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Feynman rules for an intrinsic gauge model $SU(N)\otimes SU(N)$

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A composite non-abelian model $SU(N)\otimes SU(N)$ is proposed as an extension possible of the Yang-Mills usual symmetry. In particular it yields a possibility to go beyond *QCD* by reinterpreting the $SU(3)_c$ color symmetry group as a combination $SU(3)_c\otimes SU(3)_c$. Consequently, it yields the presence of massive gluons together with the usual massless case in the gauge field sector. This work also studies the corresponding Feynman rules of the model. Propagators and vertices are derived on the momentum space.

5th International School on Field Theory and Gravitation
April 20-24, 2009
Cuiabá city, Brazil

*Poster Session
†Thanks CNPq for financial support.

1. A non-abelian model for symmetry $SU(N)\otimes SU(N)$

The possibility to go beyond of Yang-Mills symmetry [1] is presented in this work considering the group $SU(N)$ as combination $SU(N)\otimes SU(N)$ [2]. The description of the interactions by means of composite fields was considered by J. Schwinger [3]. Based on direct product we define this operation between two independent non-abelian groups [4, 5]. In particular we will apply it to the model of an composite quantum chromodynamics $SU_c(3)\otimes SU_c(3)$ that preserves the experimental result where quarks contain three colors. It yields the possibility to explore an extended symmetry having contributions that not arise in the Yang-Mills symmetry [2]. For example, the possible insertion of mass terms into the lagrangian without breaking gauge symmetry.

Consider a matter field χ composite by the direct product

$$\chi = \psi \otimes \phi \,, \tag{1.1}$$

in which ψ and ϕ are independents spinor and scalar fields in the fundamental representation, respectively. The fields (ψ,ϕ) have independent local transformation

$$\psi' = U_1(x)\psi \quad \text{and} \quad \phi' = U_2(x)\phi \quad , \quad \text{with} \quad U_1(x) = e^{it_1^a\,\omega_1^a(x)} \quad \text{and} \quad U_2(x) = e^{it_2^a\,\omega_2^a(x)} \,, \tag{1.2}$$

where (t_1^a, t_2^a) are independents generators of two non-abelian groups $SU(N)$, and they satisfy the commutation relation

$$[t_1^a, t_1^b] = i f^{abc} t_1^c \quad \text{and} \quad [t_2^a, t_2^b] = i f^{abc} t_2^c \,, \quad \text{with} \quad a = 1,2,\ldots,N^2-1 \,, \tag{1.3}$$

and ω_1 and ω_2 are real functions. Using properties of the direct products and the transformations above, we obtain the local transformation $SU(N)\otimes SU(N)$

$$\chi' = U(x)\chi \quad , \quad U(x) = U_1 \otimes U_2 \,. \tag{1.4}$$

We propose a covariant derivative based on representation product

$$D_\mu(A,B) = D_\mu(A)\otimes \mathbf{1} + \mathbf{1}\otimes D_\mu(B) \,, \tag{1.5}$$

where each covariant derivative $D_\mu(A)$ and $D_\mu(B)$ act on ψ and ϕ, respectively

$$D_\mu(A)\psi = (\partial_\mu + ig_1 t_1^a A_\mu^a)\,\psi \quad \text{and} \quad D_\mu(B)\phi = (\partial_\mu + ig_2 t_2^a B_\mu^a)\,\phi \,, \tag{1.6}$$

and the gauge fields (A_μ^a, B_μ^a) transform in accord with

$$A'^{a}_\mu t_1^a = U_1\, A_\mu\, U_1^{-1} + \frac{i}{g_1}\,(\partial_\mu U_1)U_1^{-1} \quad \text{and} \quad B'^{a}_\mu t_2^a = U_2\, B_\mu\, U_2^{-1} + \frac{i}{g_2}\,(\partial_\mu U_2)U_2^{-1} \,. \tag{1.7}$$

Given the definition for composite covariant derivative (1.5) and some properties of direct product we find the following gauge transformation

$$[D_\mu(A,B)]' = U D_\mu(A,B) U^{-1} \,. \tag{1.8}$$

For a physical interpretation one has to make the variable change

$$g_1 A_\mu^a = g_1 G_\mu^a + g_2 C_\mu^a \qquad \text{and} \qquad g_2 B_\mu^a = g_1 G_\mu^a - g_2 C_\mu^a \,, \tag{1.9}$$

in which (G^a_μ, C^a_μ) are the physical fields that we are interested. Consequently the new configuration for covariant derivative (1.6) in terms of the fields (G^a_μ, C^a_μ) is

$$D_\mu(A,B)\chi = D_\mu(G,C)\chi = (\partial_\mu + ig_1 G_\mu^a T^a + ig_2 C^a_\mu t^a)\,\chi\,, \tag{1.10}$$

the equation (1.10) shows the presence of generators $\{T^a\}$ and $\{t^a\}$ associated to the new fields (G^a_μ, C^a_μ), respectively, expressed as

$$T^a = t^a_1 \otimes \mathbf{1} + \mathbf{1} \otimes t^a_2 \quad \text{and} \quad t^a = t^a_1 \otimes \mathbf{1} - \mathbf{1} \otimes t^a_2\,, \tag{1.11}$$

and satisfying the following commutation relation

$$[T^a, T^b] = i f^{abc} T^c \quad , \quad [t^a, t^b] = i f^{abc} T^c \quad \text{and} \quad [T^a, t^b] = i f^{abc} t^c \,, \tag{1.12}$$

in accord with the Lie algebra. Notice that the coupling constants g_1 and g_2 are associated to G_μ and C_μ fields.

The next step is in obtain the gauge transformation for the extrinsic fields (G^a_μ, C^a_μ), then we consider the gauge transformation

$$D_\mu(G,C)' = U D_\mu(G,C) U^{-1}\,, \tag{1.13}$$

and using above relationships for $\omega_1^a = \omega_2^a = \omega^a$, it seems to write the above transformation as

$$G'^a_\mu T^a = U\, G^a_\mu T^a\, U^{-1} + \frac{i}{g_1}\,(\partial_\mu U)U^{-1} \quad \text{and} \quad C'^a_\mu t^a = U\, C^a_\mu t^a\, U^{-1}\,, \tag{1.14}$$

where equation (1.14) shows the presence of a massless and massive gluons.

For deriving the correspondent invariant lagrangian to such gauge transformation (1.14), we construct the following field strength tensors

$$F_{\mu\nu}(G) = [D_\mu(G), D_\nu(G)]\,, \quad f_{\mu\nu}(G,C) = [D_\mu(G), C_\nu] \quad \text{and} \quad C_{\mu\nu} = g_3 C_\mu C_\nu\,, \tag{1.15}$$

where we have defined $D_\mu(G) = \partial_\mu + ig_1 G_\mu$. By rewriting those tensors into the components notation we first obtain

$$F_{\mu\nu} = F^a_{\mu\nu} T^a \quad \text{where} \quad F^{\;a}_{\mu\nu} = \partial_\mu G^a_\nu - \partial_\nu G^a_\mu - g_1 f^{abc} G^b_\mu G^c_\nu\,. \tag{1.16}$$

The second tensor has the mix between G_μ and C_μ

$$f_{\mu\nu} = f^a_{\mu\nu} t^a\,, \quad \text{with} \quad f^a_{\mu\nu} = \partial_\mu C^a_\nu - g_1 f^{abc} G^b_\mu C^c_\mu\,, \tag{1.17}$$

in which it is split into the antisymmetric and symmetric parts

$$f^{\;a}_{[\mu\nu]} = \partial_\mu C^a_\nu - \partial_\nu C^a_\mu - g_1 f^{abc} G^b_\mu C^c_\nu - g_1 f^{abc} C^b_\mu G^c_\nu\,, \;\text{and}\; f^{\;a}_{(\mu\nu)} = \partial_\mu C^a_\nu + \partial_\nu C^a_\mu - g_1 f^{abc} G^b_\mu C^c_\nu + g_1 f^{abc} C^b_\mu G^c_\nu\,, \tag{1.18}$$

respectively. The third tensor is obtained only in terms of C_μ

$$C_{[\mu\nu]} = C^{\;a}_{[\mu\nu]} T^a\,, \quad \text{with} \quad C^a_{[\mu\nu]} = g_3 f^{abc} C^b_\mu C^c_\nu\,, \tag{1.19}$$

and the symmetric part

$$C_{(\mu\nu)} = g_3\left\{C_\mu, C_\nu\right\} = g_3 C^a_\mu C^b_\nu \left\{t^a, t^b\right\} = g_3 C^a_\mu C^b_\nu (4\delta^{ab} - 2t^a_1 \otimes t^b_2 - 2t^b_1 \otimes t^a_2 + d^{abc}T^c)\ , \tag{1.20}$$

in which g_3 is the constant coupling associated to self-interactions of massive gluons. Defining the general tensor $Z_{\mu\nu}$

$$Z_{[\mu\nu]} = F_{\mu\nu} + a f_{[\mu\nu]} + bC_{[\mu\nu]} \quad \text{and} \quad Z_{(\mu\nu)} = c f_{(\mu\nu)} + dC_{(\mu\nu)}\ , \tag{1.21}$$

in which (a,b,c,d) are real parameters, one gets

$$Z_{[\mu\nu]} = Z^{(T)a}_{[\mu\nu]}T^a + Z^{(t)a}_{[\mu\nu]}t^a\ , \tag{1.22}$$

where

$$Z^{(T)a}_{[\mu\nu]} = F^a_{\mu\nu} + bC^a_{[\mu\nu]} \quad \text{and} \quad Z^{(t)a}_{[\mu\nu]} = a f^a_{[\mu\nu]}\ . \tag{1.23}$$

Similarly

$$Z_{(\mu\nu)} = Z^{(T)a}_{(\mu\nu)}T^a + Z^{(t)a}_{(\mu\nu)}t^a + Z^{(\Lambda)ab}_{(\mu\nu)}\Lambda^{ab}\ , \tag{1.24}$$

where

$$Z^{(T)a}_{(\mu\nu)} = dg_3 d^{abc}C^b_\mu C^c_\nu\ , \quad Z^{(t)a}_{(\mu\nu)} = c f^a_{(\mu\nu)}\ , \quad Z^{(\Lambda)\,ab}_{(\mu\nu)} = dg_3 C^a_\mu C^b_\nu\ ,$$
$$\text{and} \quad \Lambda^{ab} = 4\delta^{ab} - 2t^a_1 \otimes t^b_2 - 2t^b_1 \otimes t^a_2\ . \tag{1.25}$$

It yields the complete lagrangian as

$$\mathcal{L} = -\frac{1}{4}\mathrm{tr}(Z_{\mu\nu}Z^{\mu\nu}) - \frac{1}{4}\mathrm{tr}(\tilde{Z}_{\mu\nu}Z^{\mu\nu}) + \frac{1}{2}m^2\mathrm{tr}(C_\mu C^\mu) - \frac{1}{2\xi}\mathrm{tr}\left[(\partial_\mu G_\mu + \sigma\partial_\mu C_\mu)^2\right]\ , \tag{1.26}$$

where we have taken into account the semi-topological term $\tilde{Z}_{\mu\nu} = \frac{1}{2}\,\varepsilon_{\mu\nu\alpha\beta}Z^{\alpha\beta}$. Using the traces relations

$$\mathrm{tr}(T^aT^b) = \mathrm{tr}(t^at^b) = N\delta^{ab}\ \ ,\ \ \mathrm{tr}(T^at^b) = \mathrm{tr}(T^a\Lambda^{bc}) = \mathrm{tr}(t^a\Lambda^{bc}) = 0$$
$$\text{and} \quad \mathrm{tr}(\Lambda^{ab}\Lambda^{cd}) = 4\delta^{ab}\delta^{cd} + 2\delta^{ac}\delta^{bd} + 2\delta^{ad}\delta^{bc}\ , \tag{1.27}$$

one gets the free part

$$\mathcal{L}_{0G} = -\frac{1}{4}(\partial_\mu G^a_\nu - \partial_\nu G^a_\mu)^2 - \frac{1}{2\xi}(\partial_\mu G^{\mu\,a})^2\ ,$$
$$\mathcal{L}_{0C} = -\frac{a^2}{4}(\partial_\mu C^a_\nu - \partial_\nu C^a_\mu)^2 - \frac{c^2}{4}(\partial_\mu C^a_\nu + \partial_\nu C^a_\mu)^2 - \frac{\sigma^2}{2\xi}(\partial_\mu C^{\mu\,a})^2 + \frac{1}{2}m^2\, C^a_\mu C^{\mu\,a}\ ,$$
$$\mathcal{L}_{0FP} = \bar{\zeta}^a(\delta^{ab}\Box)\zeta^b\ , \tag{1.28}$$

and the interactions part, splitting it into the antisymmetric

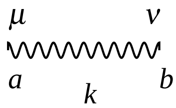
$$\mathcal{L}^{A\,(3)}_I = g_1 f^{abc}\partial_\mu G^a_\nu G^{\mu\,b}G^{\nu\,c} + a^2 g_1 f^{abc}\partial_\mu C^a_\nu G^{\mu\,b}C^{\nu\,c} + a^2 g_1 f^{abc}\partial_\mu C^a_\nu C^{\mu\,b}G^{\nu\,c}$$
$$+bg_3 f^{abc}\partial_\mu G^a_\nu C^{\mu\,b}C^{\nu\,c} + a^2 g_1 f^{abc}\varepsilon^{\mu\nu\alpha\beta}\partial_\mu C^a_\nu G^b_\alpha C^c_\beta\ ,$$
$$\mathcal{L}^{A\,(4)}_I = -\frac{1}{4}g_1^2 f^{abc} f^{ade} G^b_\mu G^c_\nu G^{\mu\,d}G^{\nu\,e} - \frac{a^2}{2}g_1^2 f^{abc} f^{ade} G^b_\mu C^c_\nu G^{\mu\,d}C^{\nu\,e}$$
$$-\frac{a^2}{2}g_1^2 f^{abc} f^{ade} C^b_\mu G^c_\nu G^{\mu\,d}C^{\nu\,e} - \frac{b}{2}g_1 g_3 f^{abc} f^{ade} G^b_\mu G^c_\nu C^{\mu\,d}C^{\nu\,e}$$
$$-\frac{b^2}{4}g_3^2 f^{abc} f^{ade} C^b_\mu C^c_\nu C^{\mu\,d}C^{\nu\,e} - \frac{a^2}{2}g_1^2 f^{abc} f^{ade}\varepsilon^{\mu\nu\alpha\beta} G^b_\mu C^c_\nu G^d_\alpha C^e_\beta\ , \tag{1.29}$$

symmetric parts and Faddeev Popov interaction

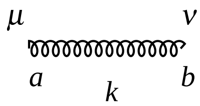
$$
\begin{aligned}
\mathcal{L}_I^{S\,(3)} &= c^2 g_1 f^{abc}\partial_\mu C_\nu^a G^{\mu\, b} C^{\nu\, c} - c^2 g_1 f^{abc}\partial_\mu C_\nu^a C^{\mu\, b} G^{\nu\, c}\,,\\
\mathcal{L}_I^{S\,(4)} &= -\frac{c^2}{2} g_1^2 f^{abc} f^{ade} G_\mu^b C_\nu^c G^{\mu\, d} C^{\nu\, e} + \frac{c^2}{2} g_1^2 f^{abc} f^{ade} G_\mu^b C_\nu^c C^{\mu\, d} G^{\nu\, e}\\
&-\frac{3d^2}{2N} g_3^2 C_\mu^a C_\nu^a C^{\mu b} C^{\nu b} - \frac{d^2}{2N} g_3^2 C_\mu^a C_\nu^b C^{\mu a} C^{\nu b} - \frac{d^2}{4} g_3^2 d^{abc} d^{ade} C_\mu^b C_\nu^c C^{\mu\, d} C^{\nu\, e}\,,\\
\mathcal{L}_I^{FP} &= g_1 f^{abc}\partial_\mu \bar{\chi}^a G^{\mu\, c}\chi^b + \sigma g_1 f^{abc}\partial_\mu \bar{\chi}^a C^{\mu\, c}\chi^b\,,
\end{aligned}
\tag{1.30}
$$

respectively. Thus one derives the objective of this work which is the Feynman rules. Considering the propagators, one gets

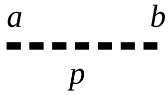
$$\langle G_\mu^a G_\nu^b\rangle = -\frac{i\delta^{ab}}{k^2}\left[g_{\mu\nu} + (\xi-1)\frac{k_\mu k_\nu}{k^2}\right]$$



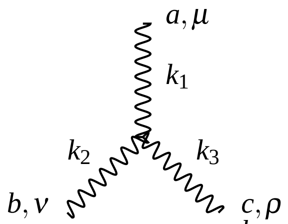
$$\langle C_\mu^a C_\nu^b\rangle = -\frac{i\delta^{ab}}{(a^2+c^2)k^2-m^2}\left[g_{\mu\nu} + \left(a^2-c^2-\frac{\sigma^2}{\xi}\right)\frac{k_\mu k_\nu}{\left(2c^2+\frac{\sigma^2}{\xi}\right)k^2-m^2}\right]$$



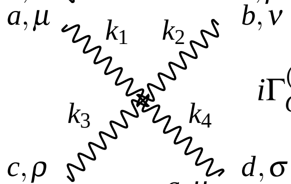
$$\langle \bar{\zeta}^a \zeta^b\rangle = -i\frac{\delta^{ab}}{p^2}$$



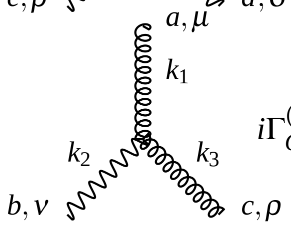
The Feynman rules for vertex are obtained on momentum space



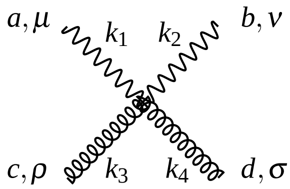
$$i\Gamma^{(3)}_{GGG} = g_1 f^{abc}\left[g^{\mu\nu}(k_1-k_2)^\rho + g^{\nu\rho}(k_2-k_3)^\mu + g^{\mu\rho}(k_3-k_1)^\nu\right]$$



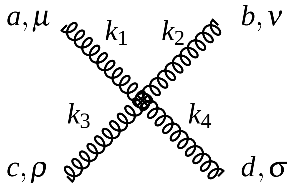
$$
\begin{aligned}
i\Gamma^{(4)}_{GGGG} = -ig_1^2\,[\,& f^{eab} f^{ecd}(g^{\mu\rho}g^{\nu\sigma} - g^{\mu\sigma}g^{\nu\rho}) + f^{eac} f^{ebd}(g^{\mu\nu}g^{\rho\sigma} - g^{\mu\sigma}g^{\rho\nu})\\
&+ f^{ead} f^{ebc}(g^{\mu\nu}g^{\rho\sigma} - g^{\mu\rho}g^{\nu\sigma})]
\end{aligned}
$$



$$
\begin{aligned}
i\Gamma^{(3)}_{GCC} = &\ bg_3 f^{abc}(g^{\nu\rho}k_2^\mu - g^{\mu\nu}k_2^\rho) + (a^2+c^2)g_1 f^{abc} g^{\mu\rho}(k_1-k_3)^\nu\\
&+(a^2-c^2)g_1 f^{abc}(g^{\mu\rho}k_1^\nu - g^{\nu\rho}k_2^\mu)
\end{aligned}
$$



$$
\begin{aligned}
i\Gamma^{(4)}_{GGCC} &= big_1 g_3 f^{eab} f^{ecd}(g^{\mu\rho}g^{\nu\sigma} - g^{\mu\sigma}g^{\nu\rho})\\
&-(a^2+c^2)ig_1^2\, g^{\mu\nu}g^{\rho\sigma}(f^{eac}f^{ebd} + f^{ead}f^{ebc})\\
&+(a^2-c^2)ig_1^2\,(f^{eac}f^{ebd}g^{\mu\sigma}g^{\nu\rho} + f^{ead}f^{ebc}g^{\mu\rho}g^{\nu\sigma})
\end{aligned}
$$

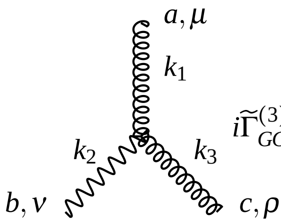


$$i\Gamma^{(4)}_{CCCC} = -ib^2 g_3^2 \,[\, f^{eab} f^{ecd} (g^{\mu\rho} g^{\nu\sigma} - g^{\mu\sigma} g^{\nu\rho}) + f^{eac} f^{ebd} (g^{\mu\nu} g^{\rho\sigma} - g^{\mu\sigma} g^{\rho\nu})$$

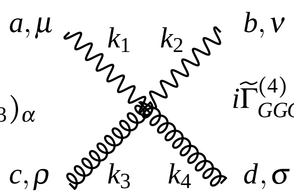
$$+ f^{ead} f^{ebc} (g^{\mu\nu} g^{\rho\sigma} - g^{\mu\rho} g^{\nu\sigma})] - \frac{6d^2}{N} i g_3^2 \,[\, \delta^{ab}\delta^{cd} (g^{\mu\rho} g^{\nu\sigma} + g^{\mu\sigma} g^{\nu\rho}) + \delta^{ac}\delta^{bd} (g^{\mu\nu} g^{\rho\sigma} + g^{\mu\sigma} g^{\nu\rho})$$

$$+ \delta^{ad}\delta^{bc} (g^{\mu\nu} g^{\rho\sigma} + g^{\mu\rho} g^{\nu\sigma})] - \frac{4d^2}{N} i g_3^2 \,(\delta^{ab}\delta^{cd} g^{\mu\nu} g^{\rho\sigma} + \delta^{ac}\delta^{bd} g^{\mu\rho} g^{\nu\sigma} + \delta^{ad}\delta^{bc} g^{\mu\sigma} g^{\nu\rho})$$

$$-d^2 i g_3^2 [d^{eab} d^{ecd} (g^{\mu\rho} g^{\nu\sigma} + g^{\mu\sigma} g^{\nu\rho}) + d^{eac} d^{ebd} (g^{\mu\nu} g^{\rho\sigma} + g^{\mu\sigma} g^{\rho\nu}) + d^{ead} d^{ebc} (g^{\mu\nu} g^{\rho\sigma} + g^{\mu\rho} g^{\nu\sigma})]$$

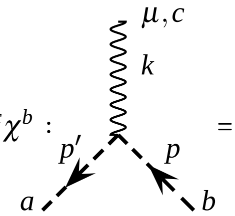


$$i\widetilde{\Gamma}^{(3)}_{GCC} = a^2 g_1 f^{abc} \varepsilon^{\alpha\mu\nu\rho} (k_1 + k_3)_\alpha$$



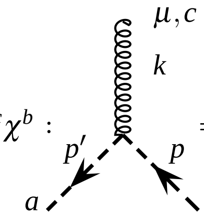
$$i\widetilde{\Gamma}^{(4)}_{GGCC} = -a^2 i g_1^2 \varepsilon^{\mu\nu\rho\sigma} (f^{ead} f^{ebc} - f^{eac} f^{ebd})$$

$g_1 f^{abc} \partial_\mu \bar{\chi}^a G^{\mu\, c} \chi^b$:



$= g_1 f^{abc} p^\mu$

$\sigma g_1 f^{abc} \partial_\mu \bar{\chi}^a C^{\mu\, c} \chi^b$:



$= \sigma g_1 f^{abc} p^\mu$

2. Conclusions

The natural continuation of this paper is the analysis on renormalizability and unitarity of the model. The effort here was just to extend *QCD* for massless and massive gluons by introducing quarks as composite fields. The quark section will be analyzed in more details [6], as well as some classical properties no established here.

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