PROCEEDINGS OF SCIENCE

Torsion Influence on Braneworld Scenarios

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In this paper we present the torsion influence in a braneworld scenario, developing the bulk metric Taylor expansion around the brane. This generalization is presented in order to better probe braneworld properties in a Riemann-Cartan framework, and it is also shown how the factors involving contorsion change the effective Einstein equation on the brane, the effective cosmological constant, and their consequence in a Taylor expansion of the bulk metric around the brane.

5th International School on Field Theory and Gravitation,
April 20th - 24th, 2009
Cuiabá, Brazil

*Poster Section. CNPq 472903/2008-0.
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PoS(ISFTG)026

1. Braneworld Scenarios

Hereon $\{e_\mu\}, \mu = 0,1,2,3$ $[\{e_A\}, A = 0,1,2,3,4]$ denotes a basis for the tangent space T_xM at a point x in M, where M denotes the manifold modelling a brane embedded in a bulk. Naturally the cotangent space at x has an orthonormal basis $\{\theta^\mu\}$ $[\{\theta^A\}]$ such that $\theta^\mu(e_\nu) = \delta^\mu_\nu$. If we choose a local coordinate chart, it is possible to represent $e_A = \partial/\partial x^A \equiv \partial_A$ and $\theta^A = dx^A$. Given the extrinsic curvature $\Xi = \Xi_{AB}\theta^A \wedge \theta^B$, it is possible to project the bulk curvature on the brane. Take $n = n^A e_A$ a vector orthogonal to T_xM and let y be the Gaussian coordinate orthogonal to the T_xM, on the brane at x, indicating how much an observer upheave out the brane into the bulk. In particular we have $n_A dX^A = dy$. A vector $v = v^A e_A$ in the bulk is split in components in the brane and orthogonal to the brane, respectively as $v = v^\mu e_\mu + y e_4 = (v^\mu, y)$. Since the bulk is endowed with a metric g that can be written in a coordinate basis, where $e^A := dx^A$, $e_A = \partial/\partial x^A$, as $g = g_{AB} dx^A \otimes dx^B$, the components of the metric on the brane and on the bulk are hereon denoted respectively by g_{AB} and ${}^{(5)}g_{AB}$, and related by

$$ {}^{(5)}g_{AB} = g_{AB} + n_A n_B\,, \quad {}^{(5)} \qquad ds^2 = g_{\mu\nu}(x^\alpha, y)dx^\mu dx^\nu + dy^2. \tag{1.1}$$

The extrinsic curvature of $\{y = \text{ const}\}$ surfaces describes the embedding of these surfaces. It can be defined via the Lie derivative or via the covariant derivative:

$$\Xi_{AB} = \frac{1}{2}\pounds_{\mathbf{n}}\, g_{AB} = g_A{}^C\, {}^{(5)}\nabla_C n_B\,, \tag{1.2}$$

so that $\Xi_{[AB]} = 0 = \Xi_{AB}n^B$. Using Einstein equations, it is possible to write the field equations in the bulk

$$ {}^{(5)}G_{AB} = -\frac{1}{2}\Lambda_5\, {}^{(5)}g_{AB} + \kappa_5^2\, {}^{(5)}T_{AB}, \tag{1.3}$$

where Λ_5 denotes the cosmological constant in the bulk, which prevents gravity from leaking into the extra dimension at low energies. The field equations on the brane can be written as [9]

$$\begin{aligned} G_{\mu\nu} \;=\; & -\frac{1}{2}\Lambda_5 g_{\mu\nu} + \kappa_5^2\,{}^{(5)}T_{\mu\nu} + {}^{(5)}R_{BD}n^B n^D g_{\mu\nu} + \frac{1}{2}{}^{(5)}R_C{}^A n_A n^C g_{\mu\nu} - \frac{1}{2}g_{\mu\nu}((\text{Tr }\Xi)^2 - (\text{Tr }\Xi^2)). \\ & - {}^{(5)}R^A{}_{BCD} n_A n^C g^B{}_\mu g^D{}_\nu - \frac{1}{2}{}^{(5)}R^A{}_{BCD}n^B n^C n^D n_A g_{\mu\nu} + (\text{Tr }\Xi)\Xi_{\mu\nu} - \Xi^C_\nu \Xi_{\mu C} \end{aligned} \tag{1.4}$$

where $\kappa_5 = 8\pi G_5$. The Riemann tensor can be expressed as a combination of the Weyl tensor, the Ricci tensor and the Ricci scalar, as ${}^{(5)}R^A{}_{BCD} = {}^{(5)}C^A{}_{BCD} + {}^{(5)}D^A{}_{BCD}$, where ${}^{(5)}C^A{}_{BCD}$ denotes the Weyl tensor and ${}^{(5)}D^A{}_{BCD}$ can be written as [1]

$$\begin{aligned} {}^{(5)}D^A{}_{BCD} \;=\; & \frac{2}{3}\left({}^{(5)}g^A{}_C\,{}^{(5)}R_{DB} - {}^{(5)}g^A{}_D\,{}^{(5)}R_{CB} - {}^{(5)}g_{BC}\,{}^{(5)}R^A{}_D + {}^{(5)}g_{BD}\,{}^{(5)}R^A{}_C\right) \\ & -\frac{1}{6}\left({}^{(5)}g^A{}_C\,{}^{(5)}g_{BD} - {}^{(5)}g^A{}_D\,{}^{(5)}g_{BC}\right){}^{(5)}R. \end{aligned} \tag{1.5}$$

From eqs.(1.3) and (1.5) it follows that

$$\begin{aligned} G_{\mu\nu} \;=\; & -\frac{1}{2}\Lambda_5 g_{\mu\nu} + \frac{2}{3}\kappa_5^2\left[{}^{(5)}T_{AB}g^A{}_\mu g^B{}_\nu + \left({}^{(5)}T_{AB}n^A n^B - \frac{1}{4}(\text{Tr }{}^{(5)}T)\right)g_{\mu\nu}\right] \\ & + (\text{Tr }\Xi)\Xi_{\mu\nu} - \Xi^C_\nu\Xi_{\mu C} + \frac{1}{2}g_{\mu\nu}[(\text{Tr }\Xi)^2 - (\text{Tr }\Xi^2)] - E_{\mu\nu}. \end{aligned} \tag{1.6}$$

PoS(ISFTG)026

The term $E_{\mu\nu}$ is the projection of the bulk Weyl tensor on the brane $E_{\mu\nu} = {}^{(5)}C_{ACBD}n^C n^D g^A{}_\mu g^B{}_\nu$..

The Israel-Darmois conditions [3, 4, 7] are obtained assuming that in the bulk field equations (1.3) the momentum-energy tensor ${}^{(5)}T_{AB}$ is a sum of a bulk intrinsic momentum-energy tensor and a brane momentum-energy tensor integrated in a brane neighbour region (from $y = -\varepsilon$ to $y = +\varepsilon$). The Israel-Darmois conditions are given by $g^+_{\mu\nu} - g^-_{\mu\nu} = 0$ and $\Xi^+_{\mu\nu} - \Xi^-_{\mu\nu} = -\kappa_5^2\left(T^{\rm brane}_{\mu\nu} - \frac{1}{3}T^{\rm brane}g_{\mu\nu}\right)$, where $T^{\rm brane} = (\mathrm{Tr}\; T^{\rm brane}_{\mu\nu})$. In order to write $T^{\rm brane}_{\mu\nu}$ using entities realized in the brane, we use $T^{\rm brane}_{\mu\nu} = T_{\mu\nu} - \lambda g_{\mu\nu}$, where λ is defined as the tension on the brane. The second assumption is the $\mathbb{Z}_2$-symmetry, resulting in the conditions $\Xi^+_{\mu\nu} = -\Xi^-_{\mu\nu} = \Xi_{\mu\nu}$. Substituting these expressions in the Israel-Darmois conditions it follows that $\Xi_{\mu\nu} = -\frac{1}{2}\kappa_5^2\left(T_{\mu\nu} + \frac{1}{3}(\lambda - T)g_{\mu\nu}\right)$. and

$$\begin{aligned} G_{\mu\nu} &= -\frac{1}{2}\Lambda_5 g_{\mu\nu} + \frac{2}{3}\kappa_5^2\left[{}^{(5)}T_{AB}g^A_\mu g^B{}_\nu + \left({}^{(5)}T_{AB}n^A n^B - \frac{1}{4}(\mathrm{Tr}\;{}^{(5)}T)\right)g_{\mu\nu}\right] \\ &+ \frac{1}{4}\kappa_5^4\left[TT_{\mu\nu} - T^C_\nu T_{\mu C} + \frac{1}{2}g_{\mu\nu}((\mathrm{Tr}\,T)^2 - (\mathrm{Tr}\,T^2))\right] - E_{\mu\nu}. \end{aligned} \tag{1.7}$$

This equation can be further simplified if the momentum-energy conservation law $T_{\mu\nu}{}^{;\nu} = 0$ is valid on the brane.

An expression for $T_{\mu\nu}{}^{;\nu}$ in terms of ${}^{(5)}T_{AB}$ can be found by combining the following equations, respectively describing the Israel-Darmois junction conditions and the Gauss-Codazzi equations:

$$\Xi_{\mu\nu} = -\frac{1}{2}\kappa_5^2\left[T_{\mu\nu} + \frac{1}{3}(\lambda - T)g_{\mu\nu}\right], \tag{1.8}$$

$$\Xi_A{}^B{}_{;B} - \Xi_{;A} = {}^{(5)}R_{BC}g^B{}_A n^C. \tag{1.9}$$

Performing the covariant derivative of the curvatures and using Eq.(1.8) it follows that

$$\Xi_{\mu\nu}^{;\nu} - \Xi^{;\nu} = -\frac{1}{2}\kappa_5^2 T_{\mu\nu}{}^{;\nu}, \tag{1.10}$$

and using Eq.(1.9) it follows that ${}^{(5)}R_{AB}g^A{}_\nu n^B = \Xi_\nu^A{}_{;A} - \Xi_{;\nu}$. But from Eq.(1.8), the expression ${}^{(5)}R_{AB} = -\Lambda_5^{(5)}g_{AB} + \kappa_5^{2(5)}T_{AB} + \frac{1}{2}{}^{(5)}g_{AB}{}^{(5)}R$ holds, which implies that

$$\Xi_\nu^A{}_{;A} - \Xi_{;\nu} = \left(-\Lambda_5^{(5)}g_{AB} + \kappa_5^{2\,(5)}T_{AB} + \frac{1}{2}{}^{(5)}g_{AB}{}^{(5)}R\right)g_{A\nu}n_B. \tag{1.11}$$

Now, introducing Eq.(1.1) in Eq.(1.11) and attempting to the fact that the projections of the Ricci tensors in the n^A direction are zero, we get from Eq.(1.10) that $T_\mu{}^A{}_{;A} = -2\,{}^{(5)}T_{AB}g_\mu^A n^B$, and assuming that $T_\mu{}^A{}_{;A} = 0$ in the brane, it is immediate that ${}^{(5)}T_{AB} = 0$. This means the bulk is in complete vacuum and the particles are in fact on the brane. Now, the field equations reduce to

$$G_{\mu\nu} = -\frac{1}{2}\Lambda_5 g_{\mu\nu} + \frac{1}{4}\kappa_5^4\left[TT_{\mu\nu} - T^C_\nu T_{\mu C} + \frac{1}{2}g_{\mu\nu}((\mathrm{Tr}\,T)^2 - (\mathrm{Tr}\,T^2))\right] - E_{\mu\nu}, \tag{1.12}$$

showing the contribution of the bulk on the brane is only due to the Weyl tensor.

2. Torsion corrections in the bulk metric Taylor expansion

In a previous paper [5] we proved that although the presence of torsion terms in the connection does not modify the Israel-Darmois matching conditions, despite of the modification in the extrinsic curvature and in the connection, the Einstein equation obtained using the Gauss-Codazzi formalism is extended. The factors involving contorsion change drastically the effective Einstein equation on the brane, as well as the effective cosmological constant.

We shall use such results to extend the Taylor expansion of the bulk metric in terms of the brane metric, in a direction orthogonal to the brane. Besides a curvature associated with the connection that endows the bulk, in a Riemann-Cartan manifold the torsion associated with the connection is in general non zero. Its components can be written in terms of the connection components $\Gamma^\rho{}_{\beta\alpha}$ as

$$T^\rho{}_{\alpha\beta} = \Gamma^\rho{}_{\beta\alpha} - \Gamma^\rho{}_{\alpha\beta}. \tag{2.1}$$

The general connection components are related to the Levi-Civita connection components $\mathring{\Gamma}^\rho{}_{\alpha\beta}$ — associated with the spacetime metric $g_{\alpha\beta}$ components — through $\Gamma^\rho{}_{\alpha\beta} = \mathring{\Gamma}^\rho{}_{\alpha\beta} + K^\rho{}_{\alpha\beta}$, where $K^\rho{}_{\alpha\beta} = \frac{1}{2}\left(T_\alpha{}^\rho{}_\beta + T_\beta{}^\rho{}_\alpha - T^\rho{}_{\alpha\beta}\right)$ denotes the contorsion tensor components.

We have investigated the matching conditions in the presence of torsion terms, and under the assumptions of discontinuity across the brane, showing that both junctions conditions are shown to be the same as the usual case, and by the fact that the covariant derivative is modified by the (con)torsion, the extrinsic curvature is also modified, and then the conventional arguments point in the direction of some modification in the matching conditions. However, it seems that the role of torsion terms in the braneworld picture is restricted to the geometric part of effective Einstein equation on the brane [5]. More explicitly, looking at the equation that relates the Einstein equation in four dimensions with bulk quantities [1] it follows that

$$^{(4)}G_{\rho\sigma} = \frac{2k_5^2}{3}\left(T_{\alpha\beta}g_\rho^\alpha g_\sigma^\beta + (T_{\alpha\beta}n^\alpha n^\beta - \frac{1}{4}T)g_{\rho\sigma}\right) + \Xi\Xi_{\rho\sigma} - \Xi_\rho{}^\alpha\Xi_{\alpha\sigma} \tag{2.2}$$

$$-\frac{1}{2}g_{\rho\sigma}(\Xi^2 - \Xi^{\alpha\beta}\Xi_{\alpha\beta}) - {}^{(5)}C^\alpha_{\beta\gamma\varepsilon}n_\alpha n^\gamma g_\rho^\beta g_\sigma^\varepsilon, \tag{2.3}$$

where $\Xi_{\rho\sigma} = g_\rho^\alpha g_\sigma^\beta \nabla_\alpha n_\beta$ is the extrinsic curvature. By restricting to quantities evaluated on the brane, or tending to the brane, we see that the only way to get some contribution from torsion terms is via the term $^{(4)}G_{\rho\sigma}$, and also via the Weyl tensor.

Supposing $\mathbb{Z}_2$-symmetry, the extrinsic curvature reads

$$\Xi^+_{\alpha\beta} = -\Xi^-_{\alpha\beta} = -2G_N\left(\pi_{\alpha\beta} - \frac{g_{\alpha\beta}\pi^\gamma_\gamma}{4}\right), \tag{2.4}$$

and consequently $2G_N\pi_{ab} = -[\Xi_{ab}] + [\Xi]g_{ab}$ reads $\Xi_{\alpha\beta} = -G_N(\pi_{\alpha\beta} - g_{\alpha\beta}\pi^\gamma_\gamma/4)$.

Decomposing the stress-tensor associated with the bulk in $T_{\alpha\beta} = -\Lambda^{(5)}g_{\alpha\beta} + \delta S_{\alpha\beta}$ and $S_{\alpha\beta} = -\lambda g_{\alpha\beta} + \pi_{\alpha\beta}$, where Λ is the bulk cosmological constant, and substituting into Eq.(2.3) it follows that

$$^{(4)}G_{\mu\nu} = -\Lambda_4 q_{\mu\nu} + 8\pi G_N\pi_{\mu\nu} + k_5^4 Y_{\mu\nu} - E_{\mu\nu}, \tag{2.5}$$

where $E_{\mu\nu} = {}^{(5)}C^{\alpha}_{\beta\gamma\sigma} n_\alpha n^\gamma q^\beta_\mu q^\sigma_\nu$ encodes the Weyl tensor contribution, $G_N = \frac{\lambda k_5^4}{48\pi}$ is the analogous of the Newton gravitational constant, the tensor $Y_{\mu\nu}$ is quadratic in the brane stress-tensor and given by $Y_{\mu\nu} = -\frac{1}{4}\pi_{\mu\alpha}\pi^\alpha_\nu + \frac{1}{12}\pi^\gamma_\gamma \pi_{\mu\nu} + \frac{1}{8}g_{\mu\nu}\pi_{\alpha\beta}\pi^{\alpha\beta} - \frac{1}{2}g_{\mu\nu}(\pi^\gamma_\gamma)^2$, and $\Lambda_4 = \frac{k_5^2}{2}\left(\Lambda + \frac{1}{6}k_5^2\lambda^2\right)$ is the effective brane cosmological constant.

Using the Einstein tensor on the brane encoding torsion terms, the $E_{\mu\nu}$ tensor can be expressed in terms of the bulk contorsion terms by

$$\begin{aligned} E_{\kappa\delta} &= \mathring{E}_{\kappa\delta} + \left(\nabla_{[\nu} K^{\mu}{}_{\alpha\beta]} + K^{\mu}{}_{\gamma[\nu} K^{\gamma}{}_{\alpha\beta]}\right) n_\mu n^\nu g^\alpha_\kappa g^\beta_\delta - \frac{2}{3}(g^\alpha_\kappa g^\beta_\delta + n^\alpha n^\beta g_{\kappa\delta})\left(\nabla_{[\lambda} K^{\lambda}{}_{\beta\alpha]} + K^{\lambda}{}_{\gamma\lambda} K^{\gamma}{}_{\beta\alpha} - K^{\sigma}{}_{\beta\gamma} K^{\gamma}{}_{\sigma\alpha}\right) \\ &\quad + \frac{1}{6} g_{\kappa\delta}\left(2\nabla^\lambda K^{\tau}{}_{\lambda\tau} - K_{\tau\lambda}{}^{\lambda} K^{\tau\gamma}{}_{\gamma} + K_{\tau\gamma\lambda} K^{\tau\lambda\gamma}\right) \end{aligned} \tag{2.6}$$

where ∇_μ is the bulk covariant derivative. Now, the explicit influence of the contorsion terms in the Einstein brane equation can be appreciated. From Eqs.(2.5) and (2.6), it reads

$$\begin{aligned} &{}^{(4)}\mathring{G}_{\mu\nu} + D_{[\lambda}\, {}^{(4)}K^{\lambda}{}_{\mu\nu]} + {}^{(4)}K^{\delta}{}_{\gamma\delta}\, {}^{(4)}K^{\lambda}{}_{\mu\nu} - {}^{(4)}K^{\sigma}{}_{\nu\gamma}\, {}^{(4)}K^{\gamma}{}_{\sigma\mu} = -\tilde{\Lambda}_4 q_{\mu\nu} + 8\pi G_N \pi_{\mu\nu} + k_5^4 Y_{\mu\nu} - \mathring{E}_{\mu\nu} \\ &+ g^\alpha_\mu g^\beta_\nu \left[\frac{2}{3}\left(\nabla_{[\lambda} K^{\lambda}{}_{\beta\alpha]} + K^{\sigma}{}_{\gamma\sigma} K^{\gamma}{}_{\beta\alpha} - K^{\lambda}{}_{\beta\gamma} K^{\gamma}{}_{\lambda\alpha}\right) - n_\rho n^\sigma \left(\nabla_{[\sigma} K^{\rho}{}_{\alpha\beta]} + K^{\rho}{}_{\gamma[\sigma} K^{\gamma}{}_{\alpha\beta]}\right)\right] \end{aligned} \tag{2.7}$$

where the new effective cosmological constant is given by

$$\begin{aligned} \tilde{\Lambda}_4 &\equiv \Lambda_4 - D^\lambda\, {}^{(4)}K^{\tau}{}_{\lambda\tau} + \frac{1}{2}\, {}^{(4)}K_{\tau\alpha}{}^{\alpha}\, {}^{(4)}K^{\tau\lambda}{}_{\lambda} - \frac{1}{2}\, {}^{(4)}K_{\tau\gamma\lambda}\, {}^{(4)}K^{\tau\lambda\gamma} - \frac{2}{3} n^\alpha n^\beta \left(\nabla_\lambda K^{\lambda}{}_{\beta\alpha} - \nabla_\alpha K^{\lambda}{}_{\beta\lambda}\right. \\ &\quad \left. + K^{\lambda}{}_{\gamma\lambda} K^{\gamma}{}_{\beta\alpha} - K^{\sigma}{}_{\beta\gamma} K^{\gamma}{}_{\sigma\alpha}\right) + \frac{1}{6}\left(2\nabla^\lambda K^{\tau}{}_{\lambda\tau} - K_{\tau\alpha}{}^{\alpha} K^{\tau\lambda}{}_{\lambda} + K_{\tau\gamma\lambda} K^{\tau\lambda\gamma}\right). \end{aligned} \tag{2.8}$$

Eqs. (2.7) and (2.8) shows that the factors involving both contorsion, in four and in five dimensions, change drastically the effective Einstein equation on the brane, as well as the effective cosmological constant. The effective field equations are not a closed system. One needs to supplement them by 5D equations governing $\mathscr{E}_{\mu\nu}$, which are obtained from the 5D Einstein and Bianchi equations. This leads to the coupled system [8]

$$\pounds_{\mathbf{n}} \Xi_{\mu\nu} = \Xi_{\mu\alpha} \Xi^{\alpha}{}_{\nu} - \mathscr{E}_{\mu\nu} - \frac{1}{6}\Lambda_5 g_{\mu\nu} \tag{2.9}$$

The above equations have been used to develop a covariant analysis of the weak field [2]. They can also be used to develop a Taylor expansion of the metric about the brane. In Gaussian normal coordinates, Eq. (1.1), we have $\pounds_{\mathbf{n}} = \partial/\partial y$. Then we find from Eq.(1.2)

$$\begin{aligned} g_{\mu\nu}(x,y) &= g_{\mu\nu}(x,0) - \kappa_5^2\left[T_{\mu\nu} + \frac{1}{3}(\lambda - T) g_{\mu\nu}\right]_{y=0+} |y| \;+ \Bigg[- \mathring{\mathscr{E}}_{\kappa\delta} + \left(\nabla_{[\nu} K^{\mu}{}_{\alpha\beta]} + K^{\mu}{}_{\gamma[\nu} K^{\gamma}{}_{\alpha\beta]}\right) n_\mu n^\nu q^\alpha_\kappa q^\beta_\delta \\ &- \frac{2}{3}(q^\alpha_\kappa q^\beta_\delta + n^\alpha n^\beta q_{\kappa\delta})\left(\nabla_{[\lambda} K^{\lambda}{}_{\beta\alpha]} + K^{\lambda}{}_{\gamma\lambda} K^{\gamma}{}_{\beta\alpha} - K^{\sigma}{}_{\beta\gamma} K^{\gamma}{}_{\sigma\alpha}\right) + \frac{1}{6} q_{\kappa\delta}\left(2\nabla^\lambda K^{\tau}{}_{\lambda\tau} - K_{\tau\lambda}{}^{\lambda} K^{\tau\gamma}{}_{\gamma} + K_{\tau\gamma\lambda} K^{\tau\lambda\gamma}\right) \\ &+ \frac{1}{4}\kappa_5^4\left\{T_{\mu\alpha} T^{\alpha}{}_{\nu} + \frac{2}{3}(\lambda - T) T_{\mu\nu}\right\} + \frac{1}{6}\Bigg\{\frac{1}{6}\kappa_5^4(\lambda - T)^2 - 2\Bigg(\Lambda_4 - D^\lambda\, {}^{(4)}K^{\tau}{}_{\lambda\tau} + \frac{1}{2}\, {}^{(4)}K_{\tau\alpha}{}^{\alpha}\, {}^{(4)}K^{\tau\lambda}{}_{\lambda} \\ &- \frac{1}{2}\, {}^{(4)}K_{\tau\gamma\lambda}\, {}^{(4)}K^{\tau\lambda\gamma} - \frac{2}{3} n^\alpha n^\beta \left(\nabla_\lambda K^{\lambda}{}_{\beta\alpha} - \nabla_\alpha K^{\lambda}{}_{\beta\lambda} + K^{\lambda}{}_{\gamma\lambda} K^{\gamma}{}_{\beta\alpha} - K^{\sigma}{}_{\beta\gamma} K^{\gamma}{}_{\sigma\alpha}\right) + \frac{1}{6}\Big(2\nabla^\lambda K^{\tau}{}_{\lambda\tau} \\ &- K_{\tau\alpha}{}^{\alpha} K^{\tau\lambda}{}_{\lambda} + K_{\tau\gamma\lambda} K^{\tau\lambda\gamma}\Big)\Bigg) - k^2\lambda\Bigg\} g_{\mu\nu}\Bigg]_{y=0+} y^2 + \cdots \end{aligned} \tag{2.10}$$

where ∇_μ is the bulk covariant derivative.

Since the term $g_{\theta\theta}$ determines the change in the area of a black string horizon along the extra dimension [10, 11, 12], it shows how the contorsion and its derivatives affect the horizon. Also, the change in the black string properties can be extracted. The torsion corrections arise only from the order of y^2 on. When the torsion — and consequently the contorsion — goes to zero, the expression is the same as the one previously obtained in, e.g., [6, 8]. Eq.(2.10) completely comprises how the corrections that contorsion terms impinge on braneworld scenarios and and how a Riemann-Cartan framework can influence corrected braneworld properties.

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