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Old and new results from the Wilsonian approach to gravity

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We summarize recent results on the application of Wilsonian renormalization group techniques to gravity and discuss the existence and properties of a nontrivial fixed point. We analyze different approximation schemes in pure gravity and gravity coupled minimally to matter. We compare with earlier results in perturbation theory and point out the differences which indicate that gravity might be nonperturbatively renormalizable.

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1. Introduction

At low energies, gravity can be described very well by effective Quantum Field Theory (QFT) methods, and quantum corrections can be calculated reliably though being strongly suppressed [1]. Proposals where quantum gravity effects can be constrained or even could become visible in upcoming experiments exist e.g. in the context of additional space dimensions [2], and deformed [3] or violated Lorentz symmetry [4], in the context of renormalization group see also [5]. However, the perturbative QFT approach faces serious problems when one tries to remove the ultraviolet regulator [6]. Therefore it remains a challenging question if perturbative QFT methods can describe gravity at very high energies correctly or if the use of nonperturbative techniques is required. Nonperturbative tools are provided for example in Loop Quantum Gravity [7] based on canonical quantization methods together with the introduction of new variables, spin foam models [8], and Regge calculus [9] and dynamical triangulations [10] as discrete nonperturbative approximations to gravity, see also the corresponding contributions in this volume. The latter discretized versions of gravity hint at a nontrivial fixed point scaling of gravity at high energies.

If gravity does display a nontrivial fixed point, also more conventional, continuum, covariant quantum field theory methods are applicable by the use of Wilsonian renormalization group methods along the "asymptotic safety" program [11, 12]. Loosely speaking, a QFT is said to be asymptotically safe if there exists a finite dimensional space of action functionals (called the ultraviolet critical surface) which in the continuum limit are attracted towards a Fixed Point (FP) of the Renormalization Group (RG) flow. For example, a free theory has vanishing beta functions, so it has a FP called the Gaussian FP. Perturbation theory describes a neighbourhood of this point. In a perturbatively renormalizable and asymptotically free QFT such as QCD, the UV critical surface is parameterized by the couplings that have positive or zero mass dimension. Such couplings are called "renormalizable" or "relevant". Asymptotic safety is a generalization of this behaviour outside the perturbative domain.

In this talk we will summarize results from [28, 29] where strong evidence for the asymptotic safety of gravity has been obtained in different approximation schemes for pure gravity as well as gravity coupled minimally to matter.

2. Renormalization Group and Asymptotic Safety

New support for the asymptotic safety of gravity was obtained since the application of a form of Wilsonian Exact RG Equation (ERGE), which describes the dependence of a coarse–grained effective action functional $\Gamma_k[\Phi]$ on an infrared momentum cutoff scale k inducing an RG flow [13]. To do so, one introduces a momentum dependent regulator term $R_k(q^2)$ interpolating between k^2 and 0 so that the propagation of fields with momenta q lower than k is strongly suppressed and functional integration will extend over fields with momenta higher than k only. Additionally one requires that its derivative with respect to k is peaked sharply around k. Doing so, one can define an action functional Γ_k which fulfills exactly the partial functional differential equation

$$\partial_t \Gamma_k = \frac{1}{2}\mathrm{STr}\left(\frac{\delta^2\Gamma_k}{\delta\Phi\delta\Phi}+R_k\right)^{-1}\partial_t R_k \tag{2.1}$$

where $t = \log(k/k_0)$, Φ are all the fields present in the theory, STr is a generalized functional trace including a minus sign for fermionic variables and a factor 2 for complex variables, and R_k is the regulator that suppresses the contribution to the trace of fluctuations with momenta below k. For $R_k = k^2$ the RG flow would basically correspond to the one as obtained from the Callan-Symanzik equations, the RG-time derivative would however not be peaked around k [14]. This peak of $\partial_t R_k$ around k is necessary to certify that the trace will give finite contributions. Then also beta functions obtained from the ERGE will stay finite. In gravity, this equation has been applied in a number of works [15, 16], for reviews see [17, 14].[1]

Γ_k will include all coupling constants, so also all beta functions will be obtained from this equation. To solve this equation, a suitable method is to expand $\Gamma_k[\Phi]$ in field monomials with the general form

$$\Gamma_k[\Phi] = \sum_i g_i(k)\mathscr{O}_i[\Phi] \tag{2.2}$$

where $\mathscr{O}_i[\Phi]$ are operators constructed with the fields and their derivatives that have the required symmetries and g_i are running couplings of mass dimension d_i.[2] Then

$$\partial_t\Gamma_k[\Phi] = \sum_i \beta_i(k)\mathscr{O}_i[\Phi] \tag{2.3}$$

where $\beta_i = \partial_t g_i$. In general the functional (2.1) will contain infinitely many terms and infinitely many couplings; the easiest way of extracting nonperturbative information from the ERGE is to retain only a finite number of terms, introduce them in (2.1), evaluate the trace and read off the beta functions β_i. To analyze the fixed point structure of the RG flow, one has to measure the in general dimensionful couplings g_i with respect to some mass scale. One can choose for example the momentum cutoff scale k which leads to the dimensionless quantities $\tilde{g}_i = g_i k^{-d_i}$ and

$$\tilde{\beta}_i = \partial_t \tilde{g}_i = -d_i\tilde{g}_i + \beta_i k^{-d_i} , \tag{2.4}$$

where the first term comes from the classical, canonical dimension.[3] A FP is defined by the condition $\tilde{\beta}_i = 0$. Its existence is essential to give a well-defined behaviour to the coupling constants up to arbitrarily high energy scales. In general, physical observables will remain finite if the coupling constants do so.

Apart from the well-defined behaviour assured by the existence of a FP, it is also important that a theory depends only on a finite number of parameters (introducing an infinite number of counterterms to absorb all occuring infinities is not helpful). Asymptotic safety therefore requires a second condition which is that the surface called the UV critical surface obtained from those points whose trajectories are attracted towards the FP when $t \to \infty$, has finite dimensionality. If this condition is met, the requirement of being attracted to the FP, which guarantees a sensible UV behaviour, fixes all couplings up to a finite number of free parameters that have to be determined by experiment. This ensures that the theory will be predictive.[4]

[1] An exhaustive list of references can be found also on http://www.percacci.it/roberto/physics/as/biblio.html

[2] Another way is to reduce the symmetries of the theory as in the 2-Killing-vector reduction [19].

[3] A discussion on the role of systems of units can be found in [18].

[4] For a discussion regarding how much of the nontrivial FP structure can be seen already in perturbation theory see the contribution of M. Niedermaier in this volume.

The attractivity properties of a FP are determined by the signs of the critical exponents ϑ_i, defined to be minus the eigenvalues of the linearized flow matrix

$$M_{ij} = \frac{\partial \tilde{\beta}_i}{\partial \tilde{g}_j}\Big|_* . \tag{2.5}$$

The couplings corresponding to negative eigenvalues (positive critical exponent) are called relevant and parameterize the UV critical surface; they are attracted towards the FP in the UV and can have arbitrary values. The ones that correspond to positive eigenvalues (negative critical exponents) are called irrelevant; they are repelled by the FP and must be set to zero. One can show from (2.4) that at the Gaussian FP $\vartheta_i = d_i$, so the relevant couplings are the ones that are power–counting renormalizable (or marginally renormalizable). In a local theory they are usually finite in number. The structure of a nontrivial FP should agree in continuum formulations and the continuum limits obtained from discrete or lattice approaches. This should be the same for the continuum formulation we are using here for gravity and the discretized versions of Regge calculus and dynamical triangulations.

At a nontrivial FP the canonical dimensions receive loop corrections. However, such corrections are expected to be finite, in which case at most finitely many critical exponents could have different sign from the canonical dimension d_i. Therefore, it is generically expected that at any FP in a local theory there will only be a finite number of relevant couplings [11].

3. Einstein–Hilbert action

As we are not able to solve the ERGE exactly to obtain the scale-dependent action Γ_k, this action has to be approached successively. A good first approximation is the known low-energy action for gravity, the Einstein-Hilbert action. The stability of the results obtained in this approximation and the following ones against the addition of further couplings will indicate its quality. Working with a background gauge (the metric is split into a background field $\bar{g}_{\mu\nu}$ and a (not necessarily small) quantum field $h_{\mu\nu}$: $g_{\mu\nu} = \bar{g}_{\mu\nu} + h_{\mu\nu}$) one starts by inserting into the ERGE an action of the form

$$\Gamma_k[h,c,\bar{c}] = \frac{1}{16\pi G}\int d^dx\sqrt{g}(2\Lambda - R) + S_{GF} + S_c \tag{3.1}$$

where the couplings are scale-dependent. The gauge fixing will have the general form

$$S_{GF} = \frac{1}{32\pi G\alpha}\int d^dx\sqrt{\bar{g}}\,\chi_\mu \bar{g}^{\mu\nu}\chi_\nu \tag{3.2}$$

where $\chi_\nu = \nabla^\mu h_{\mu\nu} - \frac{1+\rho}{4}\nabla_\nu h^\mu_\mu$ (all covariant derivatives are with respect to the background metric). The ghost action contains the Fadeev–Popov term

$$S_c = \int d^dx\sqrt{\bar{g}}\,\bar{c}_\nu(\nabla^2\delta^\nu_\mu + R^\nu_{\ \mu})c^\mu . \tag{3.3}$$

Performing the calculation for different cutoff schemes and gauges leads to the general form

$$\partial_t\left(\frac{2\Lambda}{16\pi G}\right) = \frac{k^d}{16\pi}(A_1 + A_2\eta + A_3\partial_t\Lambda) \tag{3.4}$$

$$-\partial_t\left(\frac{1}{16\pi G}\right) = \frac{k^{d-2}}{16\pi}(B_1 + B_2\eta + B_3\partial_t\Lambda) \tag{3.5}$$

by comparing the coefficients, where A_i and B_i are rational functions of the couplings and $\eta = -\partial_t G/G$. Note that the one-loop approximation will follow by setting $\eta = 0$, $\partial_t \Lambda = 0$ on the right hand side of the equation. Separating the beta-functions and the fixed point analysis were the topic of a number of publications during the last years [15, 16]. Different types of regulator-functions R_k and gauges have been tried. In [29] we classified and analyzed different ways of choosing R_k to include or exclude curvature terms or couplings in the denominator $\Gamma_k^{(2)} + R_k$ on the r.h.s. of the ERGE and modifying the RG-scale dependence of R_k by including more or less couplings. The results presented there are in the gauge $\alpha = 1$, $\rho = 1$. The quite general result that is again and again confirmed in the different cutoff schemes and gauges is the existence of a FP with two attractive directions whose eigenvalues form a complex conjugated pair indicating a spiralling around the non-Gaussian FP.

4. Curvature squared terms

In the next approximation step, one will include further curvature terms into the action, e.g. curvature squared terms. To study the influence of the Newton constant on the different curvature squared couplings all couplings except the Newton constant are set to zero. If one neglects also its renormalization group time derivatives on the r.h.s., corresponding to the one-loop approximation, one obtains in the gauge $\alpha = 1$, $\rho = 1$ the famous result

$$\frac{d\Gamma_k}{dt}|_{R^2} = \frac{1}{16\pi^2}\int d^4x\sqrt{g}\left[\frac{7}{10}R_{\mu\nu}R^{\mu\nu} + \frac{1}{60}R^2 + \frac{53}{45}E - \frac{19}{15}\nabla^2 R\right] \tag{4.1}$$

as e.g. in [21, 22] which corresponds to the logarithmic divergencies of the curvature squared couplings. Note that [21] found that pure gravity was one-loop renormalizable whereas the addition of matter spoiled the renormalizability. Will the non-Gaussian FP exist in this case?

The full beta functions obtained with the ansatz

$$\frac{d\Gamma_k}{dt} = \int d^4x\sqrt{g}\left[\frac{1}{16\pi G}(2\Lambda - R) + \frac{1}{2\lambda}C^2 + \frac{1}{\xi}R^2 + \frac{1}{\rho}E + \frac{1}{\tau}\nabla^2 R\right] \tag{4.2}$$

have been calculated in [20]. There it has been found that the beta functions for the dimensionless couplings agree with those obtained in dimensional regularization at one-loop (are asymptotically free), whereas the beta functions for Λ and G contain additional terms coming from the B_0 and B_2 heat-kernel coefficients. However, only with these additional terms exists a non-Gaussian FP where all coupling directions are attractive.

How will the inclusion of matter effect these results? Adding minimally coupled scalar, Dirac, and Maxwell fields

$$\Gamma_k[g,\phi,\psi,A]|_{\text{matter}} = \int d^4x\sqrt{g}\left[\frac{1}{2}\nabla_\mu\phi\nabla^\mu\phi + \bar{\psi}\gamma^\mu\nabla_\mu\psi + \frac{1}{4}F_{\mu\nu}F^{\mu\nu}\right] \tag{4.3}$$

to the curvature squared action will give the following contribution to the r.h.s. of the ERGE

$$\begin{aligned}\frac{d\Gamma_k}{dt}|_{\text{matter}} = {} & \frac{n_S}{2}\mathrm{Tr}_{(S)}\left(\frac{\partial_t R_k}{\frac{\delta^2\Gamma_k}{\delta\phi\delta\phi} + R_k}\right) - \frac{n_D}{2}\mathrm{Tr}_{(D)}\left(\frac{\partial_t R_k}{\frac{\delta^2\Gamma_k}{\delta\psi\delta\psi} + R_k + \frac{R}{4}}\right) \\ & + \frac{n_M}{2}\mathrm{Tr}_{(M)}\left(\frac{\partial_t R_k}{\frac{\delta^2\Gamma_k}{\delta A\delta A} + R_k + \mathrm{Ricci}}\right) - n_M\mathrm{Tr}_{(gh)}\left(\frac{\partial_t R_k}{\frac{\delta^2\Gamma_k}{\delta g\delta g} + R_k}\right)\end{aligned} \tag{4.4}$$

where the last term stems from the ghost fields g obtained from the Maxwell fields. Then the beta functions β_i receive corrections from the matter content proportional to $\tilde{g}_i\, a_i$ where the coefficients a_i are functions of the numbers of the different particle species. The result is that the FP still exists and the R^2–couplings remain asymptotically free. This is an important result: at a level where perturbation theory indicated the breakdown of the theory the non-Gaussian FP does exist.

At the current stage we are not yet able to perform the exact comparison with the results by Goroff-Sagnotti [6] where it is shown that also pure gravity is not perturbatively renormalizable at two-loop level. However, we take our results as a good indication that the non-Gaussian FP could also exist at that level.

Our analysis showed the existence of the FP for several types of approximations to the full action. At this stage we still ended up with as many relevant couplings as included into the approximation. But the necessary conditions for asymptotic safety are of course not only the existence of the FP, but also that there will only remain a finite number of relevant couplings leaving only a finite number of free parameters in the theory. Therefore we concentrated further on pure gravity and included higher curvature terms. From this calculation it will be possible to conclude that indeed many of the higher-curvature couplings will become irrelevant.

5. f(R)-gravity

Including higher curvature operators we restrict here to spherical backgrounds. Then in the trace arguments of the ERGE will only occur Laplacians as differential operators and the heat-kernel expansion for the trace evaluation can be used. Then all curvature invariants reduce to the Ricci scalar times a numerical factor so that one has to take into account only operators of the type $\mathscr{O}_i = \int d^4x\sqrt{g}R^i$ with some power of the Ricci scalar R. Such theories belong to the $f(R)$-type and have attracted much attention recently in cosmological applications (see e.g. [23] and references therein). The quantization of such theories at one-loop has been discussed in [24]. Here we analyze the RG flow of this type of theories, assuming that f is a polynomial of order $n \leq 8$. The (Euclidean) action is approximated by

$$\Gamma_k[\Phi] = \sum_{i=0}^{n} g_i(k) \int d^4x\,\sqrt{g}R^i + S_{GF} + S_c\ , \tag{5.1}$$

where $\Phi = \{h_{\mu\nu}, c_\mu, \bar{c}_\nu\}$ and the last two terms correspond to the gauge fixing and the ghost sector as given in eqs. (3.2) and (3.3) and again the background gauge condition is used. In the ansatz (5.1) the beta functions can be obtained from a calculation of the trace in the r.h.s. of (2.1) on a spherical (Euclidean de Sitter) background.

The propagator can be partly diagonalized by the decomposition

$$h_{\mu\nu} = h^{TT}_{\mu\nu} + \nabla_\mu\xi_\nu + \nabla_\nu\xi_\mu + \nabla_\mu\nabla_\nu\sigma + \frac{1}{4}g_{\mu\nu}(h - \nabla^2\sigma). \tag{5.2}$$

The inverse propagator, including the Jacobians due to the change of variables (5.2), is given explicitly in [24]. The Jacobians can be formally exponentiated introducing appropriate auxiliary fields and a cutoff is introduced on these variables, too.

The cutoff operators are chosen so that the modified inverse propagator is identical to the inverse propagator except for the replacement of $z = -\nabla^2$ by $P_k(z) = z + R_k(z)$; we use exclusively the

n	$\tilde{\Lambda}_*$	$\tilde{G}_*$	$\Lambda_* G_*$	$10^3\times$								
				$\tilde{g}_{0*}$	$\tilde{g}_{1*}$	$\tilde{g}_{2*}$	$\tilde{g}_{3*}$	$\tilde{g}_{4*}$	$\tilde{g}_{5*}$	$\tilde{g}_{6*}$	$\tilde{g}_{7*}$	$\tilde{g}_{8*}$
1	0.130	0.990	0.128	5.23	-20.1							
2	0.130	1.566	0.202	3.29	-12.7	1.51						
3	0.132	1.015	0.134	5.18	-19.6	0.70	-9.7					
4	0.123	0.966	0.119	5.06	-20.6	0.27	-11.0	-8.65				
5	0.124	0.969	0.120	5.07	-20.5	0.27	-9.7	-8.03	-3.35			
6	0.122	0.958	0.117	5.05	-20.8	0.14	-10.2	-9.57	-3.59	2.46		
7	0.120	0.949	0.114	5.04	-21.0	0.03	-9.78	-10.5	-6.05	3.42	5.91	
8	0.122	0.959	0.117	5.07	-20.7	0.09	-8.58	-8.93	-6.81	1.17	6.20	4.70

Table 1: Position of the FP for increasing number n of couplings included. The first three columns give the FP values in the form of cosmological and Newton constant and their dimensionless product. The values g_{i*} (and only them) have been rescaled by a factor 1000.

optimized cutoff functions $R_k(z) = (k^2 - z)\theta(k^2 - z)$ [25]. This has the advantage that knowledge of the heat kernel coefficients which contain at most R^4 and which we take from [27] is sufficient to calculate all the beta functions. A major simplification can be performed by choosing another gauge than in the previous sections by setting $\rho = 0$, $\alpha = 0$.[5] This gauge has the advantage that many of the field components cancel away with each other. Details have been given elsewhere [28, 29].

This simplified form allows us to calculate the r.h.s. of (2.1) in de Sitter space exactly: it is a rational function of R and the couplings $\tilde{g}_i$. The beta functions can be extracted from this function by comparing equal powers of curvature on each side. This has been done using algebraic manipulation software, and the limit $n \leq 8$ was set by the hardware (a standard single–processor machine).

The result is that a nontrivial FP does indeed exist. Its position and the corresponding critical exponents are given in tables I and II respectively for actions ranging from $n = 1$ (the Einstein–Hilbert action) to $n = 8$. For convenience, we give also the FP values for the cosmological constant and the Newton constant related to the couplings g_0 and g_1 by $\Lambda = -g_0/(2g_1)$ and $G = -1/(16\pi g_1)$ as well as the dimensionless product ΛG at the FP which remains very stable under all changes in the approximation. These results have been confirmed in [30]. The slight numerical differences arise from a different treatment of zero-modes in the traces over the contributions from the ghost fields. Both methods have been used repeatedly in the literature. The one applied in [28, 29] and presented also here is chosen because with that choice the cancelation between equal contributions from the $h_{\mu\nu}$-decomposition and the ghost parts is complete.

One sees that a FP with the desired properties exists under the inclusion of more and more couplings. When a new coupling is added, new unphysical FPs tend to appear; this is due to the approximation of f by polynomials. However, among the FPs it has always been possible to find one for which the lower couplings and critical exponents have values that are close to those of the

[5] Note that $\alpha = 0$ corresponding to Landau gauge is a fixed point of the RG flow, for a proof in Yang-Mills theories see e.g. [26].

n	ϑ'	ϑ''	ϑ_2	ϑ_3	ϑ_4	ϑ_5	ϑ_6	ϑ_7	ϑ_8
1	2.38	2.17							
2	1.38	2.32	26.9						
3	2.71	2.27	2.07	-4.23					
4	2.86	2.45	1.55	-3.91	-5.22				
5	2.53	2.69	1.78	-4.36	-3.76	-4.88			
6	2.41	2.42	1.50	-4.11	-4.42	-5.98	-8.58		
7	2.51	2.44	1.24	-3.97	-4.57	-4.93	-7.57	-11.1	
8	2.41	2.54	1.40	-4.17	-3.52	-5.15	-7.46	-10.2	-12.3

Table 2: Critical exponents for increasing number n of couplings included. The first two critical exponents are a complex conjugate pair of the form $\vartheta' \pm \vartheta'' i$. The same is the case for the fourth and fifth critical exponent $\vartheta_4 \pm \vartheta_5 i$.

previous action ansatz. That FP is then identified as the nontrivial FP for the action including more couplings.

Looking at the columns of Tables I and II we see that in general the properties of the FP are remarkably stable under improvement of the approximation. Especially the FP values for the couplings g_0 and g_1, including the cosmological constant and Newton's constant, remain extremely stable against the inclusion of more couplings, indicating that an important part of the physics can be catched already by these two couplings in agreement with claims made in [16] about the validity of the Einstein–Hilbert action. The stability of the results under variation of the gauge parameters has been checked, further details can be found in [29].

The most important result of this calculation is that for all actions the operators from R^3 upwards are irrelevant. One can conclude that in this class of action ansatz the UV critical surface is three–dimensional. Its tangent space at the FP is spanned by the three eigenvectors corresponding to the eigenvalues with negative real part. In the parametrization (5.1), it is the three–dimensional subspace in $\mathbf{R}^9$ defined by the equation:

$$
\begin{aligned}
\tilde{g}_3 &= +0.00061243 + 0.06817374\tilde{g}_0 + 0.46351960\tilde{g}_1 + 0.89500872\tilde{g}_2 \\
\tilde{g}_4 &= -0.00916502 - 0.83651466\tilde{g}_0 - 0.20894019\tilde{g}_1 + 1.62075130\tilde{g}_2 \\
\tilde{g}_5 &= -0.01569175 - 1.23487788\tilde{g}_0 - 0.72544946\tilde{g}_1 + 1.01749695\tilde{g}_2 \\
\tilde{g}_6 &= -0.01271954 - 0.62264827\tilde{g}_0 - 0.82401181\tilde{g}_1 - 0.64680416\tilde{g}_2 \\
\tilde{g}_7 &= -0.00083040 + 0.81387198\tilde{g}_0 - 0.14843134\tilde{g}_1 - 2.01811163\tilde{g}_2 \\
\tilde{g}_8 &= +0.00905830 + 1.25429854\tilde{g}_0 + 0.50854002\tilde{g}_1 - 1.90116584\tilde{g}_2
\end{aligned} \tag{5.3}
$$

Of course, we cannot yet conclude from this calculation that the operators $\mathcal{O}_i$ with $i \geq 3$ would be irrelevant if one considered more general actions than here. The couplings used here are combinations of all the couplings of different curvature invariants which are proportional to the Ricci scalar on a spherical background. But the definite conclusion is that at least the number of relevant couplings will be significantly lower than the number of higher curvature invariants.[6]

[6]Recently, the authors of [31] succeeded to distinguish between the different curvature squared invariants and found

With a finite dimensional critical surface, one can make definite predictions in quantum gravity. The real world must correspond to one of the trajectories that emanate from the FP and lie in the critical surface. Thus, at some sufficiently large but finite value of k one can choose arbitrarily three couplings, for example $\tilde{g}_0$, $\tilde{g}_1$, $\tilde{g}_2$ and the remaining six are then determined by (5.3). These couplings could then be used to compute the probabilities of physical processes, and the relations (5.3), in principle, could be tested by experiments. The linear approximation is valid only at very high energies, but it should be possible to numerically solve the flow equations and study the critical surface further away from the FP.

Extending the results to higher polynomial $f(R)$-actions seems to be only a matter of computing power. In view of the results obtained here, we expect that for this class of action ansatz a FP with three attractive directions will be maintained.

6. Conclusions

At this stage, one can conclude from the systematic analysis of the ERGE in gravity that a nontrivial FP does exist in a large variety of approximation schemes. Our work pointed out that this is the case even in examples where the perturbative treatment gives nonrenormalizability as for the one-loop calculation for matter coupled minimally to gravity. Further more we gave substantial evidence to the fact that the UV critical surface tends to remain finite dimensional - introducing more and more couplings into the approximation scheme would not require more and more free parameters in the theory. This gives strong optimism that gravity can indeed be asymptotically safe.

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that also one of those couplings is irrelevant. Their calculation was made possible by a different heat-kernel expansion of the trace in the ERGE which is based on Liechnerowitz-operators. Ours are based on Laplace-operators which are the remaining ones on a spherical background.

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