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An abelian model $U(1) \otimes U(1)$

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Based on product representation a new performance for abelian symmetry is developed. It yields a new gauge model $U(1)$ diverse from Stueckelberg formalism. It generates two potential fields that describes a massive non-linear photon plus the usual photon field. We derive the lagrangian corresponding for the symmetry $U(1) \otimes U(1)$ that describes an extended QED with one massive boson gauge invariant and the usual photon massless case. Also a leptonic sector with a field $W_{\mu}^{\pm}$ is considered. As result we obtain the Feynman rules for propagators and vertices of this lagrangian on the momentum space.

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A gauge field theory for abelian symmetry $U(1) \otimes U(1)$

An abelian model for composite leptons is presented as a extension of the quantum electrodynamics symmetry. It yields the possibility to explore an $U(1)$ symmetry enlargement, as a particular case of those extended Yang-Mills symmetry $SU(N) \otimes SU(N)$ [1]. As a first consequence, it yields a possible insertion of mass terms into the lagrangian without requiring the Stueckelberg formalism or breaking gauge symmetry [2, 3].

Based on mathematics formalism involving direct product [4] between fields we construct an abelian gauge symmetry $U(1) \otimes U(1)$ for matter composite fields [1]. It appears an extended symmetry gauge for quantum electrodynamics and seems to introduce a massive vectorial boson beside the usual massless photon.

The description of the interactions by means of composite fields already was considered by J. Schwinger [5]. A model of leptons also was developed, notably by S. Weinberg, S. Glashow and A. Salam as a description of the weak interactions by the composite symmetry $SU(2) \times U(1)$ [6]. For a review of electroweak theory and applications, *see* [7].

In our approach, firstly we consider the direct product between spinor and scalar fields (ψ, ϕ)

$$\chi = \psi \otimes \phi \;, \tag{1}$$

respectively, in which it transforms independently in accord with

$$\psi' = U_1(x)\psi \quad \text{and} \quad \phi' = U_2(x)\phi \;. \tag{2}$$

The groups U_1 and U_2 are given by local phase transformation for the abelian groups $U(1)$

$$U_1(x) = e^{i\omega_1(x)} \quad \text{and} \quad U_2(x) = e^{i\omega_2(x)} \;, \tag{3}$$

in which ω_1 and ω_2 are real functions.Thus we define gauge fields B_μ and C_μ independents and associated to gauge transformation (2)

$$B'_\mu = B_\mu + \frac{1}{e}\,\partial_\mu \omega_1 \quad \text{and} \quad C'_\mu = C_\mu + \frac{1}{g_1}\,\partial_\mu \omega_2 \;, \tag{4}$$

where e and g_1 are coupling constants. Using the direct products properties, the compose matter field has the following transformation

$$\chi' = U(x)\chi \quad \text{with} \quad U(x) = U_1 \otimes U_2 \;. \tag{5}$$

The associated composite covariant derivative is

$$D_\mu(B,C) = \alpha D_\mu(B) \otimes \mathbf{1} + \beta \mathbf{1} \otimes D_\mu(C) \;, \tag{6}$$

where $D_\mu(B)$ and $D_\mu(C)$ are defined as usual, and (α, β) are real parameters. Now we making the variables changing

$$(B_\mu, C_\mu) \longmapsto (A_\mu, Z_\mu) \;, \tag{7}$$

where

$$eA_\mu = eB_\mu + g_1 C_\mu \quad \text{and} \quad g_1 Z_\mu = eB_\mu - g_1 C_\mu \;, \tag{8}$$

and we will implement the composite abelian gauge theory with the new fields (A_μ, Z_μ). Moreover using (4), we obtain the transformation for the physical fields (A_μ, Z_μ)

$$A'_\mu = A_\mu + \frac{1}{e}\,\partial_\mu(\omega_1+\omega_2) \quad \text{and} \quad Z'_\mu = Z_\mu + \frac{1}{g_1}\,\partial_\mu(\omega_1-\omega_2)\,, \tag{9}$$

respectively. The corresponding covariant derivative will be redefined as

$$D_\mu(A,Z) = \partial_\mu + ieA_\mu + ig_1 Z_\mu\,, \tag{10}$$

in which we have redefined the constants coupling. Thus the main property of such gauge composite fields is the expansion of the symmetry benefits. This means that it allows a new distribution of a given symmetry for more fields, coupling constants, conservation laws and son on. For this one has to take $\omega_1 = \omega_2 = \omega$, consequently the transformation (9) takes the form

$$A'_\mu = A_\mu + \frac{1}{e}\,\partial_\mu \omega \quad \text{and} \quad Z'_\mu = Z_\mu\,. \tag{11}$$

which yields a new abelian lagrangian given by

$$\mathcal{L} = -\frac{1}{4}G_{\mu\nu}G^{\mu\nu} + \frac{1}{2}m^2 Z^\mu Z_\mu\,, \tag{12}$$

in which $G_{\mu\nu}$ is invariant by (11) and split antisymmetric and symmetric parts

$$G_{[\mu\nu]} = F_{\mu\nu} + a\,(\partial_\mu Z_\nu - \partial_\nu Z_\mu) \quad \text{and} \quad G_{(\mu\nu)} = b\,(\partial_\mu Z_\nu + \partial_\nu Z_\mu) + c\, g_2 Z_\mu Z_\nu\,, \tag{13}$$

respectively, where

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu\,, \tag{14}$$

with a, b and c real parameters. The constant coupling g_2 from (13) sets the self-interaction between massive vectorial bosons. Adding to the lagrangian a covariant fixing gauge, one gets

$$\mathcal{L} = -\frac{1}{4}G_{[\mu\nu]}G^{[\mu\nu]} - \frac{1}{4}G_{(\mu\nu)}G^{(\mu\nu)} + \frac{1}{2}m^2 Z^\mu Z_\mu - \frac{1}{2\xi}(\partial_\mu A^\mu + \sigma\partial_\mu Z^\mu)^2\,, \tag{15}$$

then

$$\begin{aligned}\mathcal{L} = &-\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{2\xi}(\partial_\mu A^\mu)^2 - aF_{\mu\nu}\partial^\mu Z^\nu - \frac{\sigma}{\xi}\partial_\mu A^\mu \partial_\nu Z^\nu - \frac{1}{2}(a^2+b^2)(\partial_\mu Z_\nu)^2 \\ &+\frac{1}{2}m^2 Z_\mu Z^\mu + \frac{1}{2}\left(a^2 - b^2 - \frac{\sigma^2}{\xi}\right)(\partial_\mu Z^\mu)^2 - bc\, g_2\, \partial_\mu Z_\nu Z^\mu Z^\nu - \frac{1}{4}c^2\, g_2^2\,(Z_\mu Z^\mu)^2\,,\end{aligned} \tag{16}$$

in which ξ and σ are real parameters. The equation (16) is showing a different approach from Stueckelberg for introducing a mass term in an abelian group.

For inserting the composite leptonic sector in this model, we define the spinor and scalar fields from (1) as associated to two independents abelian groups $U(1)$, where they can be written as column matrixes

$$\psi = \begin{pmatrix}\psi_1 \\ \psi_2\end{pmatrix} \quad \text{and} \quad \phi = \begin{pmatrix}\phi_1 \\ \phi_2\end{pmatrix}\,, \tag{17}$$

and performing the direct product from (1), one gets

$$\chi = \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix} \otimes \begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix} = \begin{pmatrix} \psi_1\phi_1 \\ \psi_1\phi_2 \\ \phi_1\psi_2 \\ \phi_2\psi_2 \end{pmatrix}, \tag{18}$$

which suggests to define the left-handed leptonic doublet

$$\Psi_\ell^L := \chi = \begin{pmatrix} \psi_\ell^L \\ \psi_{\nu_\ell}^L \end{pmatrix}. \tag{19}$$

Consequently these doublet transforms by $U(1)$ symmetry as

$$\begin{pmatrix} \psi_\ell^L \\ \psi_{\nu_\ell}^L \end{pmatrix} \longmapsto \begin{pmatrix} \psi_\ell^L \\ \psi_{\nu_\ell}^L \end{pmatrix}' = e^{i\omega(x)} \begin{pmatrix} \psi_\ell^L \\ \psi_{\nu_\ell}^L \end{pmatrix}, \tag{20}$$

independently we define the right-handed sector in accord with transformations

$$\psi_\ell^R \longmapsto \psi_\ell^{R\,\prime} = e^{i\omega(x)}\psi_\ell^R \quad , \quad \psi_{\nu_\ell}^R \longmapsto \psi_{\nu_\ell}^{R\,\prime} = \psi_{\nu_\ell}^R \,. \tag{21}$$

Here the index ℓ indicates all the known leptons $\ell = (e, \mu, \tau)$ and ν_ℓ their neutrinos, respectively. It gives that the leptonic sector lagrangian is

$$\mathcal{L}_\ell = \bar{\Psi}_\ell i\gamma^\mu D_\mu(A,Z)\Psi_\ell - m_\ell \bar{\Psi}_\ell \Psi_\ell + \bar{\Psi}_{\nu_\ell} i\gamma^\mu D_\mu(Z)\Psi_{\nu_\ell} - \frac{e}{2}\bar{\Psi}_{\nu_\ell}\gamma^\mu A_\mu (1-\gamma^5)\Psi_{\nu_\ell}\,, \tag{22}$$

where Ψ_ℓ and Ψ_{ν_ℓ} is the composition of left-handed plus right-handed leptons and neutrinos, respectively. Notice that the constant coupling g_1 is associated to interaction between leptonic matter fields and massive boson field.

Now the complete lagrangian $U(1)\otimes U(1)$ can be split into the form

$$\mathcal{L} = \mathcal{L}_{0A} + \mathcal{L}_{0Z} + \mathcal{L}_{0AZ} + \mathcal{L}_{0\ell} + \mathcal{L}_{int}\,, \tag{23}$$

with free terms

$$\begin{aligned} \mathcal{L}_{0A} &= A^\mu \,\frac{1}{2}\left[g_{\mu\nu}\Box - \left(1 - \frac{1}{\xi}\right)\partial_\mu\partial_\nu \right] A^\nu\,, \\ \mathcal{L}_{0Z} &= Z^\mu \,\frac{1}{2}\left[g_{\mu\nu}\left((a^2+b^2)\Box + m^2 \right) - \left(a^2 - b^2 - \frac{\sigma^2}{\xi}\right)\partial_\mu\partial_\nu \right] Z^\nu\,, \\ \mathcal{L}_{0AZ} &= A^\mu \left[a g_{\mu\nu}\Box - \left(a - \frac{\sigma}{\xi}\right)\partial_\mu\partial_\nu \right] Z^\nu\,, \\ \mathcal{L}_{0\ell} &= \bar{\Psi}_\ell \left(i\gamma^\mu\partial_\mu - m_\ell \right)\Psi_\ell + \bar{\Psi}_{\nu_\ell} i\gamma^\mu \partial_\mu \Psi_{\nu_\ell}\,, \end{aligned} \tag{24}$$

and the interaction terms

$$\mathcal{L}_{int} = -e\bar{\Psi}_\ell\gamma^\mu A_\mu \Psi_\ell - g_1\bar{\Psi}_\ell\gamma^\mu Z_\mu\Psi_\ell - g_1\bar{\Psi}_{\nu_\ell}\gamma^\mu Z_\mu \Psi_{\nu_\ell} - \frac{e}{2}\bar{\Psi}_{\nu_\ell}\gamma^\mu A_\mu(1-\gamma^5)\Psi_{\nu_\ell} - bc\,g_2\partial_\mu Z_\nu Z^\mu Z^\nu - \frac{1}{4}c^2 g_2^2 (Z_\mu Z^\mu)^2\,. \tag{25}$$

From (24) one gets the propagators on momentum space

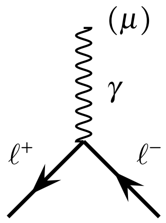
$$
\begin{aligned}
\langle A_\mu A_\nu\rangle &= -\frac{i}{k^2}\left[g_{\mu\nu}+(\xi-1)\frac{k_\mu k_\nu}{k^2}+\frac{a^2k^2}{b^2k^2-m^2}\left(g_{\mu\nu}-\frac{k_\mu k_\nu}{k^2}\right)-\sigma^2\,\frac{k_\mu k_\nu}{2b^2k^2-m^2}\right],\\
\langle Z_\mu Z_\nu\rangle &= -\frac{i}{b^2k^2-m^2}\left(g_{\mu\nu}-b^2\frac{k_\mu k_\nu}{2b^2k^2-m^2}\right),\\
\langle A_\mu Z_\nu\rangle = \langle Z_\mu A_\nu\rangle &= \frac{i}{b^2k^2-m^2}\left[a\left(g_{\mu\nu}-\frac{k_\mu k_\nu}{k^2}\right)+\sigma\frac{b^2k^2-m^2}{2b^2k^2-m^2}\,\frac{k_\mu k_\nu}{k^2}\right],
\end{aligned}
\tag{26}
$$

which show renormalizable massive propagators. Different from Proca this model is able to obtain a health power counting. The free propagators of leptons and neutrinos are

$$
\langle\bar\Psi_\ell\,\Psi_\ell\rangle = \frac{i}{\not p - m_\ell} \qquad \text{and} \qquad \langle\bar\Psi_{\nu_\ell}\,\Psi_{\nu_\ell}\rangle = \frac{i}{\not p}\,. \tag{27}
$$

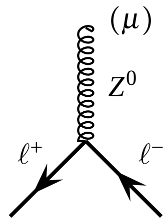
The vertex on the momentum space are

$-e\bar\Psi_\ell\gamma^\mu A_\mu\Psi_\ell$:



$= -ie\gamma^\mu$

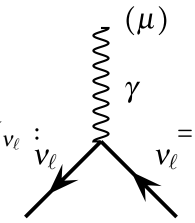
$-g_1\bar\Psi_\ell\gamma^\mu Z_\mu\Psi_\ell$:



$= -ig_1\gamma^\mu$

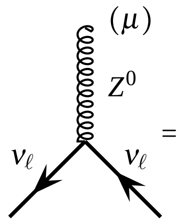
for the interactions of leptons with photons and massive vectorial boson, and in the neutrinos case

$-\frac{e}{2}\bar\Psi_{\nu_\ell}\gamma^\mu A_\mu(1-\gamma^5)\Psi_{\nu_\ell}$:



$= -\frac{ie}{2}\gamma^\mu(1-\gamma^5)$

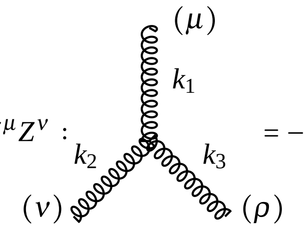
$-g_1\bar\Psi_{\nu_\ell}\gamma^\mu Z_\mu\Psi_{\nu_\ell}$:



$= -ig_1\gamma^\mu$

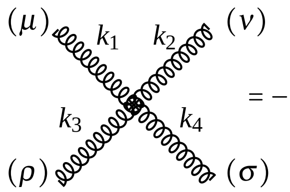
The self-interaction of the massive vectorial bosons are given by

$-bc\, g_2\partial_\mu Z_\nu Z^\mu Z^\nu$:



$= -bcg_2\left[g_{\mu\nu}(k_1+k_2)_\rho+g_{\nu\rho}(k_2+k_3)_\mu+g_{\mu\rho}(k_1+k_3)_\nu\right]$

$-\frac{1}{4}c^2g_2^2(Z_\mu Z^\mu)^2$:



$= -2ic^2g_2^2\left(g^{\mu\nu}g^{\rho\sigma}+g^{\mu\rho}g^{\nu\sigma}+g^{\mu\sigma}g^{\nu\rho}\right)$

We still can add charged massive field under this $U(1)\otimes U(1)$ context by adding a complex field W_μ under the local transformation

$$W_\mu \quad \longmapsto \quad W_\mu' = W_\mu \, e^{i\omega(x)} \; . \tag{28}$$

It interacts with photon field A_μ and the massive neutral boson Z_μ by substituting covariant derivative

$$W_{\mu\nu} = \partial_\mu W_\nu - \partial_\nu W_\mu \;\longmapsto\; W_{\mu\nu} = D_\mu(A,Z) W_\nu - D_\nu(A,Z) W_\mu \; , \tag{29}$$

in which $D_\mu(A,Z)$ is (10), so the charged massive field has the lagrangian

$$\mathcal{L}_{0W} = -\frac{1}{2} W_{\mu\nu}^{\dagger} W^{\mu\nu} + m_W^2 W_\mu^{\dagger} W^\mu \; . \tag{30}$$

Therefore the interaction lagrangian (31) is modified for

$$\begin{aligned}
\mathcal{L}_{int} = & -e\overline{\Psi}_\ell \gamma^\mu A_\mu \Psi_\ell - g_1 \overline{\Psi}_\ell \gamma^\mu Z_\mu \Psi_\ell - g_1 \overline{\Psi}_{\nu_\ell} \gamma^\mu Z_\mu \Psi_{\nu_\ell} - \frac{e}{2} \overline{\Psi}_{\nu_\ell} \gamma^\mu A_\mu (1-\gamma^5) \Psi_{\nu_\ell} \\
& - bcg_2 \partial_\mu Z_\nu Z^\mu Z^\nu - \frac{1}{4} c^2 g_2^2 (Z_\mu Z^\mu)^2 - \frac{1}{4} g_W^2 (W_\mu^\dagger W^\mu)^2 \\
& - ie\partial_\mu W_\nu^{\dagger} (A^\mu W^\nu - W^\mu A^\nu) - ig_1 \partial_\mu W_\nu^{\dagger} (Z^\mu W^\nu - W^\mu Z^\nu) - ie\partial^\mu W^\nu (W_\mu^{\dagger} A_\nu - A_\mu W_\nu^{\dagger}) \\
& - ig_1 \partial^\mu W^\nu (W_\mu^{\dagger} Z_\nu - Z_\mu W_\nu^{\dagger}) - e^2 W_\mu^\dagger W^\mu A_\nu A^\nu + e^2 W_\mu^\dagger A^\mu W_\nu A^\nu - g_1^2 W_\mu^\dagger W^\mu Z_\nu Z^\nu \\
& + g_1^2 W_\mu^\dagger Z^\mu W_\nu Z^\nu - 2eg_1 A_\mu Z^\mu W_\nu^{\dagger} W_\nu + eg_1 A_\mu W^\mu W_\nu^{\dagger} Z^\nu + eg_1 W_\mu^{\dagger} A_\mu W_\nu Z^\nu - ieG^{\mu\nu} W_\mu^\dagger W_\nu \; .
\end{aligned} \tag{31}$$

Thus based on symmetry $U(1)\otimes U(1)$ one gets a model including photon, weak bosons and the leptonic sector. In a further work we will study on its renormalizability and unitary. As first aspect, one notices the photon-neutrino interactions, also found in Stueckelberg formalism [2], which although is not included in the standard model it can be assumed through the meaning of *ubiquous lux*, which says that being an Lorentz invariant the photon should cover an universal interaction. Secondly, one can calculate the influence of the boson Z in electron gyromagnetic factor, as well as the gyromagnetic factor calculus of Z as a possible test of the model.

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