

Automatic calculation of one-loop amplitudes

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An algorithm to automatically compute any one-loop amplitude for all momentum, color, and helicity configurations of the external particles is presented. It has been implemented using the tools HELAC for the evaluation of tree-level off-shell currents, CutTools for OPP-reduction and OneLoop for the evaluation of one-loop scalar integrals. It proves to be able to deal with at least all sub-processes included in the 2007 Les Houches wish list for LHC involving 3 or 4 particles in the final state.

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1. Introduction

In order to deal with the data from the experiments at LHC for the study of elementary particles, signals and potential backgrounds for new physics have to be under control at sufficient accuracy [1]. In particular, hard processes with high multiplicities, involving many particles or partons, cannot be neglected. On top of that, such processes have to be dealt with at the next-to-leading order (NLO) level to, for example, reduce the scale dependence of observables and to have a better description of the shape of their distributions.

At the leading order in perturbation theory, many tools are already available that are able to simulate any scattering process involving up to several partons [2]. These tools are highly automated and they have been widely used [3]. At the next-to-leading order the situation is currently less advanced. MCFM [4] is able to produce results at NLO accuracy for specific scattering processes, based on analytic calculations. Regarding the calculation of one-loop amplitudes, the only automatic tool available for some time now was `FeynCalc` [5] and `FormCalc` [6]. These tools rely heavily on the use of computer algebra. For processes with two particles in the final state, their performance is very satisfactory. There exist several important calculations that make use of these automatic packages and `FeynArts` [7], `QGRAF` [8], producing results with up to four particles in the final state [9], but for the moment no publicly available automatic tool exists. Recently a program called `GOLEM` [10] has been presented, that is able to deal with processes up to six external legs. It will also provide an alternative to compute automatically one-loop amplitudes [11].

The aforementioned programs express one-loop amplitudes in Feynman graphs, which again are expressed in terms of one-loop tensor integrals. These are then calculated using universal reduction techniques, independent of the amplitude, by expressing them in terms of scalar integrals. In a very different line of thinking, starting from the pioneering work in [12], a new approach has been set forward, known under the name of unitarity approach, which has been proven very powerful in computing multi-parton amplitudes in QCD that seemed to be impossible with the traditional Feynman graph approach. The reason is that one-loop amplitudes are calculated by using tree-order building blocks, that are either known analytically with very compact expressions, or can be evaluated using fast recursive equations. Nevertheless a systematic framework to develop a generic computation of any one-loop amplitude was missing, limiting the applicability of the method.

Using the crucial input from [13], this problem has been first solved in [14], introducing a systematic framework in order to calculate all coefficients of the scalar integrals, as well as part of the so-called rational contribution, related to the occurrence of UV-divergences. The remaining rational part can be reproduced by counter terms encoded in tree-like Feynman rules involving up to four fields [15]. Therefore, this method, known as the OPP method, provides a self-contained framework for the evaluation of the full one-loop amplitude. In [16], the OPP method was applied within the so-called generalized unitarity approach [13, 17] in order to get also the full rational contribution to the amplitude, paying the price to work with tree-amplitudes in higher dimensions.

The systematic extraction of all coefficients and of the rational term opened the road for the construction of tools that are able to compute one-loop amplitudes with any number of particles. `BlackHat` [18] and `Rocket` [19] were the first tools to realize such a possibility. In the following, we report on the development of a new program.

2. The program

For any one-loop amplitude, a number n of propagator denominators can be identified such that the amplitude can be represented in the form

$$\mathcal{A} = \sum_{I \in \{1, \dots, n\}} \int \frac{\mu^{4-d} d^d \bar{q}}{(2\pi)^d} \frac{\bar{N}_I(\bar{q})}{\prod_{i \in I} \bar{D}_i(\bar{q})}, \quad \bar{D}_i(\bar{q}) = (\bar{q} + p_i)^2 - m_i^2, \quad (2.1)$$

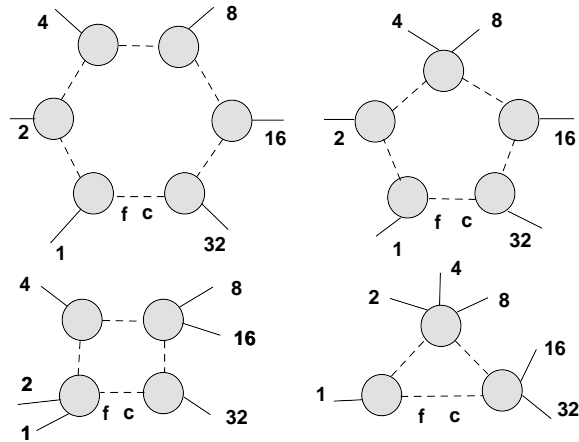
where the loop momentum \bar{q} and the numerator polynomials $\bar{N}_I(\bar{q})$ are considered to be evaluated in d dimensions. The p_i are combinations of the external momenta, and the m_i are the masses of particles running a loop. It is well-known that for $d \rightarrow 4$, each such term can be cast into the form

$$\int \frac{\mu^{4-d} d^d \bar{q}}{(2\pi)^d} \frac{\bar{N}_I(\bar{q})}{\prod_{i \in I} \bar{D}_i(\bar{q})} = \sum_j d_j \text{Box}_j + \sum_j c_j \text{Tri}_j + \sum_j b_j \text{Bub}_j + \sum_j a_j \text{Tad}_j + R, \quad (2.2)$$

where Box, Tri, Bub and Tad refer to the well known one-loop scalar integrals and $R = R_1 + R_2$ is the rational term. Given the momenta p_i , the masses m_i and a function evaluating the 4-dimensional numerator N_I as function of q , the program `CutTools` [20] identifies the scalar integrals to be evaluated, calculates the coefficients, and determines R_1 following the OPP method. The scalar integrals can be evaluated with the tools from [21] or with our own code `OneLoop`.

For the evaluation of the numerator, the program `HELAC` [22] is used. It efficiently calculates tree-level amplitudes by applying recursive relations for off-shell currents [23]. The convenience of its applicability stems from the freedom in the choice of the initial decomposition represented by the summation in Eq. (2.1). In this application it is a sum over all topologically inequivalent partitions of the external particles.

A few examples are depicted on the right, each graph corresponds to an instance of the label I in Eq. (2.1). The external particles are labelled by powers of 2, and the blobs do not contain propagators depending on the integration momentum q . `HELAC` can easily evaluate such a term by considering one line to be cut, and summing over all possible internal spin, flavor and/or color states of that cut line. Each term in this sum is an amplitude with two more external particles, restricted such that it only



contains Feynman graphs including all the propagators eventually forming the loop. This can easily be accomplished by putting together the necessary off-shell currents provided by `HELAC`.

Regarding the color-treatment, `HELAC` uses the color-connection representation. External quarks and anti-quarks are represented by a color or anti-color index, gluons are represented by a pair of color/anti-color indices, and `HELAC` calculates all non-zero tree-level color-connections. The same is done to for the amplitudes with two more particles necessary to calculate the one-loop numerators. The contributions from non-planar loops are obtained by taking into account different orderings of these extra particles among the other external particles.

The evaluation of the rational part R_2 , finally, follows the same line as the calculation of the counter terms necessary for renormalization, and is comparably trivial.

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