# Spin Density Matrix Elements (SDMEs) and Helicity Amplitude Ratios in Exclusive $\rho^{0}$ Electroproduction at HERMES 

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Spin Density Matrix Elements (SDMEs) describing the angular distribution of exclusive $\rho^{0}$ electroproduction and decay are determined in the HERMES experiment with 27.5 GeV beam energy on unpolarized hydrogen and deuterium targets. Those are extracted in the kinematic region

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## 1. Introduction

Electroproduction of neutral vector mesons $(V)$ from nucleons is described by the interaction with the nucleon of a $q \bar{q}$ pair created by the virtual photon. The reaction $e+N \rightarrow e^{\prime}+V+N^{\prime}$ in the single-photon approximation is equivalent to $\gamma^{*}+N \rightarrow V+N^{\prime}$, in which the virtual photon is characterized by the negative squared four-momentum $Q^{2}$ and the polarization parameter $\varepsilon$ is the ratio of fluxes of longitudinal and transverse virtual photons. The spin transfer from the virtual photon $\gamma^{*}$ to the vector meson is commonly described [1, 2] in terms of Spin Density Matrix Elements (SDMEs). The experimentally determined sets of 23 SDMEs for the $\rho^{0}$ meson are presented here in the Schilling-Wolf [1] representation. Using the measured values of the SDMEs $s$-Channel Helicity Conservation (SCHC) in the transition $\gamma^{*} \rightarrow V$ can be studied.

It is well known $[1,2,3]$ that SDMEs can be expressed through bilinear products of helicity amplitudes for $\rho^{0}$ electroproduction. The number of independent helicity amplitudes of the process $\gamma^{*}+N \rightarrow V+N^{\prime}$ is equal to 18 . Since the spin density matrix elements are dimensionless they may be expressed through ratios of the amplitudes rather than amplitudes themselves. The ratios are complex numbers, hence the total amount of independent real parameters is equal to 34. In general SDMEs cannot be considered as independent quantities since their number is, generally speaking, greater than 34 as it depends on the beam and target polarization [2]. Hence the helicity amplitudes provide more economic basis for description of electroproduction than the spin density matrix elements. In addition to SDMEs, the $Q^{2}$ dependence of the ratio of the real and imaginary part of the helicity-conserving amplitudes $T_{11} / T_{00}$ is presented. Those amplitudes correspond to the transitions $\gamma_{T}^{*} \rightarrow V_{T}$ and $\gamma_{L}^{*} \rightarrow V_{L}$ mediated by the natural parity exchange.

The fractional contribution of Unnatural-Parity-Exchange (UPE) amplitudes to the process $\gamma^{*}+N \rightarrow V+N$ in comparison with Natural-Parity-Exchange (NPE) amplitudes was derived from SDMEs measurements [4]. Form the measurements of the helicity amplitudes this contribution was obtained with higher accuracy. We remind that NPE amplitudes describe the exchange of a particle of "natural" parity ( $\left.J^{P}=0^{+}, 1^{-}, 2^{+}, \ldots\right)$, while UPE amplitudes describe the exchange of a particle of "unnatural" parity ( $\left.J^{P}=0^{-}, 1^{+}, \ldots\right)$.

## 2. Selection of exclusive $\rho^{0}$ mesons

The experiment was performed with longitudinally polarized electron and positron beams at an energy of 27.5 GeV using unpolarized hydrogen or deuterium gas targets. The $\rho^{0}$ mesons are observed in the HERMES spectrometer [5] by detecting their decay products in the channel $\rho^{0} \rightarrow \pi^{+} \pi^{-}(100 \%)$. The $\rho^{0}$ mesons are identified [4] by requiring $0.6<M_{\pi \pi}<1 \mathrm{GeV}$, with $M_{\pi \pi}$ being the invariant mass of the $\pi^{+} \pi^{-}$system. The $\phi \rightarrow K^{+} K^{-}$background in the $\rho^{0}$ spectra is removed by the requirement that $M_{K K}>1.04 \mathrm{GeV}$, if the hadrons are assumed to be kaons.

Exclusive events were selected by the requirement [4]: $\Delta E=\left(M_{X}^{2}-M^{2}\right) /(2 M) \leq 0.6 \mathrm{GeV}$, where $M$ is the mass of the nucleon and $M_{X}$ is the missing mass of the reaction. Diffractive events were selected by the constraint $-t^{\prime}<0.4 \mathrm{GeV}^{2}$. Here, $t^{\prime}=t-t_{0}$, with $t$ being the squared fourmomentum transfer from the virtual photon to the vector meson and $-t_{0}$ the smallest kinematically allowed value of $-t$ at fixed $Q^{2}$ and energy in the $\gamma^{*} N$ center-of-mass system $(W)$. A contamination
of semi-inclusive deep-inelastic scattering events in the chosen kinematic region was determined to be $8 \%$.

## 3. Spin Density Matrix Elements

The 3-dimensional angular distribution of the scattered lepton and the decay products is described by the following angles (see detailed definitions in [6]): $\Phi$ is the angle between the $\rho^{0}$ meson production and lepton scattering planes in the $\gamma^{*} \mathrm{p}$ center-of-mass system. The polar $\phi$ angle and the azimuthal $\Theta$ angle of $\pi^{+}$meson from $\rho^{0}$ decay are measured in the $\rho^{0}$ meson rest frame.

The SDMEs are obtained by minimizing the difference between the 3-dimensional $(\cos \Theta, \phi, \Phi)$ matrix of the data and the analogous matrix from fully reconstructed simulated events. An $8 \times 8 \times 8$ binning was used for the variables $\cos \Theta, \phi, \Phi$. The simulated events were generated with uniform angular distributions and re-weighted in an iterative procedure with the angular distribution $\mathscr{W}\left(\cos \Theta, \phi, \Phi, r_{i j}^{\alpha}\right)[1]$, where the spin density matrix elements $r_{i j}^{\alpha}$ were treated as free parameters. The best fit parameters were determined using a binned maximum log-likelihood method. The minimization itself and the error calculation were performed by MINUIT.

The extracted SDMEs will be presented based on a hierarchy of NPE helicity amplitudes: $\left|T_{00}\right| \sim\left|T_{11}\right| \gg\left|T_{01}\right|>\left|T_{10}\right| \sim\left|T_{1-1}\right|$, established for the first time in Ref. [7, 8] for $\rho^{0}$ production. This hierarchy was experimentally confirmed for exclusive $\rho^{0}$ production at HERMES kinematics [4]. The SDMEs are categorized into five classes according to this hierarchy. Classes A and B describe only SCHC transitions. Classes from C to E contain also spin flip transitions. Class A comprises SDMEs with dominant contributions proportional to $\left|T_{00}\right|^{2}$ or $\left|T_{11}\right|^{2}$, which are the helicity-conserving amplitudes describing the transitions $\gamma_{L}^{*} \rightarrow V_{L}$ and $\gamma_{T}^{*} \rightarrow V_{T}$. Class B SDMEs correspond to the interference of $T_{00}$ and $T_{11}$ amplitudes. The main terms for the unpolarized (polarized) SDMEs are proportional to the real (imaginary) part of $T_{00} T_{11}^{*}$. So called (un)polarized SDMEs correspond to (un)polarized photon spin density matrix [4] and are presented in (un)shaded areas of Fig. 1 In fact, as a general rule for the classes B to E, the dominant contribution of the unpolarized (polarized) SDMEs is proportional to the real (imaginary) part of the product of two amplitudes. Class $C$ contains SDMEs with dominant terms that are products of the $s$-channel helicity non-conserving amplitude $T_{01}$ (corresponding to the $\gamma_{T}^{*} \rightarrow V_{L}$ transition), and $T_{00}^{*}$ or $T_{11}^{*}$ (for $r_{00}^{1}$ the $T_{01}$ contribution is actually quadratic). Classes D and E are composed of SDMEs in which the main terms contain a product of the small helicity-flip amplitudes $T_{10}\left(\gamma_{L}^{*} \rightarrow V_{T}\right)$ and $T_{-11}$ $\left(\gamma_{T}^{*} \rightarrow V_{-T}\right)$, respectively, multiplied by $T_{11}^{*}$.

The SDMEs extracted in the kinematic region $1<Q^{2}<7 \mathrm{GeV}^{2}, 3<W<6.3 \mathrm{GeV}$, and $0<-t^{\prime}<0.4 \mathrm{GeV}^{2}$, are presented for $\rho^{0}$ meson data in Fig. 1. The average kinematic values are $\left\langle Q^{2}\right\rangle=1.9 \mathrm{GeV}^{2},\langle W\rangle=4.8 \mathrm{GeV}$ and $\left\langle-t^{\prime}\right\rangle=0.13 \mathrm{GeV}^{2}$. The experimental uncertainties are larger for the eight polarized SDMEs due to the imperfect lepton beam polarization (0.53), and the small kinematic factor $\sqrt{1-\varepsilon}$, with $\langle\varepsilon\rangle=0.8$, by which they are multiplied.

In Fig. 1, the SDMEs are shown multiplied by certain factors to make the coefficients of the dominant amplitude products equal to unity. The elements of class $A$ are presented in the figure in such a way that their main terms are proportional to $\left|T_{11}\right|^{2}$, in particular $1-r_{00}^{04}$ is chosen. The SDMEs of class A are similar for $\rho^{0}$. The elements of class B are also close to each other for both vector mesons. For $\rho^{0}$ mesons the elements of classes $\mathrm{C}, \mathrm{D}, \mathrm{E}$ with significantly non-zero


Figure 1: The 23 SDME's extracted for $\rho^{0}$ production on the proton (squares) and deuteron (circles) [4]. The inner error bars represent the statistical uncertainties, while the outer ones indicate the statistical and systematic uncertainties added in quadrature. The unshaded (shaded) areas indicate unpolarized (polarized) SDMEs. For easier interpretation the set of SDMEs was divided in five classes (see text).
values indicate that there exists also a production mechanism which does not conserve $s$-channel helicity. We note that a small violation of SCHC in $\rho^{0}$ production was observed by the H1 and ZEUS Collaborations [9, 10].

## 4. Ratios of Helicity Amplitudes

The SDMEs are the ratios of two sums of the bilinear products of the helicity amplitudes which are presented in Eqs. $(A .1 \div A .23)$ of Ref. [4]. Dividing both the numerators and the denominators of these equations by $T_{00}$, the formulas for the SDMEs expressed through the amplitude ratios were obtained. Putting these expressions for the SDMEs into Eqs. ( $37 \div 39$ ) of Ref. [4], the measured angular distributions were fitted considering the amplitude ratios as free complex parameters. According to the results of $\rho^{0}$ SDME analysis [4], the nucleon spin flip in the NPE amplitudes is neglected in our consideration and only $U_{11}$ amplitude from the UPE amplitudes is taken into ac-
count. Hence nine real free parameters $A_{j}$ are considered, namely the ratios: $\quad A_{1}=\operatorname{Re}\left\{T_{11} / T_{00}\right\}$, $A_{2}=\operatorname{Im}\left\{T_{11} / T_{00}\right\}, A_{3}=\operatorname{Re}\left\{T_{01} / T_{00}\right\}, A_{4}=\operatorname{Im}\left\{T_{01} / T_{00}\right\}, A_{5}=\operatorname{Re}\left\{T_{10} / T_{00}\right\}$, $A_{6}=\operatorname{Im}\left\{T_{10} / T_{00}\right\}, \quad A_{7}=\operatorname{Re}\left\{T_{1-1} / T_{00}\right\}, \quad A_{8}=\operatorname{Im}\left\{T_{1-1} / T_{00}\right\}, \quad A_{9}=\left|U_{11} / T_{00}\right|$, where $\left|U_{11}\right|^{2} \equiv\left|U_{1 \frac{1}{2} 1-\frac{1}{2}}\right|^{2}+\left|U_{1 \frac{1}{2} 1 \frac{1}{2}}\right|^{2}$. The short notations $T_{00} \equiv T_{0 \frac{1}{2} 0 \frac{1}{2}}, T_{11} \equiv T_{1 \frac{1}{2} \frac{1}{2}}$, etc. are used, see Ref. [4] for details.

The results of the fit to the amplitude ratios for spin conserving NPE and UPE amplitudes are presented in Figs. 2, 3 for the Hydrogen and Deuterium target respectively. The data are subdivided on four bins in $Q^{2}\left(0.5 \div 1 \div 1.4 \div 2 \div 7 \mathrm{GeV}^{2}\right)$ or four bins of $t^{\prime}\left(0 \div 0.04 \div 0.1 \div 0.2 \div 0.4 \mathrm{GeV}^{2}\right)$. The ratios $T_{11} / T_{00}$ and $\left|U_{11} / T_{00}\right|$ for the proton are compatible with those for the deuteron.

We note that the real part of the ratio $T_{11} / T_{00}$ follows the asymptotic behavior $\propto 1 / Q$ predicted theoretically [7,8] within the perturbative QCD framework, while the imaginary part of $T_{11} / T_{00}$ grows with $Q^{2}$ which contradicts with the high- $Q^{2}$ asymptotic in pQCD. This dependence is in full agreement with the published result from the analysis of SDMEs which are presented in Ref. [4] in terms of the phase difference $\delta$ between the amplitudes $T_{11}$ and $T_{00}$. In HERMES kinematic conditions, $\delta$ grows with $Q^{2}$ and has a mean value is about 30 degrees. For the first time, the $Q^{2}$ dependence of $\delta$ [4] is explained to be caused by the increase with $Q^{2}$ of the imaginary part of the ratio of $\rho^{0}$ leptoproduction amplitudes $T_{11} / T_{00}$. In addition, it contradicts the calculations [7, 8, 11] based on pQCD.

The $Q^{2}$ and $t^{\prime}$ dependence of the ratio $\left|U_{11} / T_{00}\right|$ is presented in Fig. 3. It shows that $\left|U_{11}\right|$ is only by a factor of approximately 2.5 smaller than $\left|T_{11}\right|$ and $\left|U_{11} / T_{00}\right|$ ratio does not depend on $Q^{2}$ and $t^{\prime}$. There is no explanation of the absence neither the $Q^{2}$ nor $t^{\prime}$ dependence of the ratio $\left|U_{11} / T_{00}\right|$, while the latter one is expected from the Regge phenomenology [12]. Signal of unnatural parity exchange, extracted from the helicity amplitude ratios, manifests itself with significance of more than $20 \sigma_{t o t}$ from the fit of $\left|U_{11} / T_{00}\right|$ ratio, while in the SDME approach existence of UPE was established with significance of $3 \sigma_{t o t}$ for the combined proton and deuteron data [4].

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Figure 2: The $Q^{2}$ dependence of the ratios of amplitudes $\operatorname{Re}\left\{T_{11} / T_{00}\right\}$ (left) and $\operatorname{Im}\left\{T_{11} / T_{00}\right\}$ (right). Red circles show data on proton and blue squares on deuteron. The inner error bars represent the statistical uncertainties, while the outer ones indicate the statistical and systematic uncertainties added in quadrature.


Figure 3: The $Q^{2}$ (left) and $t^{\prime}$ dependence (right) of the amplitude ratio $\left|U_{11} / T_{00}\right|$. Red circles show data on proton and blue squares on deuteron. The inner error bars represent the statistical uncertainties, while the outer ones indicate the statistical and systematic uncertainties added in quadrature.


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