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Top Quark Production at Hadron Colliders: an Overview

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The study of the properties of the top quark is one of the main goals of the Large Hadron Collider (LHC) physics program. The experimental precision expected at the LHC requires the calculation of several top-quark related observables beyond leading order in the strong coupling constant. In this work we briefly review the status of the theoretical predictions for the top-quark production processes at hadron colliders. Special attention is devoted to recent progress in the calculation of next-to-next-to-leading-order corrections to the top-quark pair production cross section.

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1. Introduction

With a mass $m_t = 173.1 \pm 1.3$ GeV, the top quark is the heaviest elementary[1] particle produced at colliders. Due to their very large mass, top quarks have the unique property of decaying through weak interactions before they can form bound states. For the same reason, top quarks should couple strongly with the particles responsible for the electroweak symmetry breaking and they are expected to play a key role in the search for the Higgs boson and in investigating the origin of particle masses. To date the properties and quantum numbers of the top quark could be experimentally studied only at the Tevatron, where a few thousand top quarks were produced since the particle discovery in 1995. The situation will change drastically when the LHC will start operations: the new proton-proton collider is expected to produce millions of top quarks per year already in the first low luminosity phase ($\mathscr{L} \sim 10\,\text{fb}^{-1}/\text{year}$). The large numbers of events involving top quarks at the LHC will allow precise measurements of their properties and production cross sections. Precise measurements demand for equally precise theoretical predictions of the measured observables in the Standard Model (SM) and its extensions. Precise theoretical predictions are obtained by pushing the calculation of the physical observables to the next-to-leading-order (NLO) or, if necessary, even to the next-to-next-to-leading-order (NNLO) in perturbation theory, as well as by applying resummation techniques in specific phase-space regions. Top quarks appear in processes studied at colliders as virtual particles. However, due to the small ratio between the top-quark width and mass, it is possible to factor the cross section of processes involving top quarks into the product of the production cross section for on-shell top quarks and the top-quark decay. The purpose of this short write-up is to review the status of the theoretical predictions for the top-quark production cross sections relevant at the LHC. We will focus primarily on inclusive cross sections. For a comprehensive (and excellent) review of many other aspects of top-quark phenomenology, see [1].

2. Top-Quark Pair Production

At hadron colliders top quarks are primarily produced in pairs with their antiparticle. In about $5 \cdot 10^{-25}$ seconds the top (antitop) quark decays in a W-boson and a b-quark ($\bar{b}$-quark). The W-boson can either decay leptonically or give origin to a pair of light-quark jets.

The production of an on-shell top-antitop pair is dominated by strong interactions. The inclusive top-quark pair production cross section can be written as

$$\sigma^{t\bar{t}}_{h_1,h_2}(s_{\text{had}}, m_t^2) = \sum_{ij} \int_{4m_t^2}^{s_{\text{had}}} d\hat{s}\; \underbrace{L_{ij}\left(\hat{s}, s_{\text{had}}, \mu_f^2\right)}_{\text{partonic luminosity}} \; \overbrace{\hat{\sigma}_{ij}(\hat{s}, m_t^2, \mu_f^2, \mu_r^2)}^{\text{partonic cross section}}\,, \tag{2.1}$$

where the hard scattering of the partons i and j ($i, j \in \{q, \bar{q}, g\}$) at a partonic center of mass (c. m.) energy $\hat{s}$ is described by the partonic cross section, which can be calculated in perturbative QCD. The process independent partonic luminosity describes the probability of finding, in the hadrons h_1 and h_2 ($h_1, h_2 = p, \bar{p}$ at the Tevatron, $h_1, h_2 = p, p$ at the LHC), an initial state involving partons i and j with the given partonic energy $\hat{s}$. The integration extends up to the hadronic c. m. energy s_{had}

[1] at least according to the Standard Model (SM) and its most popular extensions, such as SUSY models.

($s_{\text{had}} = 1.96$ TeV at the Tevatron, $s_{\text{had}} = 14$ TeV at the LHC). The renormalization and factorization scales, μ_f and μ_r, are usually set to be equal.

There are two production channels contributing to the partonic cross section at the tree level: the quark-antiquark channel $q\bar{q} \to t\bar{t}$ and the gluon fusion channel $gg \to t\bar{t}$. Because of the interplay between parton luminosity and partonic cross sections, the quark-antiquark channel dominates at the Tevatron, where it gives origin to $\sim 85\%$ of the inclusive cross section. On the contrary, at the LHC the inclusive cross section is largely dominated by gluon fusion events, which contribute $\sim 90\%$ of the total cross section. NLO QCD corrections to both channels have been known for two decades [2] and were recently obtained in analytic form [3]; these corrections are very large (see Fig. 1). Mixed QCD-EW corrections are also available [4], but they have a negligible impact on the inclusive cross section. NLO QCD effects involve logarithmic terms which become numerically sizable near the production threshold. Up to date theoretical predictions for the top-quark pair-production cross section include effects originating from the resummation of these logarithms (see Section 2.1), which are particularly relevant at the Tevatron, where the bulk of the inclusive cross section arises from events with a partonic c. m. energy close to the production threshold $\sqrt{\hat{s}} \sim 2m_t$. The inclusive cross section is sensitive to the top-quark mass ($\Delta\sigma^{t\bar{t}}/\sigma^{t\bar{t}} \sim -5\Delta m_t/m_t$). Current theoretical predictions (including NLO corrections and next-to-leading-logarithm resummation) indicate a cross section $\sigma^{t\bar{t}} \sim 7.6$ pb with a relative uncertainty of $\sim 11\%$ at the Tevatron and $\sigma^{t\bar{t}} \sim 908$ pb with a relative uncertainty of $\sim 13\%$ at the LHC, for $m_t \sim 171$ GeV [5]. These results are obtained using the CTEQ6.5 set of PDFs [6]. Using the set MRST2006nnlo [7], a sizable difference is observed. Calculations including approximated NNLO corrections [8, 9] indicate smaller uncertainties. At the Tevatron, as at the LHC, the theoretical uncertainty is dominated by unknown higher order corrections, whose magnitude is estimated through the residual scale dependence of the cross section. The uncertainty affecting the parton luminosity is the other source of the theoretical uncertainty. The theoretical uncertainty should be compared with the expected accuracy of the experimental measurements at the LHC, where the inclusive top-quark pair production cross section is likely to be measured with a relative error of $5-10\%$ [1]. Such precise measurements demand for equally precise cross-section predictions, which can be achieved only by including the effects of the NNLO QCD corrections (see Section 2.2).

The sensitivity of the inclusive cross section to the top-quark mass allows one to obtain from the cross-section measurement a numerical value for a well-defined short distance mass, such as the running $\overline{\text{MS}}$ mass. (This is not possible with the current mass measurements based on kinematic reconstruction of the events.) In [10], by employing the approximated NNLO corrections of [8, 10] and the cross-section measurement $\sigma^{t\bar{t}}_{\text{EXP}} = 8.18^{+0.98}_{-0.87}$ pb [11], the $\overline{\text{MS}}$ mass of the top quark was calculated to be equal to $\overline{m}_t(m_t) = 160.0^{+3.3}_{-3.2}$ GeV.

2.1 Soft-Gluon Resummation

The calculation of any physical observable in perturbative QCD is naively organized in terms of a series of increasing powers of the coupling constant α_S. Formally, the nth-order corrections are suppressed with respect to the corrections of order $n-1$ by a power of α_S and, therefore, they are sub-leading. However, in particular regions of the phase space this classification can fail, and corrections of higher order in α_S can be of the same numerical size as the leading ones.

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LHC ($s_{\text{had}} = 14$ TeV)	σ^{tt}	σ^{t} $t-$channel	σ^{t} $s-$channel	σ^{t} associated tW
NLO QCD	$\sim +50\%$ [2]	$\sim +5\%$ [26]	$\sim +44\%$ [28]	$\sim +10\%$ [29]
EW	$\sim -0.5\%$ [4]	$< 1\%$ [27]		
MSSM	up to $\pm 5\%$ [31]	$< 1\%$ [27]		

Tevatron ($s_{\text{had}} = 1.96$ TeV)	σ^{tt}	σ^{t} $t-$channel	σ^{t} $s-$channel
NLO QCD	$\sim +25\%$ [2]	$\sim +9\%$ [26]	$\sim +47\%$ [28]
EW	$\sim -1\%$ [4]	$< 1\%$ [27]	
MSSM	up to $\pm 5\%$ [31]	$< 1\%$ [27]	

Figure 1: Approximate size of the NLO QCD and MSSM corrections to pair and single-top production cross sections with respect to the Born cross sections. The size of the electroweak (EW) corrections is expressed in % of the NLO QCD cross sections.

The QCD corrections to processes involving at least two large energy scales are characterized by a logarithmic behavior in the vicinity of the boundary of the phase space. This is precisely the case of $t\bar{t}$ production, where $\hat{s}$ and m_t^2 are such that $\hat{s}, m_t^2 \gg \Lambda_{QCD}^2$. Let ρ be the inelasticity variable of a certain process and let us consider the quasi-elastic limit in which $\rho \to 1$. The physical observable, say σ, will exhibit the following logarithmic behavior: $\sigma \sim \sum_{n,m} C_{n,m} \alpha_S^n \ln^m (1-\rho)$ with $m \leq 2n$. These logarithms come from the integration over the phase space of the plus distributions originating from the cancellation of the IR singularities in the sum of virtual and real corrections in inclusive observables. They are, therefore, a reminder of the IR divergences. Although the physical observable is formally finite, in the limit $\rho \to 1$ the virtual and real radiation are unbalanced, due to the phase space restrictions, and the terms $\alpha_S^n \ln^m (1-\rho)$ can become large, even in the perturbative regime, where $\alpha_S \ll 1$. The logarithmic terms spoil the convergence of the perturbative series and they must be resummed to all orders. The resummation can be carried out by using the general factorization properties of QCD and the behavior of the process-dependent phase space in the vicinity of the elastic region, in order to re-express the multi-gluon amplitude in terms of single soft gluon emissions. This allows an exponentiation of the logarithmic terms. The exponentiation is usually not possible in the ρ space, but it takes place in a conjugate space, for instance the Mellin space of N moments, where N is the variable conjugate to ρ. It can be proved that, in the conjugate space, the partonic cross section can be written using the generalized resummation formula [12]:

$$\sigma_N^{res} \sim \exp\left\{\ln N g_1(\alpha_S \ln N) + g_2\left(\alpha_S \ln N, Q^2/\mu^2\right) + \alpha_S g_3\left(\alpha_S \ln N, Q^2/\mu^2\right) + \ldots\right\}. \qquad (2.2)$$

In Eq. (2.2) the function g_1 resums the *leading logarithms* (LL). The function g_2, which is formally suppressed by a power of $\ln N$ with respect to g_1, resums the *next-to-leading logarithms* (NLL), g_3 the NNLL ones, and so on. The resummation procedure outlined above applies to the production of a heavy-quark pair near the production threshold ($\rho = 4m_t^2/\hat{s}$) [13, 14]. In this case, the soft radiation is emitted by both the initial and the final state, and color correlations invalidate a simple c-number exponentiation of the form shown in Eq. (2.2). However, in the threshold limit, it can be shown that, at the NLL level, the final $t\bar{t}$ state acts, with respect to the soft radiation, as a single colored particle, either in the singlet or in the octet state. Consequently, one can recover a c-number exponentiation as in Eq. (2.2) independently for the singlet and octet states. Recent studies considered the extension of the resummation for the $t\bar{t}$ production cross section at the NNLL level

[8, 15]. The resummed cross section has a smaller dependence on the renormalization/factorization scale in the vicinity of the elastic region. In the case of $t\bar{t}$ production at LHC, the central value of the NLO+NLL cross section increases with respect to the pure NLO result by about +4%, while the dependence on the factorization/renormalization scale variation is reduced by a couple of percents. At the Tevatron, while the central value of the cross section is increased by the same percentage as at the LHC, the dependence on the factorization/renormalization scale of the NLO+NLL cross section is roughly half of the one found in a pure NLO calculation.

2.2 NNLO Corrections

The NNLO QCD corrections to the partonic pair-production cross section can be sorted in different sets as follows: *Virtual corrections*, which include *i-a)* two-loop corrections to the processes $q\bar{q} \to t\bar{t}$ and $gg \to t\bar{t}$, to be interfered with the tree-level amplitude, and *i-b)* one-loop diagrams for the quark-antiquark and gluon fusion channels, to be interfered among themselves; *Real corrections*, including *ii-a)* one-loop diagrams with one extra parton in the final state, to be interfered with the corresponding tree-level amplitude, and *ii-b)* tree-level diagrams with a top-quark pair plus two extra partons in the final state. The Feynman diagrams belonging to the sets *ii-a)* and *ii-b)* were evaluated in the context of the calculation of the NLO corrections for the production of a top-quark pair plus one jet [16]. The implementation of these results in the inclusive calculation of the $t\bar{t}$ production cross section will require an extension of the available subtraction methods at the NNLO. The calculation of the interference of one-loop graphs (set *i-b)*) was completed last year [17]. The two-loop corrections belonging to the set *i-a)* are known in the limit in which the top-quark mass is considered much smaller that the partonic c. m. energy [18]. The latter results, which were obtained by employing the factorization theorem proposed in [19], are not sufficient for phenomenological applications, since events with a partonic c. m. energy close to the production threshold make up a sizable fraction of the inclusive pair-production cross section. In the quark-antiquark channel, the complete set of two-loop corrections was calculated numerically in [20] for arbitrary values of the Mandelstam invariants and of the top-quark mass. The technique employed was based on the reduction of the squared amplitude to Master Integrals (MIs) by means of the Laporta algorithm, followed by a numerical solution of the differential equations satisfied by the MIs (see references in [21, 22]). All the two-loop diagrams involving a closed massive or massless quark loop in the $q\bar{q}$ channel were evaluated analytically in [21]. The calculation was carried out by employing the Laporta algorithm and by solving analytically the differential equations satisfied by the MIs. With the same technique it was possible to calculate the leading color coefficient in the interference of the two-loop corrections to $q\bar{q} \to t\bar{t}$ with the tree-level amplitude [22].

Exact results for the two-loop corrections to the gluon fusion channel are not yet available. However, the analytic expression of the infrared (IR) poles in the interference of the two-loop gluon-fusion diagrams with the tree-level amplitude is known [23]. These poles were calculated by employing the expression for the IR poles for a generic two-loop amplitude in massive QCD derived in [24]. For what concerns the finite parts, it seems possible to calculate analytically part of the two-loop diagrams in the gluon fusion channel by employing the same techniques applied to the calculation of the two-loop corrections to the quark-antiquark channel. However, it is known that several two-loop diagrams (for example some box diagrams involving a closed heavy quark

cross section	t-channel (pb)	s-channel (pb)	tW mode (pb)
$\sigma^t_{\text{Tevatron}}$	1.15 ± 0.07	0.54 ± 0.04	0.14 ± 0.03
σ^t_{LHC}	150 ± 6	7.8 ± 0.7	44 ± 5
$\sigma^{\bar{t}}_{\text{LHC}}$	92 ± 4	4.3 ± 0.3	44 ± 5

Figure 2: Predictions for the single top and antitop production cross sections at the Tevatron and at the LHC for $m_t = 171.4 \pm 2.1$ GeV [25].

loop) cannot be expressed in terms of generalization of harmonic polylogarithms. For this kind of diagrams, a numerical approach could be unavoidable.

3. Single Top Quark Production

Single top quarks can be produced by flavor-changing weak interactions. Like the pair-production cross section, the single-top production cross section can be factored in the convolution of a hard partonic process and a parton luminosity. In the SM there are three main production channels relevant at the Tevatron and at the LHC: *i)* the process $q(\overline{q})b \to q'(\overline{q}')t$ in which a W boson is exchanged in the t-channel, *ii)* the mode $q\overline{q} \to \overline{b}t$, in which a W-boson is exchanged in the s-channel, and *iii)* the process $bg \to W^-t$, referred to as tW associated production channel. The tree-level SM cross sections for the three channels are all proportional to the CKM matrix element $|V_{tb}|^2$. The single-top production cross section are large enough to allow for millions of single top events at the LHC. However, precise measurements are made difficult by the large backgrounds. Tevatron experiments recently reported evidence for single top production [30]. At the LHC it should be possible to measure separately the cross section in the three different channels. The predicted value for the single-top production cross section in the three channels at the Tevatron and at the LHC are shown in Fig. 2 [25, 1]. These predictions include NLO QCD corrections [26, 28, 29]. At the LHC, the t-channel cross section is expected to be measured with a relative error of 10%, while the other two channels will be measured with larger uncertainties. It is therefore possible to conclude that the theoretical uncertainty on the single-top production cross sections is currently under control.

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