	0.0317	398	1.21	0.031

In addition, the graviton kernel has a distinctive shape which depends on the number of extra dimensions n. In the s channel at lower partonic energies, the dominant interference term between gravitons and Z/γ imply a scaling with $\mathscr{S} \sim (n-1)$. For higher partonic energies the pure graviton amplitude is dominant and the rate scales as $(n-1)^2$. This fact is demonstrated in Fig.5. The combined LHC reach in the virtual graviton channel is given in Fig.6 and is mostly independent of n for the cases $n = 3-6$.

In this section we present a rough description of the effects of a gravitational fixed point at the LHC. Many of the technical and physical details are not worked out yet, because we are talking about recent developments. However, we can convincingly argue that a gravitational UV fixed point gives a consistent as well as complete description of extra-dimensional observables at the LHC, clearly distinguishable from alternative scenarios.

4. Outlook

Large extra dimensions are a natural as well as truly minimal extension of the Standard Model addressing the hierarchy problem. If they are realized in Nature, gravity effects become relevant at the TeV scale and probing a viable theory of quantum gravity becomes an experimental endeavor. Two generic approaches are either a free number of large and flat extra dimensions (ADD model) or one extra dimension with a warped metric (RS model). In both models only gravity with its fundamental TeV-sized Planck scale propagates into the extra-dimensional bulk, while our 4-dimensional Planck scale is a derived observable. The relative size of the fundamental and 4-dimensional Planck scales can be derived by matching of the effective 4-dimensional Kaluza–Klein effective theory. The main phenomenological difference between the two models is the spacing of the Kaluza–Klein masses, which is unobservably small in the ADD model and of the order of the fundamental Planck scale in the RS model.

In particular for flat extra dimensions, the description by the KK effective theory becomes increasingly unreliable once the experimental energy reaches the TeV scale. The geometry factors from compactification imply that any kind of observation will be dominated by the respective UV tail of the graviton tower. While real graviton emission at the LHC is accidentally well described by an effective KK theory (due to sharply falling parton luminosities) the prospect of discovery via virtual graviton exchange depends on an unphysical cutoff scheme regulating the sum over the states inside the KK tower. For warped extra dimension a similar problem occurs in the width of the KK gravitons, which is determined by the UV completion of the model, *i.e.* the quantum gravity regime. Being a theoretical ambiguity this generic UV cutoff dependence can act as a platform on which to test candidate theories of quantum gravity.

This means that the moment we probe extra dimensions at the LHC we need to worry about the fundamental structure of gravity. For example, asymptotic safety or fixed-point gravity allows us to consider gravity as a UV-complete theory, without introducing additional states or ideas. For the UV regime of gravity as probed at the LHC it predicts a smooth fall-off in graviton amplitudes, clearly different from resonances as expected by a string theory completion. Once the LHC produces data we know there are a plethora of exciting possibilities — discerning among candidate theories of quantum gravity, in the fortuitous scenario of large extra dimensions, ranks among the most exciting prospects for the coming decade.

Acknowledgments

First and foremost, we are grateful to Daniel Litim for the exciting collaboration on ultraviolet fixed points at the LHC and to the BUSSTEPP school of 2006 for brokering this collaboration over great real ales. Daniel Litim also organized the school/workshop these lecture notes are based on. Moreover, TP would like to thank Gian Giudice and Tao Han for their convincing arguments on the attractiveness of extra dimensional theories. EG would like to thank the Institut für Theoretische Physik in Heidelberg for kind hospitality during the writing of these notes, as well as SUPA and the University of Heidelberg for their travel support.

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