

# Higher-order QCD corrections to vector boson production at hadron colliders.

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We present two recent results on higher-order QCD corrections to the production of vector bosons in hadron collisions.

We discuss the resummation of logarithmic-enhanced QCD corrections at small values of  $q_T$  and the matching procedure to consistently combine resummation with the fixed-order perturbative result at intermediate and large  $q_T$ . We study the perturbative uncertainty of the results and we compare our prediction with Tevatron data for Z bosons production.

Moreover we discuss a fully exclusive calculation up to next-to-next-to-leading order (NNLO) in QCD perturbation theory. The calculation is implemented in a parton level Monte Carlo program which allows the user to apply arbitrary kinematical cuts on the final-states and to compute the corresponding distributions in the form of bin histograms.

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#### 1. Transverse-momentum resummation

We are interested in the high-energy collisions of the hadrons  $h_1$  and  $h_2$  which produce a vector boson V (which decays into the lepton pair  $l_1, l_2$ ) plus an arbitrary and undetected final state X

$$h_1 + h_2 \to V(M, q_T) + X \to l_1 + l_2 + X,$$
 (1.1)

where  $q_T$  and M are respectively the transverse momentum and the invariant mass of the vector boson.

We consider the transverse-momentum distribution and we identify two different kinematical regions. In the region where  $q_T \sim m_V$ ,  $m_V$  being the mass of the vector boson ( $m_V = m_W, m_Z$ ), the QCD perturbative series is controlled by a small expansion parameter,  $\alpha_S(m_V)$ . In this region the fixed-order QCD calculations, known up to next-to-leading order (i.e.  $\mathcal{O}(\alpha_S^2)$ ) [1], are theoretically justified. In the small- $q_T$  region ( $q_T \ll m_V$ ), the convergence of the fixed-order perturbative expansion is spoiled by the presence of powers of large logarithmic terms,  $\alpha_S^n \ln^m (m_V^2/q_T^2)$ . In order to obtain reliable predictions in such region an all order resummation of these terms is mandatory.

The  $q_T$  resummation is performed at the level of the partonic cross section, which is decomposed in two terms:  $d\hat{\sigma}^V/dq_T^2 = d\hat{\sigma}^{V(\text{res.})}/dq_T^2 + d\hat{\sigma}^{V(\text{fin.})}/dq_T^2$  [2, 3]. The term  $d\hat{\sigma}^{V(\text{res.})}$  contains all the logarithmically enhanced contributions (at small  $q_T$ ) we have to resum while the term  $d\hat{\sigma}^{V(\text{fin.})}$  is free of such contributions and can be evaluated at fixed order in perturbation theory.

The resummation procedure is performed in the impact-parameter space through a Fourier-Bessel transform

$$\frac{d\hat{\sigma}^{V(\text{res.})}}{dq_T^2}(q_T, M, \hat{s}, \alpha_S) = \hat{\sigma}_{LO}^V(M) \frac{M^2}{\hat{s}} \int_0^\infty db \, \frac{b}{2} \, J_0(bq_T) \, \mathscr{W}^V(b, M, \hat{s}, \alpha_S) \,, \tag{1.2}$$

where the impact parameter *b* is the conjugate variable with respect to  $q_T$ ,  $J_0(x)$  is the 0-order Bessel function and  $\hat{\sigma}_{LO}^V$  is the Born partonic cross section. We can now write the partonic resummed component  $\mathscr{W}^V(b, M, \hat{s}, \alpha_S)$  in the exponential form by considering its *N*-moments with respect to the variable  $z = M^2/\hat{s}$  at fixed *M* 

$$\mathscr{W}_{N}^{V}(b,M,\alpha_{S}) = \mathscr{H}_{N}^{V}(\alpha_{S}) \times \exp\{\mathscr{G}_{N}(\alpha_{S},L)\}, \text{ with } L = \ln(Q^{2}b^{2}/b_{0}^{2}), b_{0} = 2e^{-\gamma_{E}}.$$
(1.3)

We have introduced in the above formula the scale  $Q \sim M \sim m_V$ , the so called resummation scale, which has a role analogous to the factorization and renormalization scales: variations of Q around  $m_V$  can be used to estimate the size of higher-order logarithmic contributions that are not explicitly resummed in a given calculation.

The process dependent function  $\mathscr{H}_N^V$  includes all the perturbative terms that behave as constants as  $q_T \to 0$ . It can thus be expanded in powers of  $\alpha_s = \alpha_s(\mu_R^2)$ :

$$\mathscr{H}_{N}^{V}(\alpha_{S}) = \left[1 + \frac{\alpha_{S}}{\pi} \mathscr{H}_{N}^{V(1)} \left(\frac{\alpha_{S}}{\pi}\right)^{2} \mathscr{H}_{N}^{V(2)} + \dots\right].$$
(1.4)

The universal exponent  $\mathscr{G}_N$  resums all the terms that order-by-order in  $\alpha_S$  are logarithmically divergent.

Finally the finite component has to be evaluated starting from the usual fixed-order perturbative truncation of the partonic cross section and subtracting the expansion of the resummed part at the

same perturbative order:  $[d\hat{\sigma}^{V(\text{fin.})}/dq_T^2]_{f.o.} = [d\hat{\sigma}^V/dq_T^2]_{f.o.} - [d\hat{\sigma}^{V(\text{res.})}/dq_T^2]_{f.o.}$ . This matching procedure is important to achieve uniform theoretical accuracy over the entire range of transverse momenta.

To perform a resummation at next-to-next-to-leading logarithmic order, the knowledge of the coefficient  $\mathscr{H}_N^{V(2)}$  is necessary. Since this coefficient has been computed only recently [4], here we limit ourselves to presenting results up to next-to-leading logarithmic accuracy matched with the leading fixed-order result (NLL+LO).

In Fig. 1 we compare our NLL+LO resummed spectrum [3] (with different values of the factorization, renormalization and resummation scale) with the Tevatron DO Run II data [5]. We find that the scale uncertainty is about  $\pm 12 - 15\%$  from the region of the peak up to the intermediate  $q_T$  region ( $q_T \sim 20$  GeV), and it is dominated by the resummation-scale uncertainty. Taking into account the scale uncertainty, we see that the resummed curve agrees reasonably well with the experimental points <sup>1</sup>. We expect a sensible reduction of the scale dependence once the complete NNLL+NLO calculation is available.



**Figure 1:** The  $q_T$ -spectrum of the Drell-Yan  $e^+e^-$  pairs produced in  $p\bar{p}$  collisions at the Tevatron Run II [5]. Theoretical results are shown at NLL+LO, including scale variations. Left side:  $m_Z/2 \le \mu_F, \mu_R \le 2 m_Z$ , with the constraint  $1/2 \le \mu_F/\mu_R \le 2$ . Right side:  $m_Z/4 \le Q \le m_Z$ 

### 2. Fully exclusive NNLO calculation

We now consider a generic observable  $d\hat{\sigma}^V$  for the process in Eq. 1.1. We present a computation of the next-to-next-to-leading order (NNLO) QCD radiative corrections for such observable with arbitrary (though infrared safe) kinematical cuts on the final-state [4]. Provided the observable is sufficiently inclusive over the small- $q_T$  region, resummation is not necessary and fixed-order perturbation theory can be used.

Following Ref. [6], we observe that, at LO, the transverse momentum  $q_T$  of V is exactly zero. This means that if  $q_T \neq 0$  the (N)NLO contributions is given by the (N)LO contribution to the final state V + jet(s):  $d\hat{\sigma}_{(N)NLO}^V|_{q_T\neq 0} = d\hat{\sigma}_{(N)LO}^{V+jets}$ . We compute  $d\hat{\sigma}_{NLO}^{V+jets}$  by using the subtraction

<sup>&</sup>lt;sup>1</sup>We note that in Fig. 1 the theoretical results are obtained in a pure perturbative framework, without introducing any models of non-perturbative contributions. These contributions can be relevant in the  $q_T$  region below the peak.

method at NLO and we treat the remaining NNLO singularities at  $q_T = 0$  by the additional subtraction of a counter-term constructed by exploiting the universality of the logarithmically-enhanced contributions to the  $q_T$  distribution (see Eq. 1.3)

$$d\hat{\sigma}_{(N)NLO}^{V} = \mathscr{H}_{(N)NLO}^{V} \otimes d\hat{\sigma}_{LO}^{V} + \left[ d\hat{\sigma}_{(N)LO}^{V+\text{jets}} - d\hat{\sigma}_{(N)LO}^{CT} \right] , \qquad (2.1)$$

where  $\mathscr{H}^{V}_{(N)NLO}$  is the process dependent coefficient function of Eq. 1.4.

We have encoded our NNLO computation in a parton level Monte Carlo event generator. The calculation includes finite-width effects, the  $\gamma - Z$  interference, the leptonic decay of the vector bosons and the corresponding spin correlations<sup>2</sup>. Our numerical code is particularly suitable for the computation of distributions in the form of bin histograms, as shown the illustrative numerical results presented in Fig. 2.



**Figure 2:** Left side: transverse mass distribution for *W* production at the Tevatron. Right side: distributions in  $p_{T min}$  and  $p_{T max}$  for the *Z* signal at the Tevatron.

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<sup>&</sup>lt;sup>2</sup>In the quantitative studies that we have carried out, our computation gives results in numerical agreement with the calculation, performed with a different method, presented in Ref. [7].