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Renormalizable noncommutative U(1) gauge theory without IR/UV mixing

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We investigate the quantum effects of the nonlocal gauge invariant operator $\frac{1}{D^2}F^{\mu\nu} * \frac{1}{D^2}F^{\mu\nu}$ in the noncommutative $U(1)$ action and its consequences to the infrared sector of the theory. Nonlocal operators of such kind were proposed to solve the infrared problem of the noncommutative gauge theories evading the questions on the explicit breaking of the Lorentz invariance. More recently, a first step in the localization of this operator was accomplished by means of the introduction of an extra tensorial matter field, and the first loop analysis was carried out $(hep-th/0901.1681v1)$. We will complete this localization avoiding the introduction of new degrees of freedom beyond those of the original action by using only BRST doublets. This will allow us to make a complete BRST algebraic study of the renormalizability of the theory, following Zwanziger's method of localization of nonlocal operators in QFT. We also give some difficulties that should be overcome in order to apply this method to the general $U(N)$ case, which will require further analysis.

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1. Introduction

One of the areas that is constantly being studied is the quantum field theory in noncommutative space-time (NCQFT). The reason of this study is the fact NCQFT emerging from a limit of low-energy in open string theory in a magnetic background field [1]. However some problems were found and one is the mixing of infrared and ultraviolet divergences [3]. For noncommutative scalar field, Wulkenhar proposed add a term with avoid this mixing [5]. However, this one are not lorentz invariant. A proposal was better add a non-local term in start action [6, 9] for it to be renormalizable at the stability point of view.

For the gauge field, would be consistent add a non-local term

$$\frac{1}{D^2}F^{\mu\nu} * \frac{1}{D^2}F^{\mu\nu} \tag{1.1}$$

This term is gauge and lorentz invariant and leads to a slight change in the propagator

$$\langle A(k)_\mu A_\nu(-k)\rangle = \left(\delta_{\mu\nu} - \frac{k_\mu k_\nu}{k^2}\right)\frac{k^2}{k^4+\gamma^4}\ , \tag{1.2}$$

avoiding the IR divergences and then decoupling the IR/UV mixing.

Our intention here will be to present an alternative scenario of localization, leading to a renormalizable noncommutative gauge field theory, but avoiding to introduce any extra degree of freedom. We present the nonlocal action, its localization via doublet fields and the resulting BRST symmetry.

After, we dedicated to the analysis of stability of this theory using alegraic methods to renormalization. The definitive form of the propagator is finally obtained, showing a modification from the classical starting one.

2. BRST in Euclidean space

The nonlocal action that we will study is

$$S_{NL} = \int d^4x\{\frac{1}{4}F_{\mu\nu} * F^{\mu\nu} + \gamma^4\frac{1}{4}\frac{1}{D^2}F_{\mu\nu} * \frac{1}{D^2}F^{\mu\nu}\}. \tag{2.1}$$

We are assuming an Euclidian signature for the space-time and an Abelian gauge group, with

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - ig[A_\mu \overset{*}{,} A_\nu], \quad D_\mu = \partial_\mu + ig[\ \overset{*}{,} A_\mu]. \tag{2.2}$$

This action gives to the gauge field propagator [1.2] a more adequate behavior in the infrared for the noncommutative space

As pointed out in [6] and [9], the infrared behavior of this kind of propagator decouples the ultraviolet and infrared regimes, and, then, the action (2.1) is a good candidate to generate a coherent quantum gauge theory in noncommutative space, without the IR/UV mix.

The action S_{NL} can be localized introducing a set of auxiliary tensorial fields. We use two pairs of complex conjugated fields $\overline{B}_{\mu\nu}$, $B_{\mu\nu}$, $\overline{\chi}_{\mu\nu}$ and $\chi_{\mu\nu}$.

Although the new action represents the nonlocal operador in a localized form, still presents the problem that new degrees of freedom are being introduced by the auxiliary fields changing the start theory.

This problem can be solved by associating a ghost for each tensorial field introduced, in a way that a BRST structure of quartets will appear. Then we have

$$
\begin{aligned}
S_{LO+G} &= S_{0+G}+S_{break} \qquad (2.3)\\
S_{0+G} &= \int d^4x\{\frac{1}{4}F_{\mu\nu}*F^{\mu\nu}+\overline{\chi}_{\mu\nu}*D^2B^{\mu\nu}\\
&+ \overline{B}_{\mu\nu}*D^2\chi^{\mu\nu}+\gamma^2\overline{\chi}_{\mu\nu}*\chi^{\mu\nu}\\
&- \overline{\psi}_{\mu\nu}*D^2\xi^{\mu\nu}-\overline{\xi}_{\mu\nu}*D^2\psi^{\mu\nu}-\gamma^2\overline{\psi}_{\mu\nu}*\psi^{\mu\nu}\}.\\
S_{break} &= \int d^4x\{-i\frac{\gamma}{2}B_{\mu\nu}*F^{\mu\nu}+i\frac{\gamma}{2}\overline{B}_{\mu\nu}*F^{\mu\nu}\} \qquad (2.4)
\end{aligned}
$$

where S_{0+G} is left invariant by the set of BRST transformations

$$
\begin{aligned}
sA_\mu &= -D_\mu c\ ,\ sc=-\frac{ig}{2}\{c\stackrel{*}{,}c\}\ ,\\
s\overline{c} &= ib\ ,\ sb=0\ ,\\
sF_{\mu\nu} &= -ig[c\stackrel{*}{,}F_{\mu\nu}]\ ,\\
s\overline{\xi}_{\mu\nu} &= \overline{B}_{\mu\nu}-ig\{c\stackrel{*}{,}\overline{\xi}_{\mu\nu}\}\ ,\ s\overline{B}_{\mu\nu}=-ig[c\stackrel{*}{,}\overline{B}_{\mu\nu}]\ ,\\
s\overline{\psi}_{\mu\nu} &= \overline{\chi}_{\mu\nu}-ig\{c\stackrel{*}{,}\overline{\psi}_{\mu\nu}\}\ ,\ s\overline{\chi}_{\mu\nu}=-ig[c\stackrel{*}{,}\overline{\chi}_{\mu\nu}]\ ,\\
sB_{\mu\nu} &= \xi_{\mu\nu}-ig[c\stackrel{*}{,}B_{\mu\nu}]\ ,\ s\xi_{\mu\nu}=-ig\{c\stackrel{*}{,}\xi_{\mu\nu}\}\ ,\\
s\chi_{\mu\nu} &= \psi_{\mu\nu}-ig[c\stackrel{*}{,}\chi_{\mu\nu}]\ ,\ s\psi_{\mu\nu}=-ig\{c\stackrel{*}{,}\psi_{\mu\nu}\}\ . \qquad (2.5)
\end{aligned}
$$

where one can see the formation of a double quartet structure. The action S_{0+G} can then be written as a BRST variation.

But, in our localized action (2.4) there is still a piece to be analyzed. The S_{break} sector of the action is not left invariant under the BRST transformations (2.5).

Then, we have:

$$
sS_{break} = \int d^4x\{-i\frac{\gamma}{2}\xi_{\mu\nu}*F^{\mu\nu}\}. \qquad (2.6)
$$

Looking at this term we see a soft break, which is a break with UV dimension less than action. The treatment of softly broken theories was recently formalized in [10]. We will need to study the renormalization of the theory together with the renormalization of the breaking itself. This is done by introducing a set of sources in a BRST doublet in such a way that the physical action is obtained when we set the sources to their physical values:

$$
\begin{aligned}
S_{break} &= S_{source}\Big|_{\text{phys}}\\
S_{source} &= \int d^4x(\overline{J}_{\mu\nu\alpha\beta}*\{B^{\mu\nu}\stackrel{*}{,}F^{\alpha\beta}\}+J_{\mu\nu\alpha\beta}*\{\overline{B}^{\mu\nu}\stackrel{*}{,}F^{\alpha\beta}\}\\
&- \overline{Q}_{\mu\nu\alpha\beta}*\{\xi^{\mu\nu}\stackrel{*}{,}F^{\alpha\beta}\}), \qquad (2.7)
\end{aligned}
$$

where by $\Big|_{\text{phys}}$ we mean that in this limit the sources attain their physical values,

$$J_{\mu\nu\alpha\beta}| = \frac{i}{8}\gamma(\delta_{\mu\alpha}\delta_{\beta\nu}-\delta_{\mu\beta}\delta_{\alpha\nu})\ \bar{J}_{\mu\nu\alpha\beta}| = -\frac{i}{8}\gamma(\delta_{\mu\alpha}\delta_{\beta\nu}-\delta_{\mu\beta}\delta_{\alpha\nu}),\ ,$$
$$Q_{\mu\nu\alpha\beta}| = 0\ ,\ \bar{Q}_{\mu\nu\alpha\beta}| = 0. \tag{2.8}$$

The BRST transformation of the sources,

$$sQ_{\mu\nu\alpha\beta} = J_{\mu\nu\alpha\beta} - ig\{c \stackrel{*}{,} Q_{\mu\nu\alpha\beta}\}\ ,\ sJ_{\mu\nu\alpha\beta} = -ig[c \stackrel{*}{,} J_{\mu\nu\alpha\beta}], \tag{2.9}$$
$$s\bar{Q}_{\mu\nu\alpha\beta} = \bar{J}_{\mu\nu\alpha\beta} - ig\{c \stackrel{*}{,} \bar{Q}_{\mu\nu\alpha\beta}\}\ ,\ s\bar{J}_{\mu\nu\alpha\beta} = -ig[c \stackrel{*}{,} \bar{J}_{\mu\nu\alpha\beta}], \tag{2.10}$$

shows the doublet structure that we have already mentioned. The action (2.7) is now easily seen as an exact BRST variation, and the process altogether is a kind of an immersion of the original theory inside this more general one.

The last steps needed for the BRST quantization are the definition of a gauge fixing (noncommutative landau gauge),

$$S_{gf} = \int d^4x\{ib * \partial_\mu A^\mu + \bar{c} * \partial^\mu D_\mu c\}. \tag{2.11}$$

and the introduction of a set of Slavnov sourcesΩ ,L, $\bar{u}$,u, $\bar{v}$,v, $\bar{P}$, P, $\bar{R}$, R, $\bar{M}$, M, $\bar{N}$, N coupled to the nonlinear BRST transformations of the field A,c,ξ , $\bar{\xi}$,B, $\bar{B}$, ψ , $\bar{\psi}$, χ , $\bar{\chi}$ and sources Q, $\bar{Q}$,J , $\bar{J}$ respectively.

The complete invariant action can then be written as

$$\Sigma = S_{0+G} + S_{source} + S_{gf} + S_{Slavnov} \tag{2.12}$$

where

$$\begin{aligned}
S_{Slavnov} = \int d^4x\{ & -\Omega_\mu * D^\mu c - \frac{i}{2}L * g\{c \stackrel{*}{,} c\} - i\bar{u}^{\mu\nu} * g\{c \stackrel{*}{,} \xi_{\mu\nu}\} \\
& + u^{\mu\nu} * (\bar{B}_{\mu\nu} - ig\{c \stackrel{*}{,} \bar{\xi}_{\mu\nu}\}) + \bar{v}^{\mu\nu} * (\xi_{\mu\nu} - ig[c \stackrel{*}{,} B_{\mu\nu}]) \\
& - iv^{\mu\nu} * g[c \stackrel{*}{,} \bar{B}_{\mu\nu}] - i\bar{P}^{\mu\nu} * g\{c \stackrel{*}{,} \psi_{\mu\nu}\} + P^{\mu\nu} * (\bar{\chi}_{\mu\nu} - ig\{c \stackrel{*}{,} \bar{\psi}_{\mu\nu}\}) \\
& + \bar{R}^{\mu\nu} * (\psi_{\mu\nu} - ig[c \stackrel{*}{,} \chi_{\mu\nu}]) - iR^{\mu\nu} * g[c \stackrel{*}{,} \bar{\chi}_{\mu\nu}] \\
& + \bar{M}^{\mu\nu\alpha\beta} * (J_{\mu\nu\alpha\beta} - ig\{c \stackrel{*}{,} Q_{\mu\nu\alpha\beta}\}) \\
& + M^{\mu\nu\alpha\beta} * (\bar{J}_{\mu\nu\alpha\beta} - ig\{c \stackrel{*}{,} \bar{Q}_{\mu\nu\alpha\beta}\}) \\
& - i\bar{N}^{\mu\nu\alpha\beta} * g[c \stackrel{*}{,} J_{\mu\nu\alpha\beta}] - iN^{\mu\nu\alpha\beta} * g[c \stackrel{*}{,} \bar{J}_{\mu\nu\alpha\beta}]\}
\end{aligned} \tag{2.13}$$

and it is ready for the BRST analysis.

3. Stability of the quantum action and the invariant counterterm

Now we can analyse the stability of the theory. The BRST analysis is to obtain equations compatible with the quantum action principle and using these equations, we impose the simetries of the classical action would be kept a quantum level. From this we construct counterterms.

In order to characterize the most general invariant counterterm which can be added freely to all orders in perturbation theory [7], we perturb the classical action Σ by adding and arbitrary integrated local polynomial Σ^{count} of dimension up-bounded by four, vanishing ghost number and Q charge. We demand that $\Gamma = \Sigma + \varepsilon\Sigma^{count} + O(\varepsilon^2)$, where ε is a small expansion parameter, satisfies the same Ward identities as Σ. This requirement provides the following constraints on the counterterm

As we have a nilpotent linearized operator (a BRST generalization for the counterterms) invoke the cohomology to construct the counterterm action as

$$\Sigma^{count} = \frac{a_0}{4}\int d^4xF_{\mu\nu} * F_{\mu\nu} + \Delta^{(0)}, \qquad \Delta^{(0)} = \mathscr{B}_\Sigma \Delta^{(-1)} , \tag{3.1}$$

where $\Delta^{(0)}$is a local integrated polynomial in all fields and sources, with ultra-violet dimension up-bounded by four, ghost number zero and vanishing Q charge and $\mathscr{B}$ is the linearized operator.

From the analisys using the Ward identities, we get constraints on the counterterms' cocicles [8]. Then, what is left for the quantum contributions for the counterterm are (we neglect here the terms which become null at the physical limit of (2.8))

$$\begin{aligned}\Sigma^{count} \;=\; & \frac{a_0}{4}\int d^4xF_{\mu\nu} * F_{\mu\nu} + (a_1\frac{\delta\Sigma}{\delta A_\mu} * A_\mu + a_1(\Omega^\mu + \partial^\mu\overline{c}) * D_\mu c \\ & + \; a_2\frac{\delta\Sigma}{\delta c}c - \frac{ig}{2}a_2L\{c \overset{*}{,} c\} + a_3\overline{\chi}_{\mu\nu} * D^2B^{\mu\nu} + a_3\overline{B}_{\mu\nu} * D^2\chi^{\mu\nu} \\ & - \; a_3\overline{\psi}_{\mu\nu} * D^2\xi^{\mu\nu} - a_3\overline{\xi}_{\mu\nu} * D^2\psi^{\mu\nu} \\ & + \; a_4\overline{\chi}_{\mu\nu} * D^2\chi^{\mu\nu} - a_4\overline{\psi}_{\mu\nu} * D^2\psi^{\mu\nu}).\end{aligned} \tag{3.2}$$

The most important point to be stressed here is that this couterterm action implies that the term $\overline{\chi}_{\mu\nu} * D^2\chi^{\mu\nu} - \overline{\psi}_{\mu\nu} * D^2\psi^{\mu\nu}$ must be in the classical starting action in order to assure its stability. This term is then responsible for a gauge propagator slightly modified in relation to that in (1.2). When the sources $J, \overline{J}, Q$ and $\overline{Q}$ are set to their physical values the propagator for gauge field takes the form:

$$\langle A(k)_\mu A_\nu(-k)\rangle = \left(\delta_{\mu\nu} - \frac{k_\mu k_\nu}{k^2}\right)\frac{k^2}{k^4 - a_4\gamma^2k^2 + \gamma^4} , \tag{3.3}$$

This means that the inclusion of all counterterms of (3.2) in the starting classical action will ensure the renormalizability of the noncommutative Maxwell theory, not only from the stability point of view as well as from the fact that the resulting propagator (3.3) still decouples the infrared and ultraviolet regims, avoiding the IR/UV mix.

4. Conclusion

We saw along this work how a nonlocal action as that in equation (2.1) can cure the infrared problem without spoiling the ultraviolet stability of a noncommutative Maxwell action. It is interesting to notice that the presence of the Moyal coupling θ with its negative mass dimension and of the infinite set of non-power-counting vertices of the noncommutative Maxwell theory make the renormalizability so obtained a result that fills the idea once proposed by Gomis and Weinberg on the possibility of renormalization of nonrenormalizable theories by the power-counting criterium [12] .

In the development of our algebraic proof, we followed the approach used by [11] , and more recently improved by Sorella and Baulieu [10] , to the study of the BRST quantization of the nonlocal action coming from Gribov's observations on the infrared properties of gauge theories. We understand that, if in the usual commutative space the use of nonlocal actions is an alternative option to the study of the infrared regime, on the other hand, in the noncommutative case this seems to be the inevitable path to solve the intrinsic problem of the IR/UV mix. Using the same analogy with these works, we can identify evidences for a confining character of this noncommutative theory as can be found in [11, 10].

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