PROCEEDINGS OF SCIENCE

# Scalar fields, density perturbations and the Chaplygin gas

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Among the several ideas being discussed presently in cosmology, one can find the interesting proposal of unifying dark matter and dark energy with the use of a single component with an 'exotic' equation of state. A somewhat popular candidate for playing the role of a unified dark matter is the Chaplygin gas family. In this work we show several relations between the so-called modified Chaplygin gas and a cosmological scalar field, indicating another road for the study of the evolution of perturbations in cosmological models of a single fluid.

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## 1. Introduction

Among the several ideas being discussed presently in cosmology, one can find the interesting proposal of unifying dark matter and dark energy with the use of a single component with an ‘exotic’ equation of state. A somewhat popular candidate for playing the role of a unified dark matter is the so-called Chaplygin gas, an exotic fluid whose main characteristic is to have the product between its pressure $p$ and its energy density $\rho$ as a negative constant. Many works can be found in the literature studying the implications of the use, as a cosmological fluid, of the Chaplygin gas and its generalizations, such as the modified Chaplygin gas, defined by the equation of state [1]

$$p = (\gamma - 1)\rho - M\rho^{-\mu}, \tag{1.1}$$

where $M$, $\mu$ and $\gamma$ are free parameters.

From the condition for conservation of energy, with an adiabatic expansion of the universe quantified through the scale factor $a$,

$$\frac{d\rho}{p+\rho} = -3\frac{da}{a}, \tag{1.2}$$

one obtains, for $\gamma \neq 0$ and $\mu \neq -1$, the expression

$$\rho = \left[A + (B-A)\, a^{-3\gamma(1+\mu)}\right]^{\frac{1}{1+\mu}}, \tag{1.3}$$

where $A \equiv M/\gamma$ and $B \equiv \rho_0^{1+\mu}$. Such result, in conjunction with the Friedmann equation,

$$H^2 + \frac{k}{a^2} = \frac{8\pi}{3}\rho, \tag{1.4}$$

where $H \equiv \dot{a}/a$, may yield solutions for $a = a(t)$.

## 2. Scalar fields

Both energy density and pressure of the modified Chaplygin gas can be related to a homogeneous scalar field $\varphi$ [2], through the transformation equations

$$\rho = \frac{\dot{\varphi}^2}{2} + V(\varphi) \tag{2.1}$$

and

$$p = \frac{\dot{\varphi}^2}{2} - V(\varphi), \tag{2.2}$$

where the first term in the right side of each equality corresponds to the kinetical energy of the field, while the second one corresponds to its potential energial.

If one assumes that the value of field decreases with the expansion of the universe, one may write

$$\dot{\varphi} = -(p+\rho)^{\frac{1}{2}}. \tag{2.3}$$

However,

$$\dot{\varphi} = \frac{d\varphi}{dt} = \frac{d\varphi}{da}\frac{da}{dt} = \frac{d\varphi}{da} aH. \tag{2.4}$$

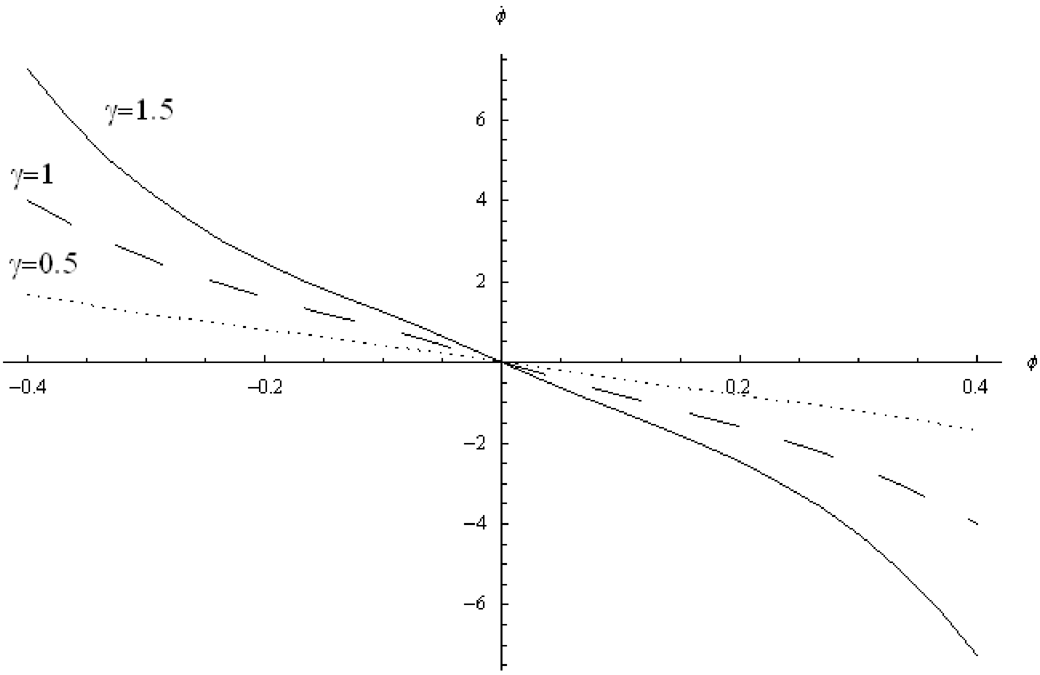


**Figure 1:** Graphs for $\dot{\phi}$ versus $\phi$, where $\phi = \varphi - \varphi_0$. In all graphs $A > 0$ and $\mu = 1$, while $\gamma$ assumes three values, 1/2, 1 and 3/2.

Finally, the Friedmann equation may be susbstituted in this last result to give

$$d\varphi = -da \left[ \frac{p\,(a) + \rho\,(a)}{\frac{8\pi}{3}\rho\,(a)\,a^2 - k} \right]^{\frac{1}{2}} . \tag{2.5}$$

The important fact to notice here is that one does not need to obtain an explicit solution for $a = a\,(t)$ in order to obtain $a = a\,(\varphi)$. This means that $\varphi$ may be seen as a surrogate quantity to be used in the place of the cosmological time $t$.

For flat spaces, $k = 0$, and then one can use the substitution

$$a^{-3\gamma(1+\mu)} = \frac{|A|\cosh 2u - A}{2\,(B-A)} , \tag{2.6}$$

valid for $\gamma \neq 0$ and $\mu \neq -1$, to easily obtain

$$\dot{\varphi} = -\left(\rho\gamma\right)^{\frac{1}{2}} \left\{ \frac{|A|\cosh 2u - A}{|A|\cosh 2u + A} \right\}^{\frac{1}{2}} \tag{2.7}$$

and

$$V\,(\varphi) = \frac{\rho}{2} \left[ 2 - \gamma \frac{|A|\cosh 2u - A}{|A|\cosh 2u + A} \right] , \tag{2.8}$$

where $u \equiv \sqrt{6\pi\gamma}\,(1+\mu)\,(\varphi - \varphi_0)$, with $\varphi_0$ being a constant of integration. Graphs for $\dot{\phi}\,(\phi)$ and $V\,(\phi)$, where $\phi \equiv \varphi - \varphi_0$, are presented in Figures 1 and 2.

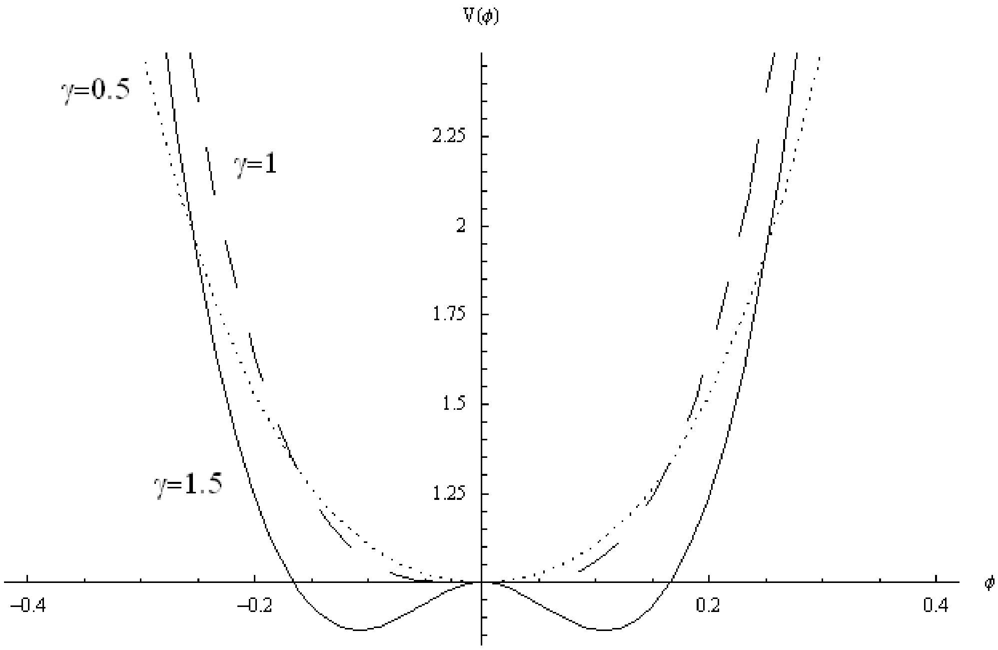


**Figure 2:** Graphs for the potential $V(\phi)$, where $\phi \equiv \varphi - \varphi_0$. In all graphs $A > 0$ and $\mu = 1$, while $\gamma$ assumes three values, 1/2, 1 and 3/2, and only for this last one the potential presents two minima.

For spaces with curvature ($k \neq 0$) the obtention of analytic solutions for the functions $a(\varphi)$ and $V(\varphi)$ seems to be feasible only for certain values of $\mu$ and $\gamma$, such as the combination $\gamma = 4/3$ and $\mu = 0$, when

$$V(\varphi) = A + (B - A)\left(3a^4\right)^{-1} . \tag{2.9}$$

This potential may have two minima $\varphi_m$ given by the conditions

$$\exp\left[\sqrt{8\pi}\,(\varphi_m - \varphi_0)\right] + \frac{3k}{16\pi\sqrt{|A|\,(B-A)}} = \pm\sqrt{\frac{A}{|A|}} \tag{2.10}$$

valid only if $A > 0$, and

$$\exp\left[2\sqrt{8\pi}\,(\varphi_m - \varphi_0)\right] = \frac{9k^2}{256\pi^2\,|A|\,(B-A)} - \frac{A}{|A|} . \tag{2.11}$$

valid only for $k \neq 0$ if $A > 0$. In the flat case only one of the above conditions may be obeyed, and only in the presence of curvature the two may be valid simultaneously.

## 3. Density perturbations

The scalar field representation may also be of some utility, for example, in the mathematical analysis of the evolution of perturbations [3], where the relevant quantity, the density contrast $\delta$, is usually seen as a function of the cosmological time $t$, the conformal time $\eta$ or the scale factor $a$.

To give an specific example, and without considering the "averaging problem" [4], one can rewrite the equation for the perturbations (cf. equation 4.122 from Padmanabhan [5]),

$$\frac{d^2\delta}{da^2}+\frac{3-15\omega+6v^2}{2a}\frac{d\delta}{da}+\frac{k^2v^2\delta}{H^2a^4}=\frac{3\delta}{2a^2}\left(1-6v^2-3\omega^2+8\omega\right), \tag{3.1}$$

where $\omega=p/\rho$, $v^2=\frac{\partial p}{\partial\rho}$ and $k$ is the wavenumber of the Fourier mode of the density perturbation in consideration.

The scalar representation suggests the use of the variable

$$w=\frac{1}{2}\left(1-\frac{|A|}{A}\cosh 2u\right)=-\left(\frac{B-A}{A}\right)a^{-3\gamma(1+\mu)}, \tag{3.2}$$

where, again,

$$u\equiv\sqrt{6\pi\gamma}(1+\mu)(\varphi-\varphi_0), \tag{3.3}$$

and, by doing this, one can obtain, for example, a very general solution for the wavemode $k=0$,

$$\begin{aligned}\delta\ &=\ c_1\frac{w^{\frac{2x}{\gamma}}}{(1-w)^x}\\ &+\ c_2\frac{w^{1+\frac{4x}{3\gamma}}}{(1-w)^x}\,{}_2F_1\left[1-\frac{2x}{3\gamma},1+x;2-\frac{2x}{3\gamma};w\right],\end{aligned} \tag{3.4}$$

where $x\equiv 1/\left[2(1+\mu)\right]$, with $c_1$ and $c_2$ being arbitrary constants.

An analogous procedure, with a change of variable motivated by the scalar field representation, allows one to obtain, for the case $\mu=0$ and $\gamma=2/3$, the analytical solution [6]

$$\delta=c_1'\delta_+ + c_2'\delta_-, \tag{3.5}$$

where $c_1'$ and $c_2'$ are arbitrary constants,

$$\delta_\pm=\frac{1}{\sinh\bar{t}\cosh^2\bar{t}}\left(\frac{\cosh\bar{t}\mp 1}{\sinh\bar{t}}\right)^\nu, \tag{3.6}$$

and where $\nu\equiv\left(1+4\tilde{k}^2\right)^{1/2}$ and $\bar{t}=(4\pi M)^{1/2}t$.

## 4. Conclusion

The relations between the modified Chaplygin gas and a cosmological scalar field shown in this work, with results which incorporate the possibility of having a negative value for $A$, indicate that models using the modified Chaplygin gas as a single fluid may also be studied using a representation in terms of a scalar field. For example, in cosmology one may be interested in solutions for the scale factor when there is curvature. Since, for cosmologies with the modified Chaplygin gas acting as a single fluid, few of such analytic solutions are known, the usage of a scalar field as an auxiliary quantity offers, at least in principle, another way for the search of new solutions. Also, the representation of the modified Chaplygin gas in terms of a scalar field $\varphi$ opens another road for the study of the evolution of perturbations, and as such it may be seen as a mathematical tool of some value.

## References

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