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The Cosmological constant and the Wheeler-DeWitt Equation

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We discuss how to extract information about the cosmological constant from the Wheeler-DeWitt equation, considered as an eigenvalue of a Sturm-Liouville problem. The equation is approximated to one loop with the help of a variational approach with Gaussian trial wave functionals. A canonical decomposition of modes is used to separate transverse-traceless tensors (graviton) from ghosts and scalar. We show that no ghosts appear in the final evaluation of the cosmological constant. A zeta function regularization is used to handle with divergences. A renormalization procedure is introduced to remove the infinities together with a renormalization group equation. A brief discussion on the extension to a f(R) theory is considered.

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1. Introduction

The Friedmann-Robertson-Walker model of the universe, based on the Einstein's field equations gives an explanation of why the Universe is in an acceleration phase. This is supported by data observations on type I supernovae[1]. Nevertheless, to obtain such an expansion we need almost 76% of what is known as *Dark Energy*. Dark Energy is based on the following equation of state $P = \omega\rho$ (where P and ρ are the pressure of the fluid and the energy density, respectively). When $\omega < -1/3$, we are in the Dark energy regime, while we have a transition to *Phantom Energy* when $\omega < -1$. The particular case of $\omega = -1$ corresponds to a cosmological constant. Nevertheless, neither Dark Energy nor Phantom Energy models appear to be satisfactory to explain the acceleration. A proposal to avoid Dark and Phantom energy comes form the so-called modified gravity theories. In particular, one could consider the following replacement in the Einstein-Hilbert action[2] $(\kappa = 8\pi G)$

$$S = \frac{1}{2\kappa}\int d^4x\sqrt{-g}R + S^{matter} \quad \rightarrow \quad S = \frac{1}{2\kappa}\int d^4x\sqrt{-g}f(R) + S^{matter}. \tag{1.1}$$

It is clear that other more complicated choices could be done in place of $f(R)$[4]. In particular, one could consider $f\left(R, R_{\mu\nu}R^{\mu\nu}, R_{\alpha\beta\gamma\delta}R^{\alpha\beta\gamma\delta}, \ldots\right)$ or $f(R,G)$ where G is the Gauss-Bonnet invariant or any combination of these quantities[1]. One of the prerogatives of a $f(R)$ theory is the explanation of the cosmological constant. Nevertheless, nothing forbids to consider a more general situation where a $f(R)$ is combined with a cosmological constant Λ_c, especially in the context of the Wheeler-DeWitt equation (WDW)[5]. For a $f(R) = R$, one gets

$$\mathscr{H} = (2\kappa)\, G_{ijkl}\pi^{ij}\pi^{kl} - \frac{\sqrt{g}}{2\kappa}\left({}^3R - 2\Lambda_c\right) = 0, \tag{1.2}$$

where ${}^3R$ is the scalar curvature in three dimensions. The main reason to work with a WDW equation becomes more transparent if we formally re-write the WDW equation as[9]

$$\frac{1}{V}\frac{\int \mathscr{D}[g_{ij}]\,\Psi^*[g_{ij}]\int_\Sigma d^3x\hat{\Lambda}_\Sigma\Psi[g_{ij}]}{\int \mathscr{D}[g_{ij}]\,\Psi^*[g_{ij}]\,\Psi[g_{ij}]} = \frac{1}{V}\frac{\left\langle \Psi \middle| \int_\Sigma d^3x\hat{\Lambda}_\Sigma \middle| \Psi \right\rangle}{\langle \Psi|\Psi\rangle} = -\frac{\Lambda_c}{\kappa}, \tag{1.3}$$

where $V = \int_\Sigma d^3x\sqrt{g}$ is the volume of the hypersurface Σ and $\hat{\Lambda}_\Sigma = (2\kappa)\, G_{ijkl}\pi^{ij}\pi^{kl} - \sqrt{g}^3R/(2\kappa)$. Eq.(1.3) represents the Sturm-Liouville problem associated with the cosmological constant. The related boundary conditions are dictated by the choice of the trial wavefunctionals which, in our case are of the Gaussian type. Different types of wavefunctionals correspond to different boundary conditions. We can gain more information if we consider $g_{ij} = \bar{g}_{ij} + h_{ij}$,where $\bar{g}_{ij}$ is the background metric and h_{ij} is a quantum fluctuation around the background. Thus Eq.(1.3) can be expanded in terms of h_{ij}. Since the kinetic part of $\hat{\Lambda}_\Sigma$ is quadratic in the momenta, we only need to expand the three-scalar curvature $\int d^3x\sqrt{g}^3R$ up to the quadratic order. However, to proceed with the computation, we also need an orthogonal decomposition on the tangent space of 3-metric deformations[10, 11]:

[1]For a recent riview on $f(R)$, see Refs.[3, 4], while a recent review on the problem of $f(G)$ and $f(R,G)$ can be found in Ref.[6, 7].

$$h_{ij}=\frac{1}{3}\left(\sigma+2\nabla\cdot\xi\right)g_{ij}+\left(L\xi\right)_{ij}+h_{ij}^{\perp}. \tag{1.4}$$

The operator L maps ξ_i into symmetric tracefree tensors $\left(L\xi\right)_{ij}=\nabla_i\xi_j+\nabla_j\xi_i-\frac{2}{3}g_{ij}\left(\nabla\cdot\xi\right)$, $h_{ij}^{\perp}$ is the traceless-transverse component of the perturbation (TT), namely $g^{ij}h_{ij}^{\perp}=0$, $\nabla^i h_{ij}^{\perp}=0$ and h is the trace of h_{ij}. It is immediate to recognize that the trace element $\sigma=h-2\left(\nabla\cdot\xi\right)$ is gauge invariant. If we perform the same decomposition also on the momentum π^{ij}, up to second order Eq.(1.3) becomes

$$\frac{1}{V}\frac{\left\langle\Psi\left|\int_\Sigma d^3x\left[\hat{\Lambda}_\Sigma^{\perp}+\hat{\Lambda}_\Sigma^{\xi}+\hat{\Lambda}_\Sigma^{\sigma}\right]^{(2)}\right|\Psi\right\rangle}{\langle\Psi|\Psi\rangle}=-\frac{\Lambda_c}{\kappa}\Psi\left[g_{ij}\right]. \tag{1.5}$$

Concerning the measure appearing in Eq.(1.3), we have to note that the decomposition (1.4) induces the following transformation on the functional measure $\mathscr{D}h_{ij}\rightarrow\mathscr{D}h_{ij}^{\perp}\mathscr{D}\xi_i\mathscr{D}\sigma J_1$, where the Jacobian related to the gauge vector variable ξ_i is

$$J_1=\left[\det\left(\triangle g^{ij}+\frac{1}{3}\nabla^i\nabla^j-R^{ij}\right)\right]^{\frac{1}{2}}. \tag{1.6}$$

This is nothing but the famous Faddev-Popov determinant. It becomes more transparent if ξ_a is further decomposed into a transverse part ξ_a^T with $\nabla^a\xi_a^T=0$ and a longitudinal part $\xi_a^{\parallel}$ with $\xi_a^{\parallel}=\nabla_a\psi$, then J_1 can be expressed by an upper triangular matrix for certain backgrounds (e.g. Schwarzschild in three dimensions). It is immediate to recognize that for an Einstein space in any dimension, cross terms vanish and J_1 can be expressed by a block diagonal matrix. Since $\det AB=\det A\det B$, the functional measure $\mathscr{D}h_{ij}$ factorizes into

$$\mathscr{D}h_{ij}=\left(\det\triangle_V^T\right)^{\frac{1}{2}}\left(\det\left[\frac{2}{3}\triangle^2+\nabla_iR^{ij}\nabla_j\right]\right)^{\frac{1}{2}}\mathscr{D}h_{ij}^{\perp}\mathscr{D}\xi^T\mathscr{D}\psi \tag{1.7}$$

with $\left(\triangle_V^{ij}\right)^T=\triangle g^{ij}-R^{ij}$ acting on transverse vectors, which is the Faddeev-Popov determinant. In writing the functional measure $\mathscr{D}h_{ij}$, we have here ignored the appearance of a multiplicative anomaly[8]. Thus the inner product can be written as

$$\int\mathscr{D}h_{ij}^{\perp}\mathscr{D}\xi^T\mathscr{D}\sigma\Psi^*\left[h_{ij}^{\perp}\right]\Psi^*\left[\xi^T\right]\Psi^*\left[\sigma\right]\Psi\left[h_{ij}^{\perp}\right]\Psi\left[\xi^T\right]\Psi\left[\sigma\right]\left(\det\triangle_V^T\right)^{\frac{1}{2}}\left(\det\left[\frac{2}{3}\triangle^2+\nabla_iR^{ij}\nabla_j\right]\right)^{\frac{1}{2}}. \tag{1.8}$$

Nevertheless, since there is no interaction between ghost fields and the other components of the perturbation at this level of approximation, the Jacobian appearing in the numerator and in the denominator simplify. The reason can be found in terms of connected and disconnected terms. The disconnected terms appear in the Faddeev-Popov determinant and these ones are not linked by the Gaussian integration. This means that disconnected terms in the numerator and the same ones appearing in the denominator cancel out. Therefore, Eq.(1.5) factorizes into three pieces. The piece containing $\hat{\Lambda}_\Sigma^{\perp}$ is the contribution of the transverse-traceless tensors (TT): essentially is the graviton contribution representing true physical degrees of freedom. Regarding the vector term $\hat{\Lambda}_\Sigma^T$, we observe that under the action of infinitesimal diffeomorphism generated by a vector field ε_i, the components of (1.4) transform as follows[10]

$$\xi_j \longrightarrow \xi_j + \varepsilon_j, \qquad h \longrightarrow h + 2\nabla\cdot\xi, \qquad h_{ij}^{\perp} \longrightarrow h_{ij}^{\perp}. \tag{1.9}$$

The Killing vectors satisfying the condition $\nabla_i\xi_j + \nabla_j\xi_i = 0$, do not change h_{ij}, and thus should be excluded from the gauge group. All other diffeomorphisms act on h_{ij} nontrivially. We need to fix the residual gauge freedom on the vector ξ_i. The simplest choice is $\xi_i = 0$.This new gauge fixing produces the same Faddeev-Popov determinant connected to the Jacobian J_1 and therefore will not contribute to the final value. We are left with

$$\frac{1}{V}\frac{\left\langle \Psi^{\perp} \left| \int_\Sigma d^3x \left[\hat{\Lambda}_\Sigma^{\perp}\right]^{(2)} \right| \Psi^{\perp} \right\rangle}{\langle \Psi^{\perp} | \Psi^{\perp} \rangle} + \frac{1}{V}\frac{\left\langle \Psi^{\sigma} \left| \int_\Sigma d^3x \left[\hat{\Lambda}_\Sigma^{\sigma}\right]^{(2)} \right| \Psi^{\sigma} \right\rangle}{\langle \Psi^{\sigma} | \Psi^{\sigma} \rangle} = -\frac{\Lambda_c}{\kappa}\Psi\left[g_{ij}\right]. \tag{1.10}$$

Note that in the expansion of $\int_\Sigma d^3x\sqrt{g}R$ to second order, a coupling term between the TT component and scalar one remains. However, the Gaussian integration does not allow such a mixing which has to be introduced with an appropriate wave functional. Extracting the TT tensor contribution from Eq.(1.3) approximated to second order in perturbation of the spatial part of the metric into a background term $\bar{g}_{ij}$, and a perturbation h_{ij}, we get

$$\hat{\Lambda}_\Sigma^{\perp} = \frac{1}{4V}\int_\Sigma d^3x\sqrt{\bar{g}}G^{ijkl}\left[(2\kappa)K^{-1\perp}(x,x)_{ijkl} + \frac{1}{(2\kappa)}\left(\tilde{\triangle}_L\right)_j^a K^{\perp}(x,x)_{iakl}\right], \tag{1.11}$$

where

$$\left(\tilde{\triangle}_L h^{\perp}\right)_{ij} = \left(\triangle_L h^{\perp}\right)_{ij} - 4R_i^k h_{kj}^{\perp} + {}^3R h_{ij}^{\perp} \tag{1.12}$$

is the modified Lichnerowicz operator and $\triangle_L$is the Lichnerowicz operator defined by

$$\left(\triangle_L h\right)_{ij} = \triangle h_{ij} - 2R_{ikjl}h^{kl} + R_{ik}h_j^k + R_{jk}h_i^k \qquad \triangle = -\nabla^a\nabla_a. \tag{1.13}$$

G^{ijkl} represents the inverse DeWitt metric and all indices run from one to three. Note that the term $-4R_i^k h_{kj}^{\perp} + {}^3R h_{ij}^{\perp}$ disappears in four dimensions. The propagator $K^{\perp}(x,x)_{iakl}$ can be represented as

$$K^{\perp}(\overrightarrow{x},\overrightarrow{y})_{iakl} = \sum_\tau \frac{h_{ia}^{(\tau)\perp}(\overrightarrow{x})\, h_{kl}^{(\tau)\perp}(\overrightarrow{y})}{2\lambda(\tau)}, \tag{1.14}$$

where $h_{ia}^{(\tau)\perp}(\overrightarrow{x})$ are the eigenfunctions of $\tilde{\triangle}_L$. τ denotes a complete set of indices and $\lambda(\tau)$ are a set of variational parameters to be determined by the minimization of Eq.(1.11). The expectation value of $\hat{\Lambda}_\Sigma^{\perp}$ is easily obtained by inserting the form of the propagator into Eq.(1.11) and minimizing with respect to the variational function $\lambda(\tau)$. Thus the total one loop energy density for TT tensors becomes

$$\frac{\Lambda}{8\pi G} = -\frac{1}{2}\sum_\tau\left[\sqrt{\omega_1^2(\tau)} + \sqrt{\omega_2^2(\tau)}\right]. \tag{1.15}$$

The above expression makes sense only for $\omega_i^2(\tau) > 0$, where ω_i are the eigenvalues of $\tilde{\triangle}_L$. Concerning the scalar contribution of Eq.(1.10), in Ref.[20] has been proved that the cosmological constant contribution is

$$\frac{\Lambda^{\sigma}}{8\pi G} = \frac{1}{4}\sqrt{\frac{2}{3}}\sum_\tau\left[\sqrt{\omega^2(\tau)}\right], \tag{1.16}$$

where $\omega(\tau)$ is the eigenvalue of the scalar part of the perturbation. In the next section, we will explictly evaluate Eqs.$(1.15, 1.16)$ for a specific background.

2. One loop energy Regularization and Renormalization for a $f(R) = R$ theory

If we consider a background of the form

$$ds^2 = -N^2(r)\,dt^2 + \frac{dr^2}{1-\frac{b(r)}{r}} + r^2\left(d\theta^2 + \sin^2\theta d\phi^2\right), \tag{2.1}$$

then, with the help of Regge and Wheeler representation[12], $\left(\tilde{\triangle}_L h^{\perp}\right)_{ij}$ can be reduced to

$$\left[-\frac{d^2}{dx^2} + \frac{l(l+1)}{r^2} + m_i^2(r)\right] f_i(x) = \omega_{i,l}^2 f_i(x) \quad i = 1,2 \quad , \tag{2.2}$$

where we have used reduced fields of the form $f_i(x) = F_i(x)/r$ and where we have defined two r-dependent effective masses $m_1^2(r)$ and $m_2^2(r)$

$$\begin{cases} m_1^2(r) = \frac{6}{r^2}\left(1-\frac{b(r)}{r}\right) + \frac{3}{2r^2}b'(r) - \frac{3}{2r^3}b(r) \\ \\ m_2^2(r) = \frac{6}{r^2}\left(1-\frac{b(r)}{r}\right) + \frac{1}{2r^2}b'(r) + \frac{3}{2r^3}b(r) \end{cases} \quad (r \equiv r(x)). \tag{2.3}$$

In order to use the WKB approximation, from Eq.(2.2) we can extract two r-dependent radial wave numbers

$$k_i^2(r,l,\omega_{i,nl}) = \omega_{i,nl}^2 - \frac{l(l+1)}{r^2} - m_i^2(r) \quad i = 1,2 \quad . \tag{2.4}$$

When $b(r) = r_t = 2MG$, the effective masses can be approximated in the range where $r \in [r_t, 5r_t/2]$ with $m_1^2(r) = -m_2^2(r) = m_0^2(r)$. Such a restriction comes from the fact that the effective masses, in this range, represent short distance contribution. Indeed, we expect to receive large contribution from quantum fluctuations at short distances. It is now possible to explicitly evaluate Eq.(1.15) in terms of the effective mass. To further proceed we use the W.K.B. method used by 't Hooft in the brick wall problem[13] and we count the number of modes with frequency less than ω_i, $i = 1,2$. This is given approximately by

$$\tilde{g}(\omega_i) = \int_0^{l_{\max}} \nu_i(l,\omega_i)(2l+1)\,dl, \tag{2.5}$$

where $\nu_i(l,\omega_i)$, $i = 1,2$ is the number of nodes in the mode with (l,ω_i), such that $(r \equiv r(x))$

$$\nu_i(l,\omega_i) = \frac{1}{\pi}\int_{-\infty}^{+\infty} dx\sqrt{k_i^2(r,l,\omega_i)}. \tag{2.6}$$

Here it is understood that the integration with respect to x and $l_{\max}$ is taken over those values which satisfy $k_i^2(r,l,\omega_i) \geq 0$, $i = 1,2$. With the help of Eqs.(2.5, 2.6), Eq.(1.15) becomes

$$\frac{\Lambda}{8\pi G} = -\frac{1}{\pi}\sum_{i=1}^{2}\int_0^{+\infty} \omega_i \frac{d\tilde{g}(\omega_i)}{d\omega_i} d\omega_i. \tag{2.7}$$

This is the graviton contribution to the induced cosmological constant to one loop. The explicit evaluation of Eq.(2.7) gives

$$\frac{\Lambda}{8\pi G} = \rho_1 + \rho_2 = -\frac{1}{4\pi^2}\sum_{i=1}^{2}\int_{\sqrt{m_i^2(r)}}^{+\infty} \omega_i^2\sqrt{\omega_i^2 - m_i^2(r)}\,d\omega_i, \tag{2.8}$$

where we have included an additional 4π coming from the angular integration. The use of the zeta function regularization method to compute the energy densities ρ_1 and ρ_2 leads to

$$\rho_i(\varepsilon) = \frac{m_i^4(r)}{64\pi^2}\left[\frac{1}{\varepsilon} + \ln\left(\frac{4\mu^2}{m_i^2(r)\sqrt{e}}\right)\right] \quad i = 1,2 \quad , \tag{2.9}$$

where we have introduced the additional mass parameter μ in order to restore the correct dimension for the regularized quantities. Such an arbitrary mass scale emerges unavoidably in any regularization scheme. The renormalization is performed via the absorption of the divergent part into the re-definition of the bare classical constant Λ, namely $\Lambda \to \Lambda_0 + \Lambda^{div}$. The remaining finite value for the cosmological constant reads

$$\frac{\Lambda_0}{8\pi G} = (\rho_1(\mu) + \rho_2(\mu)) = \rho_{eff}^{TT}(\mu, r), \tag{2.10}$$

where $\rho_i(\mu)$ has the same form of $\rho_1(\varepsilon)$ but without the divergence. The quantity in Eq.(2.10) depends on the arbitrary mass scale μ. It is appropriate to use the renormalization group equation to eliminate such a dependence. To this aim, we impose that[14]

$$\frac{1}{8\pi G}\mu\frac{\partial\Lambda_0(\mu)}{\partial\mu} = \mu\frac{d}{d\mu}\rho_{eff}^{TT}(\mu, r). \tag{2.11}$$

Solving it we find that the renormalized constant Λ_0 should be treated as a running one in the sense that it varies provided that the scale μ is changing

$$\frac{\Lambda_0(\mu, r)}{8\pi G} = \frac{\Lambda_0(\mu_0, r)}{8\pi G} + \frac{m_0^4(r)}{16\pi^2}\ln\frac{\mu}{\mu_0}. \tag{2.12}$$

Substituting Eq.(2.12) into Eq.(2.10) we find

$$\frac{\Lambda_0(\mu_0, r)}{8\pi G} = -\frac{1}{32\pi^2}\left\{m_0^4(r)\left[\ln\left(\frac{m_0^2(r)\sqrt{e}}{4\mu_0^2}\right)\right]\right\}. \tag{2.13}$$

If we go back and look at Eq.(1.3), we note that what we have actually computed is the opposite of an effective potential (better an effective energy). Therefore, we expect to find physically acceptable solutions in proximity of the extrema. We find that Eq.(2.13) has an extremum when

$$\frac{1}{e} = \frac{m_0^2(r)}{4\mu_0^2} \qquad \Longrightarrow \qquad \frac{\bar{\Lambda}_0(\mu_0, \bar{r})}{8\pi G} = \frac{m_0^4(\bar{r})}{64\pi^2} = \frac{\mu_0^4}{4\pi^2 e^2}. \tag{2.14}$$

Actually $\bar{\Lambda}_0(\mu_0, \bar{r})$ is a maximum, corresponding to a minimum of the effective energy. Note also that there exists another extremum when

$$m_0^4(r) = 0 \qquad \Longrightarrow \qquad M = 0. \tag{2.15}$$

This solution corresponds to Minkowski space, producing no effect on the vacuum. For this reason it will be discarded. On the other hand, the effect of the gravitational fluctuations is to shift the minimum of the effective energy away from the flat solution leading to an induced cosmological constant. If we apply the same procedure to the scalar part of the perturbation, we find that the only consistent solution is that $\Lambda^{\sigma}=0$. Therefore, the whole contribution is due to the physical degrees of freedom: the graviton[15]. Plugging Eq.(2.14) into Eq.(2.12), we find

$$\frac{\Lambda_0(\mu,\bar{r})}{8\pi G}=\frac{\bar{\Lambda}_0(\mu_0,\bar{r})}{8\pi G}+\frac{m_0^4(\bar{r})}{16\pi^2}\ln\frac{\mu}{\mu_0}=\frac{m_0^4(\bar{r})}{64\pi^2}\left(1+4\ln\frac{\mu}{\mu_0}\right). \tag{2.16}$$

If we set $\mu_0=m_P$, where m_P is the Planck mass, we can find that

$$\frac{\Lambda_0(\tilde{\mu},\bar{r})}{8\pi G}=0 \qquad \text{when} \qquad \tilde{\mu}=\exp\left(-\frac{1}{4}\right)\mu_0. \tag{2.17}$$

Nevertheless, $\tilde{\mu}$ is of the order of the Planck mass again, but unfortunately is a scale which is very far from the nowadays observations. However, it is interesting to note that this approach can be generalized by replacing the scalar curvature R with a generic function of R. Although a $f(R)$ theory does not need a cosmological constant, rather it should explain it, we shall consider the following Lagrangian density describing a generic $f(R)$ theory of gravity

$$\mathscr{L}=\sqrt{-g}\,(f(R)-2\Lambda), \qquad with\ f''\neq 0, \tag{2.18}$$

where $f(R)$ is an arbitrary smooth function of the scalar curvature and primes denote differentiation with respect to the scalar curvature. A cosmological term is added also in this case for the sake of generality, because in any case, Eq.(2.18) represents the most general lagrangian to examine. Obviously $f''=0$ corresponds to GR.[17]. The semi-classical procedure followed in this work relies heavily on the formalism outlined in Refs.[20, 16]. The main effect of this replacement is that at the scale μ_0, we have a shift of the old induced cosmological constant into

$$\frac{\Lambda_0'(\mu_0,r)}{8\pi G}=\frac{1}{\sqrt{h(R)}}\left[\frac{\Lambda_0(\mu_0,r)}{8\pi G}+\frac{1}{16\pi GV}\int_\Sigma d^3x\sqrt{g}\frac{Rf'(R)-f(R)}{f'(R)}\right], \tag{2.19}$$

where V is the volume of the system. Note that when $f(R)=R$, consistently it is $h(R)=1$ with

$$h(R)=1+\frac{2\left[f'(R)-1\right]}{f'(R)} \tag{2.20}$$

We can always choose the form of $f(R)$ in such a way $\Lambda_0(\mu_0,r)$. This implies

$$\frac{\Lambda_0'(\mu_0,r)}{8\pi G}=\frac{1}{\sqrt{h(R)}}\frac{1}{16\pi GV}\int_\Sigma d^3x\sqrt{g}\frac{Rf'(R)-f(R)}{f'(R)}. \tag{2.21}$$

A comment is in order. We have found that our calculation is in agreement with Ref.[15], where only the graviton contribution is fundamental. Note also the absence of a Faddeev-Popov determinant. This is in agreement with Ref.[15] but also with Ref.[10], where the Faddeev-Popov determinant appears when perturbations of the shift vectors are considered. The second comment regards our one loop computation which is deeply different form the one loop computation of Refs.[18, 19], where the analysis has been done expanding directly $f(R)$. In our case, the expansion involves only the three dimensional scalar curvature. Note that with the metric (2.1) and the effective masses (2.3), in principle, we can examine every spherically symmetric metric. Note also the absence of boundary terms in the evaluation of the induced cosmological constant.

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