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On the Consistency Conditions to Braneworlds in Scalar-Tensor Gravity for Arbitrary Dimensions

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We derive an one-parameter family of consistency conditions to braneworlds in the Brans-Dicke gravity. The General Relativity case is recovered by taking a correct limit of the Brans-Dicke parameter. We show that it is possible to build a multiple AdS brane scenario in a six-dimensional bulk only if the brane tensions are negative. Besides, in the five-dimensional case, it is showed that no fine tuning is necessary between the bulk cosmological constant and the brane tensions, in contrast to the Randall-Sundrum model.

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1. Introduction

In the recent years braneworld models are consolidating a new branch of high energy physics. Among many interesting physical insights [1], they provide an elegant solution to the hierarchy problem [2, 3, 4]. Such perspectives encourage new developments in braneworld models, even in cases when there is not an explicit analytical solution. Formal advances of string theory point into a scalar tensorial theory as the right approach to the gravitational phenomena [5]. Unification models such as supergravity, superstrings and M-theory [6] effectively predict the existence of a scalar gravitational field acting as a mediator of the gravitational interaction together with the usual purely rank-2 tensorial field. In this context, the analysis of braneworld consistency conditions — or sum rules — in scalar-tensorial theories are indeed necessary. These sum rules were obtained in ref. [7] for five dimensions in the General Relativity (GR) case and extended to an arbitrary number of dimensions in the ref. [8], also in the GR frame.

The main purpose of this work is to generalize the braneworld sum rules, already obtained in the GR set up, to the case of scalar tensorial gravity (Section II). In particular, we work with the Brans-Dicke (BD) theory [9], since it is the simplest scalar-tensor theory we have in the literature. By this generalization we have two main results: the possibility of a multiple braneworld scenario with only negative brane tensions (Section II) and the fact that for a five-dimensional case, it is not necessary — in the BD gravity framework — any fine tuning between the bulk cosmological constant and the brane tension (Section III), in contrast with the RS model.

2. Generalized Sum Rules

In this section we obtain the generalized braneworld sum rules within the BD gravity framework. Following the standard notation used in refs. [7, 8], we analyze a D-dimensional bulk spacetime endowed with a non-factorable geometry, which metric is given by

$$ds^2 = G_{AB}dX^A dX^B = W^2(r)g_{\alpha\beta}dx^\alpha dx^\beta + g_{ab}(r)dr^a dr^b, \tag{2.1}$$

where $W^2(r)$ is the warp factor, X^A denotes the coordinates of the full D-dimensional spacetime, x^α stands for the $(p+1)$ non-compact coordinates of the spacetime and r^a labels the $(D-p-1)$ directions in the internal compact space[1]. Note that this type of metric encodes the possibility of existing q-branes $(q > p)$ [8]. In this case, the $(q-p)$ extra dimensions are compactified on the brane and constitute part of the internal space. This possibility is important in the hybrid compactification models context, but we will not extend this discussion by now.

The D-dimensional spacetime Ricci tensor can be related with the brane Ricci tensor as well as with the internal space partner by the equations [7]

$$R_{\mu\nu} = \bar{R}_{\mu\nu} - \frac{g_{\mu\nu}}{(p+1)W^{p-1}}\nabla^2 W^{p+1}, \tag{2.2}$$

and

$$R_{ab} = \tilde{R}_{ab} - \frac{p+1}{W}\nabla_a\nabla_b W, \tag{2.3}$$

[1] As an example, if $D = 5$, $p = 3$ and $W(r) = e^{-2k|r|}$ one arrives at the RS model.

where $\tilde{R}_{ab}$, ∇_a and ∇^2 are respectively the Ricci tensor, the covariant derivative and the Laplacian operator constructed by the internal space metric g_{ab}. $\bar{R}_{\mu\nu}$ is the Ricci tensor derived from $g_{\mu\nu}$. Let us denote the three curvature scalars by $R = G^{AB}R_{AB}$, $\bar{R} = g^{\mu\nu}\bar{R}_{\mu\nu}$ and $\tilde{R} = g^{ab}\tilde{R}_{ab}$. Therefore, the traces of equations (2.2) and (2.3) give

$$\frac{1}{p+1}\left(W^{-2}\bar{R} - R^{\mu}_{\mu}\right) = pW^{-2}\nabla W\cdot\nabla W + W^{-1}\nabla^2 W \tag{2.4}$$

and

$$\frac{1}{p+1}\left(\tilde{R} - R^{a}_{a}\right) = W^{-1}\nabla^2 W, \tag{2.5}$$

where $R^{\mu}_{\mu} \equiv W^{-2}g^{\mu\nu}R_{\mu\nu}$ and $R^{a}_{a} \equiv g^{ab}R_{ab}$ (in such a way that $R = R^{\mu}_{\mu} + R^{a}_{a}$). It is not difficult to see that, if ξ is an arbitrary constant,

$$\nabla\cdot(W^{\xi}\nabla W) = W^{\xi+1}(\xi W^{-2}\nabla W\cdot\nabla W + W^{-1}\nabla^2 W). \tag{2.6}$$

The combination of the equations (2.4), (2.5) and (2.6) leads to

$$\nabla\cdot(W^{\xi}\nabla W) = \frac{W^{\xi+1}}{p(p+1)}[\xi\left(W^{-2}\bar{R} - R^{\mu}_{\mu}\right) + (p-\xi)\left(\tilde{R} - R^{a}_{a}\right)]. \tag{2.7}$$

Once established the usual notation and conventions, it is time to look at the scalar field — generically called the "dilaton" from now on — (ϕ) contributions. The Einstein tensor ($\mathscr{G}_{MN}$) in the Einstein-Brans-Dicke (EBD) gravity is given by

$$\mathscr{G}_{MN} = \frac{8\pi}{\phi}T_{MN} + \frac{w}{\phi^2}\left(\nabla_M\phi\nabla_N\phi - \frac{1}{2}\nabla_A\phi\nabla^A\phi G_{MN}\right) + \frac{1}{\phi}\left(\nabla_M\nabla_N\phi - \frac{8\pi}{3+2w}TG_{MN}\right), \tag{2.8}$$

where T_{MN} is the bulk stress-tensor, T is its trace and w the BD parameter. We remark that the scalar part of the BD set of equations was already taken into account in the last term of the right-hand side of eq. (2.8). From eq. (2.8), calling $T^{\mu}_{\mu} \equiv W^{-2}g^{\mu\nu}T_{\mu\nu}$, it is easy to note that

$$\begin{aligned} R^{\mu}_{\mu} &= \frac{8\pi}{\phi(D-2)(3+2w)}\Big((3D+2w(D-p-3)-2(p+1))T^{\mu}_{\mu} - 2(1+w)(p+1)T^{m}_{m}\Big) \\ &+ \frac{wW^{-2}}{\phi^2}\nabla^{\nu}\phi\nabla_{\nu}\phi + \frac{W^{-2}}{\phi}\nabla^{\nu}\nabla_{\nu}\phi, \end{aligned} \tag{2.9}$$

and

$$\begin{aligned} R^{m}_{m} &= \frac{8\pi}{\phi(D-2)(3+2w)}\Big((D+2w(p-1)-2(p-2))T^{m}_{m} - 2(1+w)(D-p-1)T^{\mu}_{\mu}\Big) \\ &+ \frac{w}{\phi^2}\nabla^{m}\phi\nabla_{m}\phi + \frac{1}{\phi}\nabla^{m}\nabla_{m}\phi. \end{aligned} \tag{2.10}$$

An important characteristic about braneworld models in the BD theory is that, generally, the scalar field depends only on the large extra dimensions [10]. This type of dependence is indeed useful since the projected EBD equations on the brane leads to important subtle modifications but still resembles the Einstein's equations [11]. Therefore, let us consider hereon the $\nabla_{\mu}\phi = 0$ case.

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In an internal compact space the following identity is respected $\oint \nabla \cdot (W^{\xi} \nabla W) = 0$. Therefore, inserting the eqs. (2.9) and (2.10) in (2.7), and taking into account the identity above, we have

$$\oint \frac{W^{\xi+1}}{\phi} \Bigg(T^{\mu}_{\mu} [\xi(5D + 4w(D-p-2) - 2(2p+5)) - 2p(1+w)(D-p-1)]$$
$$+\ T^{m}_{m}[\xi(-4wp - D + 2(1-2p)) + p(D - 2(p-2) + 2w(p-1))] - \left(\frac{8\pi}{\phi}\right)^{-1}(D-2)(3+2w)$$
$$\times\ [\xi W^{-2}\bar{R} + (p-\xi)\tilde{R}] + \left(\frac{8\pi}{\phi}\right)^{-1}(D-2)(3+2w)\left(\frac{w}{\phi^2}\nabla^m\phi\nabla_m\phi + \frac{1}{\phi}\nabla^m\nabla_m\phi\right)\Bigg) = 0. \quad (2.11)$$

The above equation provides an one-parameter family of consistency conditions in arbitrary dimensions in the scope of the BD gravity. It is important to stress that if one reintroduces the $(3+2w)^{-1}$ factored term and takes $w \to \infty$ ($\phi \to 1/G_N$), by the usual L'Hopital limit, the case analyzed in General Relativity is recovered as expected. Equation (2.11) is quite general and self-consistent. However, in that form it is not very useful. Going forward, we rewrite eq. (2.11) in terms of an adequate energy-momentum tensor form. We shall use the same stress-tensor *Ansatz* of ref. [8], since it is very complete. So, we write the stress-tensor in the form

$$T_{MN} = -\Lambda G_{MN} - \sum_i T^{(i)}_q P[G_{MN}]^{(i)}_q \Delta^{(D-q-1)}(r - r_i) + \tau_{MN}, \quad (2.12)$$

where Λ is the cosmological constant, $T^{(i)}_q$ is the i^{th} q-brane tension, $\Delta^{(D-q-1)}(r-r_i)$ is the covariant combination of delta functions which localizes the brane, $P[G_{MN}]^{(i)}_q$ is the pull-back of the bulk metric and any other matter contribution is represented by τ_{MN}. From this *Ansatz* one obtains

$$T^{\mu}_{\mu} = -(p+1)\Lambda + \tau^{\mu}_{\mu} - \sum_i T^i_q \Delta^{(D-q-1)}(r-r_i)(p+1), \quad (2.13)$$

and

$$T^{m}_{m} = -(D-p-1)\Lambda + \tau^{m}_{m} - \sum_i T^i_q \Delta^{(D-q-1)}(r-r_i)(q-p). \quad (2.14)$$

Now, substituting these expressions in equation (2.11) we get, after some algebra, the following form to the consistency conditions

$$\oint \frac{W^{\xi+1}}{\phi} \Bigg(-\Lambda(c(p+1) + bD) - \sum_i (cp + a + bq) T^{(i)}_q \Delta^{(D-q-1)}(r-r_i) + aT^{\mu}_{\mu} + bT^{m}_{m}$$
$$-\ \left(\frac{8\pi}{\phi}\right)^{-1}(D-2)(3+2w)[\xi W^{-2}\bar{R} + (p-\xi)\tilde{R}] + \left(\frac{8\pi}{\phi}\right)^{-1}(D-2)(3+2w)$$
$$\times\ \left(\frac{w}{\phi^2}\nabla^m\phi\nabla_m\phi + \frac{1}{\phi}\nabla^m\nabla_m\phi\right)\Bigg) = 0, \quad (2.15)$$

where $a \equiv \xi[5D + 4w(D-p-2) - 2(2p+5)] - 2p(1+w)(D-p-1)$, $b \equiv \xi[-4wp - D + 2(1-2p)] + p[D - 2(p-2) + 2w(p-1)]$ and $c \equiv a - b$.

Let us particularize our analysis to braneworld models inspired in one of those previously found in this framework. In the models proposed in [10], all the bulk-brane structure was generated by using local and global cosmic string as sources. The final scenario is composed by 4-branes

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embedded into a bulk of six-dimensions. Therefore, they are models of hybrid compactification. The on-brane dimension is compactified into a S^1 circle and the dilaton field depends only on the large transverse dimension. With these specifications, we have $D = 6$, $p = 3$ and $q = 4$, and consequently, $a = 2\xi(15+2w) - 2(17+6w)$, $b = -4\xi(4+3w) + 12(1+w)$ and $c = 2\xi(23+8w) - 2(23+12w)$. Then, the equation (2.15) gives

$$\oint \frac{W^{\xi+1}}{\phi}\Bigg(-4\Lambda[35\xi - w(\xi+3) - 14] - 2\sum_i [26\xi + w(\xi-9) - 31] T_4^{(i)} \Delta^{(1)}(r - r_i) + \tau_\mu^\mu[\xi(15+2w) - (17+6w)] + \tau_m^m[-2\xi(4+3w) + 6(1+w)] - 2(3+2w)\left(\frac{8\pi}{\phi}\right)^{-1} \times [\xi W^{-2}\bar{R} + (3-\xi)\tilde{R}] + 2(3+2w)(3-\xi)\left(\frac{8\pi}{\phi}\right)^{-1}\left(\frac{w}{\phi^2}\nabla^m\phi\nabla_m\phi + \frac{1}{\phi}\nabla^m\nabla_m\phi\right)\Bigg) = 0. \quad (2.16)$$

The analysis can be simplified if we assume empty bulk models and study the vacuum on the brane ($\tau_\mu^\mu = 0 = \tau_m^m$). Moreover, let us take ($\Lambda = 0$) for simplicity. Note that different choices of ξ lead to different contributions, so we begin with $\xi = -1$ since it eliminates the overall warp factor on the left-hand side of equation (2.16). With these several simplifications, the eq. (2.16) reads

$$\oint \left(\frac{w}{\phi^2}\nabla^m\phi\nabla_m\phi + \frac{1}{\phi}\nabla^m\nabla_m\phi + \frac{W^{-2}}{2}\bar{R} \right) = 8\pi\chi - \frac{4\pi(57+10w)}{3+2w}\sum_i T_4^{(i)} L_i \phi^{-1}(r_i), \quad (2.17)$$

where L_i is the area of the S^1 circle (the extra dimension compactified on the brane) and $\phi(r_i)$ is the dilaton field value on the i^{th}-brane at the r_i position. Note the appearance of the Euler character $\chi = \frac{1}{4\pi}\oint \tilde{R}$. The internal space of the model can be characterized by χ and, then, for each model this topological invariant contributes in a specific way for the sum rules. Another interesting choice for the ξ parameter is $\xi = 3$. If we reconsider all the simplifications which lead to (2.17) but with $\xi = 3$ it results in

$$\frac{3(3+2w)}{8\pi}\oint W^2\bar{R} = (6w - 47)\sum_i T_4^{(i)} W^4(r_i)\phi^{-1}(r_i) L_i. \quad (2.18)$$

Note that there is no contribution from the scalar field derivatives, as well as, from the Euler invariant. We remark that, for a negative constant curvature scalar it is possible to have a multiple brane scenario only with negative brane tensions! On the other hand, for a positive constant curvature scalar, it is possible, at least, one negative brane tension. We stress, however, that in both cases we took $\Lambda = 0$. This situation can be changed if we take into account the contributions coming from the cosmological constant terms.

3. Concluding Remarks

We generalize the braneworld sum rules to the BD gravity framework. In the interface between General Relativity and gravity coming from an effective low energy string theory, such a generalization is quite necessary. The results recover the previous case when the limit $w \to \infty$ is taken. The main goal of obtaining consistency conditions in braneworlds is the large application to several models in the BD gravity for arbitrary dimensions.

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On one hand, the constraints imposed by the equation (2.15), for example, must be respected by any braneworld model in the BD gravity in which the scalar field depends only on the large extra dimension and should be taken into account in more involved models. It is a strong imposition. On the other hand, however, as the eq. (2.15) shows, such a constraint is not too much dramatic itself.

Of course, the consistency conditions are much more attractive if one assumes a particular model. To the particular cases analyzed from eq. (2.17), it is important to emphasize the possibility of existing only positive or only negative brane tensions. It is in sharp contrast with the RS model [3], where the tension of the two branes are necessarily equal and opposite. Apart of that, we stress that it is possible to see, from eq. (2.11), that, in the five-dimensional case, a fine tuning here is not neccessary to be introduced as in the RS model. Nevertheless, we are unable to classify our space as being an AdS type or not, since we took, for simplicity, $\Lambda = 0$. Therefore, our conclusion is just partially true but lacks of deepest evaluation if we intend to go further in the analysis of the extra-dimensions compactification, the hierarchy problems, the Ads/CFT correspondence and many other issues which will demand a carefully choice of our parameters. For that, we should return back to the general result expressed by eq. (2.16).

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