PROCEEDINGS OF SCIENCE

Energy Conditions for Electromagnetic Field in presence of Cosmological Constant

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The Einstein's equations can be solved in order to find which energy-momentum tensor corresponds to a given geometry of space-time. However, these solutions sometimes have not any physical meaning. To choose which solutions have physical meaning, we use the energy conditions. This work presents the explicit expressions of the energy conditions for the electromagnetic field in the vacuum and in the presence of the cosmological constant. The Analysis of these two cases produces some bounds for the value of cosmological constant, showing a relation between it and the electromagnetic field present in the space-time. In particular, it is shown that the cosmological constant need to have a value bigger than that one considered in cosmological models in order to characterize a dark energy fluid in the presence of electromagnetic fields.

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1.Introduction

The General Relativity Theory says that the matter and the energy are always related with the geometry of the space-time, by the Einstein's Field Equations.

$$G_{\mu\nu} = kT_{\mu\nu} + \Lambda g_{\mu\nu} \,, \tag{1.1}$$

where $G_{\mu\nu}$ is the Einstein tensor, which describe the space-time geometry, $T_{\mu\nu}$ is the energy-momentum tensor, which describe the matter and the energy in this space time, and Λ is the cosmological constant.

Some solutions of the Einstein's Equations do not have any physical meaning. To choose which solutions have physical meaning we use the **energy conditions.**

Following are present the energy conditions for an anisotropic fluid , with energydensity and pressures given by ρ and p_i , respectively.

1.1. Weak Energy Condition (WEC)

This condition imposes that the energy density measured for any observer need to be always positive.

$$\rho \geq 0. \tag{1.2}$$

In terms of the energy-momentum tensor this equation can be represented by:

$$T_0^0 \geq 0. \tag{1.3}$$

Also the energy density plus the pressure in any direction need to be positive too,

$$\rho + p_i \geq 0 \tag{1.4}$$

or

$$T_0^0 - T_i^i \geq 0. \tag{1.5}$$

1.2. Dominant Energy Condition (DEC)

This condition says that the local speed of any observable fluid is always less than the local speed of light. This is assured if the local pressure of the fluid do not exceed the energy density,

$$\rho - p_i \geq 0. \tag{1.6}$$

In terms of the energy-momentum tensor:

$$T_0^0 + T_i^i \geq 0. \tag{1.7}$$

To obey the **DEC** a fluid need to obey also the **WEC**.

1.3. Strong Energy Condition (SEC)

This condition guarantes that all physically "well behaved" matter exercises an effect of convergence in the time-like and null geodesics. This is assured if:

$$\rho + \sum_{i=1}^{3} p_i \geq 0, \tag{1.8}$$

or in terms of the energy-momentum tensor:

$$T_0^0 + T_i^i \geq 0. \tag{1.9}$$

To satisfy the **SEC** a fluid need to satisfy also the equation (1.4).

The **SEC** is particularly interesting because cosmological models recently provide that must to exist a type of energy, called **Dark Energy**, which exercises in all bodies just the counter effect of the "well behaved" energy exercises. The dark energy must repel matter.

2.Energy-momentum tensor of electromagnetic field

The electromagnetic energy-momentum tensor is defined by:

$$T_{\mu\nu} = \frac{1}{4\pi}\left[-g^{\lambda\rho}\left(F_{\mu\lambda}F_{\nu\rho}\right) + \frac{1}{4}g_{\mu\nu}\left(F_{\lambda\rho}F^{\lambda\rho}\right)\right], \tag{2.1}$$

where $F_{\mu\nu}$ is the electromagnetic tensor and it is defined by:

$$F_{\mu\nu} = A_{\nu;\mu} - A_{\mu;\nu}, \tag{2.2}$$

and A_μ is the electromagnetic four-potential.

In order to simplify the calculations of the energy conditions we can rewrite the energy momentum tensor as:

$$T_\mu^\alpha = \frac{1}{4\pi}\left[-\Pi_\mu^\alpha + \frac{1}{4}\delta_\mu^\alpha \Pi\right], \tag{2.3}$$

where:

$$\Pi_\mu^\alpha = F_{\mu\lambda}F^{\alpha\lambda}, \qquad \Pi = F_{\lambda\rho}F^{\lambda\rho} = \Pi_\mu^\mu.$$

3.Energy conditions for electromagnetic field in a space-time without rotation and with cosmological constant

In a space-time without rotation, the electromagnetic field will be purely electric and $\Lambda = 0$ in equation (1.1). Once the electromagnetic energy-momentum is written as (2.3), the energy condition can be also written in function of the tensor Π_μ^α.

For diagonal energy-momentum tensor we have:

(WEC)
$$\begin{cases} \Pi_0^0 \leq \frac{1}{4}\Pi, \\ \Pi_i^i - \Pi_0^0 \geq 0. \end{cases} \tag{3.1}$$

(DEC)
$$\Pi_i^i + \Pi_0^0 \leq \frac{1}{2}\Pi. \tag{3.2}$$

(SEC)
$$\sum \Pi_i^i - \Pi_0^0 \geq \frac{1}{2}\Pi. \tag{3.3}$$

In the case of the energy-momentum tensor to be non-diagonal, the tensor Π_μ^α need to be made diagonal, so:

$$\left|\Pi_\mu^\alpha - \lambda\delta_\mu^\alpha\right| = 0. \tag{3.4}$$

Once the eigenvalues are found, the energy conditions can are written as.

$$\textbf{(WEC)} \qquad \begin{cases} \lambda_0 \leq \dfrac{1}{4}\Pi, \\ \lambda_i - \lambda_0 \geq 0. \end{cases} \tag{3.5}$$

$$\textbf{(DEC)} \qquad \lambda_i + \lambda_0 \leq \frac{1}{2}\Pi. \tag{3.6}$$

$$\textbf{(SEC)} \qquad \sum \lambda_i - \lambda_0 \geq \frac{1}{2}\Pi. \tag{3.7}$$

The more general metric for the space-time without rotation can be given by:

$$ds^2 = \alpha^2 (t, q_i) dt^2 - \sum_{i=1}^{3} \beta_i^2 (t, q_i) dq_i^2. \tag{3.8}$$

Considering the more general electric potential as:

$$A_0 = \phi(t, q_i), \tag{3.9}$$

and using the equation (2.2), (3.8) and (3.9), the components of the electromagnetic tensor are given by:

$$F_{0i} = -\frac{\partial \phi}{\partial q_i}, \qquad F^{0i} = \frac{1}{\alpha^2 \beta_i^2} \frac{\partial \phi}{\partial q_i}. \tag{3.10}$$

Using these components to find the components of the tensor Π_μ^α and then, applying this results in equation (3.4), the following eigenvalues are found:

$$\lambda_1 = \lambda_2 = 0, \tag{3.11}$$

$$\lambda_3 = \lambda_0 = -\sum_{i=1}^{3} \frac{1}{\alpha^2 \beta_i^2} \left(\frac{\partial \phi}{\partial q_i} \right)^2. \tag{3.12}$$

Applying these eigenvalues in equations (3.5), (3.6) and (3.7), all of the energy conditions are satisfied. So for a general metric without rotation in an universe without cosmological constant, always the electromagnetic field will be “well behaved”, as expected.

4. Energy conditions for electromagnetic field in the presence of cosmological constant

The equation (1.1) can be redefined by:

$$G_{\mu\nu} = kT_{\mu\nu}^{(ef)}, \tag{4.1}$$

where the effective energy-momentum tensor is given by:

$$T_{\mu\nu}^{(ef)} = T_{\mu\nu} + \Lambda_{(k)} g_{\mu\nu}, \tag{4.2}$$

with $\Lambda_{(k)} = \dfrac{\Lambda}{4\pi k}$.

Writing the $T_{\mu\nu}^{(ef)}$ analogue to the equation (2.3), the following expression can be found:

$$^{(ef)}T^{\alpha}_{\mu} = \frac{1}{4\pi}\left[-\Pi^{\alpha}_{\mu} + \delta^{\alpha}_{\mu}\left(\frac{1}{4}\Pi + \Lambda_k\right)\right]. \tag{4.3}$$

Using the same metric given by (3.8) and the same potential (3.9), and considering that the cosmological constant is just added in the diagonal of electromagnetic energy-momentum tensor, the energy conditions can be analyzed for the eigenvalues given by equations (3.11) and (3.12) plus the cosmological constant, but it is more interesting to use this energy conditions to estimate intervals at values for the cosmological constant once the purely electric field always obey the energy conditions.

The cosmological term will transform the expressions (3.5), (3.6) and (3.7) in to:

$$\textbf{(WEC)} \qquad \begin{cases} \lambda_0 \leq \frac{1}{4}\Pi + \Lambda_{(k)}, \\ \lambda_i - \lambda_0 \geq 0. \end{cases} \tag{4.4}$$

$$\textbf{(DEC)} \qquad \lambda_i + \lambda_0 \leq \frac{1}{2}\Pi + 2\Lambda_{(k)}. \tag{4.5}$$

$$\textbf{(SEC)} \qquad \sum \lambda_i - \lambda_0 \geq \frac{1}{2}\Pi + 2\Lambda_{(k)}. \tag{4.6}$$

Analyzing these expressions, the following results are found.

4.1. The intervals wich saisfy all energy conditions

To satisfy all energy conditions, the following intervals are found:

$$0 \leq \Lambda \leq \frac{k}{8\pi}\sum_{i=1}^{3}\frac{1}{A^2 B_i^2}\left(\frac{\partial\phi}{\partial q_i}\right)^2 \tag{4.7}$$

So, Λ needs to be positive, but limited for electromagnetic field. We can see that the presence of the electromagnetic field imposes an inferior limit on the cosmological constant.

4.2. The interval to brake the SEC

Today is believed that Λ is a candidate to represent a model for dark energy, for this the SEC needs to be violated, once the dark energy is an exotic fluid that exercises a repulsive gravitational force, so:

$$\Lambda > \frac{k}{8\pi}\sum_{i=1}^{3}\frac{1}{A^2 B_i^2}\left(\frac{\partial\phi}{\partial q_i}\right)^2 = \Lambda_c \tag{4.8}$$

Therefore, for Λ values higher than Λ_c, that depends on the electromagnetic field, the combination f this one with the cosmological constant would exercise a repulsive gravitational force.

5.Conclusion

In this work we show how the limits for the Λ value can change, in order to represent a dark energy fluid, due to the presence of an electromagnetic field. Once this limits of Λ depends on the type of the matter that fulfill the space-time, it is reasonable to conclude that the limit for the parameter Λ, based on the energy conditions, depends on the age of the universe, since different ages of universe are dominated by different type of matter across the time.

This work is only an example of the importance to consider all the fields of energy when the cosmological constant is taken as a candidate of dark energy. To study this more carefully it is necessary to take the energy-momentum tensor for all universe ages and to analyze each interval of Λ where the energy conditions are violated. This can constitute a way to find the time when occurred transitions between the decelerated to the accelerated phase.

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