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Accretion mechanisms onto primordial black holes

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We extend previous work on the evolution of a Primordial Black Hole (PBH) to address the presence of a general perfect fluid whose particular case is a dark energy component with a super-negative equation of state as a background. The first approach for this problem is static accretion, for which we study the competition between the radiation accretion, the Hawking evaporation and the phantom accretion. The evaporation of PBHs is quite modified at late times by these effects, and we investigate the impact of these results on the Generalized Second Law of thermodynamics. We address further generalizations of this scenario, with a preliminary treatment of the formulation of non-static accretion.

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1. Introduction

Primordial black holes play an important role in modern Cosmology. Their evolution can have noticeable consequences on the subsequent history of the universe, in particular to structure formation, as they can even serve as seeds to the formation of super-massive black holes at galactic centers. Since the physical constraints to their formation do not exclude them as a possibility in the early universe, they remain a piece to be considered to put together the cosmological puzzle.

We present here the most recent developments on the problem concerning the interaction between primordial black holes and a general form of perfect fluid, which in the simplest case we choose as phantom energy. Its accretion by primordial black holes is studied from a dynamical and thermodynamical point of view, extending previous works on the subject as we also take into account other processes. The more general case of accretion involves abandoning the assumption of a static metric, and the mathematical framework for dealing with this scenario is currently under development.

2. Static Phantom Accretion

From the conservation of the energy-momentum tensor of a perfect fluid in the Schwarzschild metric

$$T^{\mu\nu}{}_{;\nu} = 0 \tag{2.1}$$

with $T^{\mu\nu}$ given by

$$T^{\mu\nu} = (\rho + p)u^{\mu}u^{\nu} - pg^{\mu\nu} \tag{2.2}$$

and a relativistic generalization of the continuity equation

$$\frac{\mathrm{d}m}{\mathrm{d}t} = -4\pi r^2 T_0{}^1 \tag{2.3}$$

the phantom accretion term is derived by Babichev *et al* [1].

$$\dot{m} = \frac{16\pi G^2}{c^3} m^2 \left[\rho_{\mathrm{ph}} + p_{\mathrm{ph}}\right] \tag{2.4}$$

taking radiation accretion and Hawking evaporation into account, the complete evolution equation for the black hole mass is given by [2]

$$\dot{m} = -\frac{A(m)}{m^2} + \frac{G^2}{c^3}\left[27\pi\rho_{\mathrm{rad}}(T) + 16\pi(1+\omega)\rho_{\mathrm{ph}}\right] m^2. \tag{2.5}$$

The cosmological evolution of the phantom term is given by the Friedmann equations

$$\rho_{\mathrm{ph}} = \frac{\rho^0_{\mathrm{ph}}}{|1+\omega|}\left(\frac{3H_0 t}{2}\right)^{-\frac{9}{2}(1+\omega)}. \tag{2.6}$$

Combining the result from equation (2.5) with the radiation evolution, we find an instant during the matter-dominated era in which the phantom accretion becomes more relevant than the radiation accretion, making the latter negligible. We call this instant t_{ph} the *phantom time*.

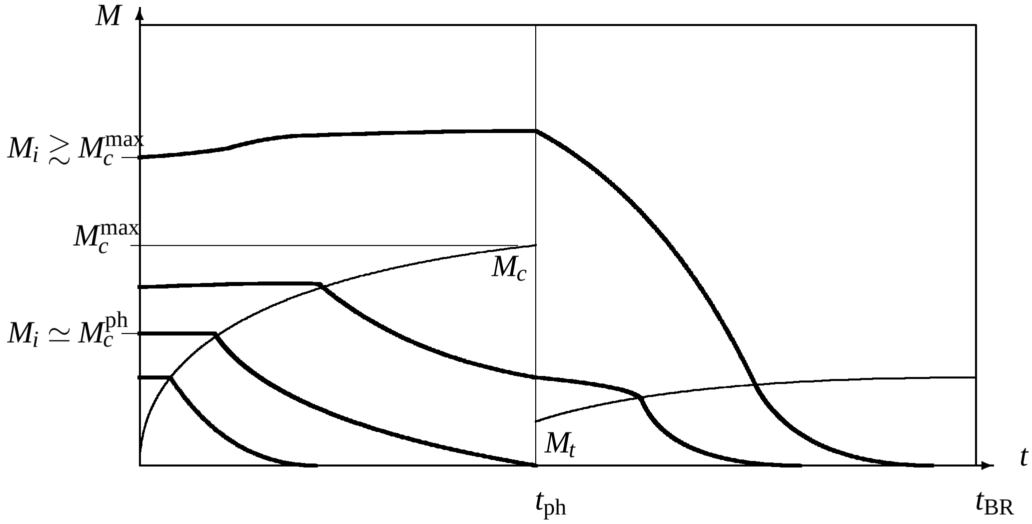

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Figure 1: Evolution of black hole masses for different initial masses along the matter and phantom energy dominated eras.

$$\frac{t_{\rm ph}}{1\ {\rm s}} = \frac{2}{3H_0}\left(\frac{16}{27}\frac{\rho^0_{\rm ph}}{\rho^0_{\rm rad}}\right)^{6-\frac{9}{2}(1+\omega)} \tag{2.7}$$

The phantom time depends on the present-day densities of the ratiation, which decreases with cosmological time, and on the density of the phantom energy, which increases with time, becoming important only on late stages of cosmological evolution.

After the phantom accretion dominates, the black hole mass monotonically decreases, and there is a competition between the two mass reduction mechanisms.

After the phantom time, the ratio between phantom accretion and Hawking evaporation indicates there is a mass threshold which marks the transition between the dominance of either regime.

$$M_t = \left[\frac{c^3}{G^2}\frac{A(m)}{|1+\omega|\,\rho_{\rm ph}}\right]^{1/4} \tag{2.8}$$

The evolution of black holes of different initial masses is shown on figure 1, taking into account all the aspects described above.

3. Phantom entropy and the Generalized Second Law of Thermodynamics

The critical mass M_c shown on figure 1 is due to the Generalized Second Law of Thermodynamics [3], which imposes a positive global entropy variation.

It can be shown that one such limit also exists on phantom energy accretion. The phantom entropy, according to the analysis by Lima *et al* [4], reads

$$S_{\rm ph} = \left[\frac{(1+\omega)\rho_0 - \mu_0 n_0}{T_0}\right]\left(\frac{T}{T_0}\right)^{\frac{1}{\omega}}. \tag{3.1}$$

with $\mu < 0$ so that both $T > 0$ and $S > 0$.

The Generalized Second Law of Thermodynamics imposes another critical mass, above which phantom accretion cannot occur [5].

$$m \geq M_{\rm crit} = \frac{1}{4\pi G(1+\omega)^2}\left[(1+\omega) - \frac{\mu_0 n_0}{\rho_0}\right]\frac{1}{T_0}\left(\frac{\rho}{\rho_0}\right)^{-\frac{\omega}{1+\omega}}. \tag{3.2}$$

4. Non-static Accretion

A possible generalization of the accretion term from (2.4) involves abandoning the assumption that the accretion takes place quasi-statically, and hence may produce a back reaction on the metric. Such a possibility has been mentioned [6], but no backup calculations have been presented so far.

The first choice for a non-static spherically symmetric space-time is the Vaidya metric

$$ds^2 = \left[1 - \frac{2m(v)}{r}\right] dv^2 - 2dv dr - r^2 d\Omega \tag{4.1}$$

which, along with the assumption that we can write for such fluids an equation of state $p(\rho)$, allows us to cast the conservation equations (2.1) as functions of the fluid four-velocity $u^0(v,r)$, its energy density $\rho(v,r)$ and the central mass $m(v)$.

The full back-reaction analysis requires us to use, instead of (2.3), the full Einstein equations for the dynamics of the fluid and the metric itself. For the Vaidya metric with free mass parameter, the only Einstein equation which is not identically satisfied is

$$-\frac{2}{r^2}\frac{dm(v)}{dv} = 8\pi G T_{00} \tag{4.2}$$

Current developments in this area involve the numerical solution of the system of coupled equations consisting of (2.1) and (4.2).

The system can be cast in a quasi-linear matrix form

$$A_{ij}\frac{\partial u_j}{\partial v} + B_{ij}\frac{\partial u_j}{\partial r} = C_i \tag{4.3}$$

with the unknowns being $\rho(v,r)$, $u^0(v,r)$ and $m(v)$, and the coefficient matrices having as entries functions of the unknowns and of the coordinates. Due to the complications which arise from the numerical computation of partial differential equations [7], we solve the system in characteristic form, provided it satisfies the necessary hyperbolicity conditions.

By this method, the coordinates are parameterized as $v = v(\eta)$ and $r = r(\eta)$, making a system of m partial equations on n dimensions become a system of $m+n$ ordinary equations.

$$\begin{aligned} m_j \frac{du_j}{d\eta} &= l_j C_j \\ \frac{dx^\mu}{d\eta} &= \alpha_j \end{aligned} \tag{4.4}$$

The solution will be able to describe the evolution of any set of initial conditions, which appear as characteristic curves on the coordinate space.

Further generalizations of this treatment involve abandoning the perfect fluid assumption. One such possibility is including a viscosity term to the energy-momentum tensor (2.2), which is to be investigated in the future.

5. Conclusions

We investigate the interaction between phantom accretion, radiation accretion and Hawking evaporation on the evolution of primordial black holes. The mass-decreasing nature of phantom energy accretion arises several regime transitions and a very interesting behavior on the later stages of the evolution of the universe.

The entropy of the phantom energy as described above not only makes this form of dark energy thermodynamically viable, but also introduces yet another transition mass on the overal picture of black hole evolution.

There is still much work to be done to put all these aspects together on a single description of the problem, and it will give us a more accurate knowledge on primordial black holes and their role on the evolution of the universe.

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References

[1] E. O. Babichev, V. I. Dokuchaev, and Yu. N. Eroshenko. Black hole mass decreasing due to phantom energy accretion. *Physical Review Letters*, 93(021102), 2004, gr-qc/0402089.

[2] Daniel C. Guariento, J. E. Horvath, P. S. Custódio, and J. E de Freitas Pacheco. Evolution of primordial black holes in a radiation and phantom energy environment. *General Relativity and Gravitation*, 40(8):1593–1602, 2008, arXiv:0711.3641 [gr-qc].

[3] P. S. Custódio and J. E. Horvath. The evolution of primordial black hole masses in the radiation-dominated era. *General Relativity and Gravitation*, 34(11):1895–1907, 2002.

[4] J. A. S. Lima and S. H. Pereira. Chemical potential and the nature of dark energy: The case of a phantom field. *Physical Review D*, 78(083504), 2008, arXiv:0801.0323 [gr-qc].

[5] J. A. S. Lima, S. H. Pereira, J. E. Horvath, and Daniel C. Guariento. Phantom accretion by black holes and the generalized second law. 2008, arXiv:0808.0860 [gr-qc].

[6] Chang Jun Gao, Xuelei Chen, Valerio Faraoni, and You-Gen Shen. Does the mass of a black hole decrease due to the accretion of phantom energy? 2008, arXiv:0802.1298 [gr-qc].

[7] Carlos Eduardo Pellicer de Oliveira. Perturbações de sistemas gravitacionais dependentes do tempo. Master’s thesis, Universidade de São Paulo – Instituto de Física, São Paulo, 2007.