

The Exclusive NLO DGLAP Kernels for Non-Singlet Evolution *

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We show for a first time ever a prototype of a fully exclusive QCD NLO parton shower for the initial state (albeit for a limited set of diagrams). It is based on the rigorous theorems of the collinear factorisation, however the standard DGLAP evolution kernels are replaced by their exclusive versions. Contrary to the standard DGLAP approach, the constructed parton shower provides fully exclusive events, i.e. four-momenta of all emitted partons. At the inclusive level it is identical to the \overline{MS} scheme of the DGLAP evolution.

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With the approach of the first data from the LHC experiments the issue of the precision of the Monte Carlo (MC) simulations of the perturbative QCD becomes more and more important. In the domain of the semi-inclusive, analytical or semi-analytical calculations the results are spectacular: the NLO accuracy is a standard and numerous quantities are calculated to much higher precision, just to mention the DGLAP kernels known to NNLO level [1]. On the contrary, all the currently available MC Parton Shower (PS) codes are of the LO or, at most, of the improved LO type (as far as the differential distributions are concerned, the overall, inclusive, normalization can easily be corrected to higher order precision), based on the methods developed in early 1980-s, cf. for example [2, 3, 4].

It is therefore justified to ask whether it is possible to develop a new, more precise scheme for the QCD MC parton shower? In this presentation we will demonstrate that yes, it is feasible. We will describe a working prototype of such a genuine NLO parton shower for the initial state QCD which we have developed recently within the KRK_{MC} project. Note that there are some other attempts to construct parton shower schemes beyond leading order as well, cf. eg. [5, 6, 7].

The KRK_{MC} parton shower fulfills the following list of requirements:

- (A) it is based firmly on Feynman Diagrams (Matrix Element) and Lorentz-invariant phase space (LIPS);
- (B) it is based rigorously on the collinear factorization [8, 9];
- (C) it implements *exactly* the NLO \overline{MS} DGLAP evolution [10];
- (D) it implements fully unintegrated parton density functions (PDFs); with NLO evolution done by MC itself, using new, exclusive NLO kernels.

The combination of the collinear factorization (item 1), which is the best proven factorization scheme, with the complete kinematical information of the generated partons (item 4), i.e. full exclusiveness of the shower is the essence of the novelty of our approach. Let us now highlight some of the important points in this new KRK_{MC} scheme. For more details we refer to Refs. [11, 12, 13].

Step 1. Re-do the calculation of the NLO kernels, based on the framework of Curci, Furmanski and Petronzio [10] but in the exclusive way. Use various types of evolution time (virtuality, k_{\perp} , rapidity . . .). The standard DGLAP kernels do not depend on the choice of the evolution time. On the contrary, the exclusive ones depend. Use \overline{MS} factorization scheme. In the Fig. 1 we show all the contributions to the non-singlet LO and NLO evolution kernels. In this work we restrict ourselves to the C_F^2 part (two “bremsstrahlung-like” graphs) labeled with the blue box. It is in a sense the most complicated part because it involves the subtraction of the soft counter-term (labeled as $1 - P$ in the plot).

Step 2. Remove the dimensional regularization, go back to four-dimensions. Use instead a geometrical cut-off Δ for the collinear singularity. But, all the time stay within the \overline{MS} scheme. This is possible because the collinear pole comes from a simple, factorisable integral of the form

$$\frac{1}{\varepsilon} \Rightarrow \int_0^{Q^2} d\left(\frac{q^2}{Q^2}\right) \left(\frac{Q^2}{q^2}\right)^{1-\varepsilon} \Rightarrow \int_{\Delta^2}^{Q^2} d\left(\frac{q^2}{Q^2}\right) \frac{Q^2}{q^2}.$$

Step 3. Re-formulate the factorization formula, because the DGLAP equation mixes orders of perturbative expansion. The NLO DGLAP kernel, denoted as P , is in fact a sum of LO and NLO pieces. Therefore P^k terms are a mixture of various orders:

$$P = \alpha P^{LO} + \alpha^2 P^{NLO} \Rightarrow P^2 = \alpha^2 (P^{LO})^2 + \alpha^3 (P^{LO} P^{NLO} + P^{NLO} P^{LO}) + \alpha^4 (P^{NLO})^2.$$

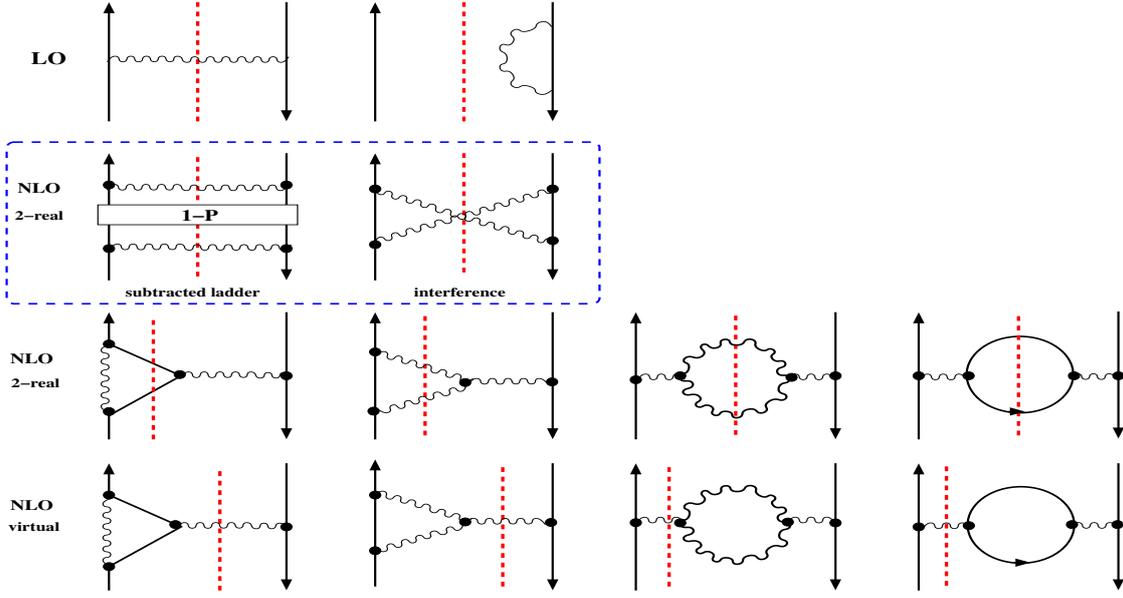


Figure 1: Contributions to the NLO non-singlet evolution kernel. The C_F^2 part label-led with the blue box.

Such a set-up, with partial, incomplete higher orders, may lead for example to the negative events in the MC and should be avoided.

Step 4. Resign from the ordering in the evolution time in the underlying LO crude MC. Use the Bose-symmetric form instead:

$$\int_{t_{\min}}^{t_{\max}} \prod_i^N dt_i \theta_{t_i > t_{i-1}} \Rightarrow \frac{1}{N!} \int_{t_{\min}}^{t_{\max}} \prod_i^N dt_i. \quad (1)$$

The ordering is an approximate feature of the LO matrix element and therefore it is not strict at the NLO level and we have to explicitly sum over entire phase space.

Step 5. Construct appropriate MC weight with the NLO exclusive kernel. It will be then applied on the top of the standard LO MC.

Step 6. Resolve the mismatch of the lower limits of the “internal NLO phase space”, i.e. of the internal degree of freedom integrated out in the construction of the inclusive kernel. In the analytical calculation of the \overline{MS} kernel the lower limit of dt_{internal} integration is set to 0, whereas in MC it is limited by some t_{\min} for all partons. In this work we resolve this conflict by “artificial” lowering of the t_{\min} limit, below the actual start of the evolution and by performing a LO pre-evolution in this extended region. This way we maintain exact agreement with the \overline{MS} result.

We have implemented the above scheme of the exclusive NLO evolution in the MC program KRKMC. In the Fig. 2 we compare its results with a standard NLO DGLAP MC evolution. Both evolutions are implemented as weights on the top of the same LO Markovian MC algorithm. The curves shown correspond to the contributions with one and two NLO “insertions”. Evolution ranges from 10 GeV to 1 TeV. LO pre-evolution starts at 1GeV from $\delta(1-x)$ distribution. As one can see the agreement is very good, within the statistical errors. This way we demonstrated for the

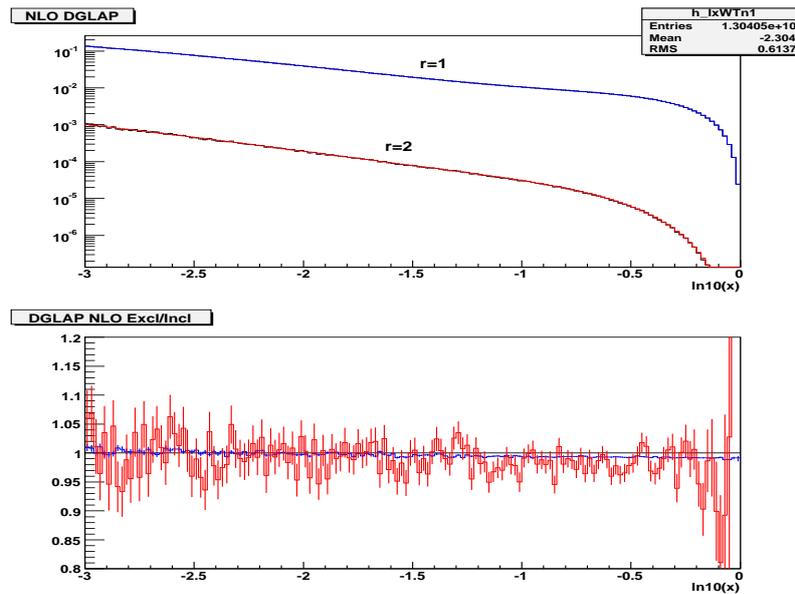


Figure 2: Comparison of exclusive and standard DGLAP NLO evolutions

first time ever that the QCD NLO parton shower can be constructed (although for a limited set of Feynman diagrams)!

We are currently in the process of adding the rest of graphs from Fig. 1, omitted in this work. This way the non-singlet evolution will be completed. Once also the singlet evolution is added, we will be ready to construct the complete event generator for the Drell-Yan-type processes at LHC or DIS at HERA.

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