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Padé approximations and non-singlet structure function up to N^3LO

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We apply a method of estimating perturbative coefficients in Quantum Field Theory using Padé approximations. In our QCD analysis we have performed this method to determine 4-loop anomalous dimension and 3-loop Wilson coefficients and found that the method works very well. By using Padé approximations, the results of our non-singlet QCD analysis for the experimental data of the deep-inelastic neutrino-nucleon scattering up to N^3LO have been calculated. The analysis is based on the associated Jacobi polynomials technique of reconstruction of the structure functions from its Mellin moments. Our results of parton densities $xu_v(x,Q^2)$ and $xd_v(x,Q^2)$, Λ_{QCD} and $\alpha_s(M_z^2)$ have been presented.

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1. Introduction

Presently the next-to-leading order is the standard approximation for most important processes but the N^{2}LO and N^{3}LO corrections need to be included, however, in order to arrive at quantitatively reliable predictions of DIS and hard hadronic scattering processes at present and future high-energy colliders. Everybody interested in more precise quantitative tests of QCD would welcome a more *precise* determination of the parton densities so perturbative QCD corrections beyond the next-to-leading order, N^{2}LO and N^{3}LO, need to be taken into account. For at least the next ten years, proton (anti-) proton colliders will continue to form the high-energy frontier in particle physics. At such machines, many quantitative studies of hard (high mass/scale) standard-model and new-physics processes require a precise understanding of the parton structure of the proton.

2. QCD formalism and Padé approximations

The results of the present analysis is based on the associated Jacobi polynomials expansion of the non–singlet structure function, the method of the structure function reconstruction over their Mellin moments [1–3]. The structure function is reconstructed from its moments by using the expansion in terms of orthogonal associated jacobi polynomials

$$xF_3(x,Q^2) = x^\beta (1-x)^\alpha \sum_{n=0}^{N_{max}} H_n^{\alpha,\beta}(x,c) \sum_{j=0}^{n} c_j^{(n)}(\alpha,\beta,c)\, M_{xF_3}(j+2,Q^2) \tag{2.1}$$

where $c_j^{(n)}(\alpha,\beta,c)$ are combinatorial coefficients, given in terms of Euler Γ-functions of the α and β weight parameters which have been fixed, $H_n^{\alpha,\beta}(x,c)$ is the associated jacobi polynomials satisfy the orthogonality [4, 5]

$$\int_0^1 x^\beta (1-x)^\alpha H_m^{(\alpha,\beta)}(x,c) H_n^{(\alpha,\beta)}(x,c) dx = \delta_{mn}\ , \tag{2.2}$$

and $x^\beta(1-x)^\alpha$ is the Jacobi weight function.

In spite of the unknown 4-loop anomalous dimensions and 3-loop Wilson coefficients, one can obtain the non-singlet parton distributions and $\Lambda_{QCD}^{\overline{MS}}$ by estimating uncalculated fourth-order corrections to the non-singlet anomalous dimension and third-order corrections to the Wilson coefficients. In this case these functions may be obtain from Padé approximations [6–8]. In the framework of this technique the values of the terms $C^3(n)$ and $\hat{P}_3^+(n)$ with the help of Padé approximations could be expressed as [9–12]

$$\begin{aligned} C^3(n) &= [C^2(n)]^2/C^1(n)\ , \\ \hat{P}_3^+(n) &= [\hat{P}_2^+(n)]^2/\hat{P}_1^+(n)\ . \end{aligned} \tag{2.3}$$

In the QCD analysis we parameterized the strong coupling constant α_s in terms of four massless flavors determining $\Lambda_{\rm QCD}$. Our results on $\Lambda_{QCD}^{\overline{MS}}$ and $\alpha_s(M_Z^2)$ up to N^{3}LO are

$$\begin{aligned} \Lambda_{\rm QCD}^{(4)\overline{\rm MS}} &= 311\ \text{MeV}, \quad \alpha_s(M_Z^2) = 0.1359,\ \ \texttt{NLO}, \\ \Lambda_{\rm QCD}^{(4)\overline{\rm MS}} &= 273\ \text{MeV}, \quad \alpha_s(M_Z^2) = 0.1147,\ \ \texttt{N}^2\texttt{LO}, \\ \Lambda_{\rm QCD}^{(4)\overline{\rm MS}} &= 277\ \text{MeV}, \quad \alpha_s(M_Z^2) = 0.1162,\ \ \texttt{N}^3\texttt{LO}. \end{aligned} \tag{2.4}$$

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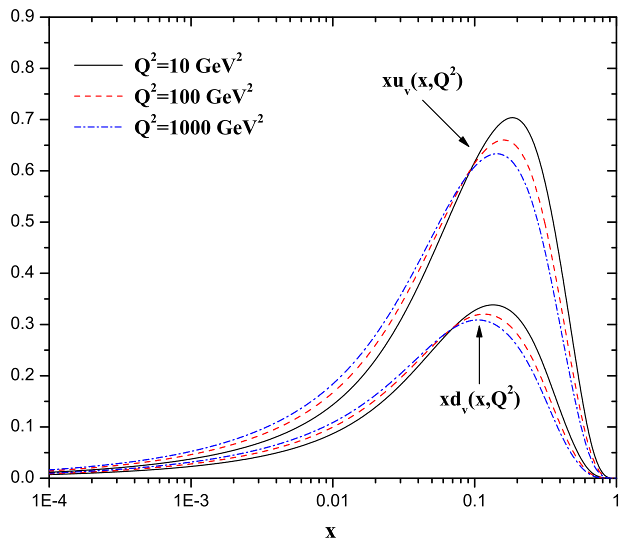

Figure 1: The parton densities xu_v and xd_v up to 10^3 GeV^2 at N^3LO.

Note that in above results we use the matching between n_f and n_{f+1} flavor couplings calculated in Ref. [13]. In Fig. (1) we show the evolution of the valence quark distributions $xu_v(x,Q^2)$ and $xd_v(x,Q^2)$ up to $Q^2 = 10^3$ GeV^2 at N^3LO.

References

[1] A. N. Khorramian and S. A. Tehrani, Phys. Rev. D **78** (2008) 074019 [arXiv:0805.3063 [hep-ph]].

[2] A. N. Khorramian, S. A. Tehrani and M. Ghominejad, Acta Phys. Polon. B **38**, 3551 (2007).

[3] A. N. Khorramian and S. Atashbar Tehrani, JHEP **0703**, 051 (2007) [arXiv:hep-ph/0610136].

[4] J. Wimp, Canadian J. Math. **39** (1987), 893-1000.

[5] J. Letessier, SIAM J. Math. Anal. (1993)

[6] M. A. Samuel, J. R. Ellis and M. Karliner, Phys. Rev. Lett. **74**, 4380 (1995) [arXiv:hep-ph/9503411].

[7] J. R. Ellis, E. Gardi, M. Karliner and M. A. Samuel, Phys. Lett. B **366**, 268 (1996) [arXiv:hep-ph/9509312].

[8] J. R. Ellis, E. Gardi, M. Karliner and M. A. Samuel, Phys. Rev. D **54**, 6986 (1996) [arXiv:hep-ph/9607404].

[9] A. L. Kataev, G. Parente and A. V. Sidorov, Phys. Part. Nucl. **34**, (2003) 20 [arXiv:hep-ph/0106221].

[10] A. L. Kataev, G. Parente and A. V. Sidorov, Nucl. Phys. B **573**, (2000) 405 [arXiv:hep-ph/9905310].

[11] A. L. Kataev, A. V. Kotikov, G. Parente and A. V. Sidorov, Phys. Lett. B **417**, (1998) 374 [arXiv:hep-ph/9706534].

[12] A. L. Kataev, G. Parente and A. V. Sidorov, arXiv:hep-ph/9809500.

[13] K. G. Chetyrkin, B. A. Kniehl and M. Steinhauser, , Phys. Rev. Lett. **79** (1997) 2184 [arXiv:hep-ph/9706430].