

Measurement of the CKM angle $\gamma (\phi_3)$ using $B \rightarrow DK$ decays

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We present the status of the measurement of the CKM CP -violating phase $\gamma (\phi_3)$ performed using $B \rightarrow D^{(*)}K^{(*)}$ decays. We review and compare the main experimental methods.

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1. Introduction

CP violation was first established in $K_L^0 \rightarrow \pi^+ \pi^-$ decays in 1964 [1] and since then it has been observed in several K and B meson weak decays [2]. In the standard model of particle physics (SM) CP violation in the quark sector of the weak interactions arises from a single irreducible phase in the Cabibbo-Kobayashi-Maskawa (CKM) matrix [3] that describes the mixing of the quarks. The unitarity of the CKM matrix imposes a set of relations among its elements, including the condition $V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0$ that defines a *unitarity triangle* in the complex plane, shown in Fig. 1. Many measurements can be conveniently displayed and compared as constraints on sides and angles of this triangle. CP violation is proportional to the area of the unitarity triangle and therefore it requires that all sides and angles be different from zero. The angle $\gamma \equiv \phi_3 \equiv \arg(-V_{ud}V_{ub}^*/V_{cd}V_{cb}^*)$ is at present the most difficult to measure.

An important goal of flavor physics is to over-constrain the CKM matrix. The reason is twofold. First, it is desirable to determine its elements as precisely as possible because their values are fundamental parameters of the SM. Second, new physics (NP) effects could manifest themselves as inconsistencies between two or more measurements of the CKM parameters [4]. The angle γ can be measured in decays mediated by tree amplitudes, such as $B \rightarrow D^0 K$: assuming that NP does not change the tree-level processes, its determination is not affected by effects beyond the SM¹ and together with the measurement of $|V_{ub}/V_{cb}|$ provides a constraint that can be compared with those potentially sensitive to NP.

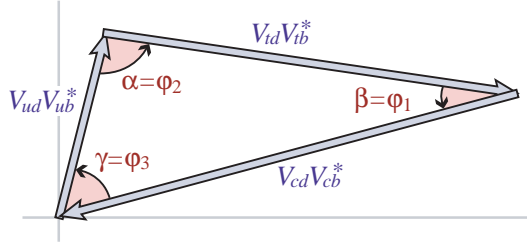


Figure 1: Graphical representation of the unitarity constraint $V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0$ as a triangle in the complex plane [2].

In the following we will describe and discuss the main techniques to measure the angle γ based on the measurement of $B \rightarrow DK$ decays and we will summarize the results.

2. Measurement of γ with flavor-tagged $B \rightarrow DK$ decays

The most powerful method used so far exploits the interference between $b \rightarrow c\bar{u}s$ and $b \rightarrow u\bar{c}s$ amplitudes in $B \rightarrow DK$ decays, whose relative weak phase is γ . In charged B decays the interference is between the $B^- \rightarrow D^0 K^-$ amplitude and the color- and Cabibbo-suppressed $B^- \rightarrow \bar{D}^0 K^-$ amplitude, when D^0 and \bar{D}^0 decay to a common final state. The leading interfering diagrams are shown in Fig. 2.

¹NP may appear in $D^0 - \bar{D}^0$ mixing but the effect is expected to be small and can be taken into account [5].

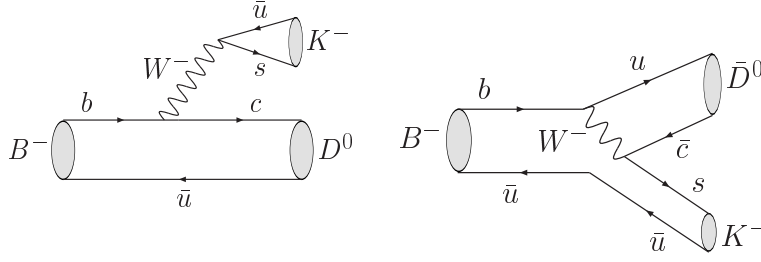


Figure 2: Main Feynman diagrams contributing to the $B^- \rightarrow DK^-$ decay. The left decay proceeds through a $b \rightarrow c\bar{u}s$ transition, while the right diagram proceeds via a $b \rightarrow u\bar{c}s$ transition and is both color- and Cabibbo-suppressed.

Let us introduce the decay amplitudes $A(B^- \rightarrow D^0 K^-) = A_c e^{i\delta_c}$, $A(B^- \rightarrow \bar{D}^0 K^-) = A_u e^{i(\delta_u - \gamma)}$, $A(D^0 \rightarrow f) = A_f e^{i\delta_f}$ and $A(\bar{D}^0 \rightarrow f) = A_{\bar{f}} e^{i\delta_{\bar{f}}}$, where f is a generic final state of the D meson. The parameters δ_c , δ_u , δ_f and $\delta_{\bar{f}}$ are strong phases, and γ is the weak phase difference between $B^- \rightarrow D^0 K^-$ and $B^- \rightarrow \bar{D}^0 K^-$. A_c , A_u , A_f and $A_{\bar{f}}$ are real and positive. The decay amplitude of $B^- \rightarrow [f]_D K^-$ is

$$A(B^- \rightarrow [f]_D K^-) = A_c A_f e^{i(\delta_c + \delta_f)} + A_u A_{\bar{f}} e^{i(\delta_u + \delta_{\bar{f}} - \gamma)}, \quad (2.1)$$

and the rate can be written as

$$\Gamma(B^- \rightarrow [f]_D K^-) = A_c^2 A_{\bar{f}}^2 \left(A_f^2 / A_{\bar{f}}^2 + r_B^2 + 2r_B A_f / A_{\bar{f}} \text{Re}(e^{i(\delta_B + \delta_D - \gamma)}) \right), \quad (2.2)$$

where $r_B = A_u / A_c$, $\delta_B = \delta_u - \delta_c$ and $\delta_D = \delta_{\bar{f}} - \delta_f$. The rate of the charge-conjugate mode is obtained from Eq. 2.2 by replacing γ with $-\gamma$. Since with excellent approximation there are no other contributions besides the ones in Fig. 2, γ and the unknown hadronic parameters r_B and δ_B can be constrained in a theoretically clean way by measuring the yields of B^+ and B^- , provided that the D decays are chosen appropriately. The same concept and equations apply to $B^\pm \rightarrow D^{*0} K^\pm$, $B^\pm \rightarrow D^0 K^{*\pm}$ and to the flavor-tagged $B^0 \rightarrow D^0 K^{*0}$ decays with small, though important, modifications[6, 7]². The parameters r_B and δ_B are not expected to have the same value in different $B \rightarrow D^{(*)0} K^{(*)}$ decays. The magnitude of r_B is crucial because it measures the interference which allows the extraction of γ . For charged B decays $r_B \approx c_F |V_{cs} V_{ub}^* / V_{us} V_{cb}^*| \sim 0.1 - 0.2$, where $c_F \sim 0.2 - 0.4$ is a color suppression factor. For neutral B decays r_B can be as large as 0.4, but the typical yields are significantly smaller making these decays less competitive at present.

In the next three sections the main experimental methods explored so far are briefly discussed.

2.1 The GLW method

In the method proposed by Gronau, London and Wyler (GLW) [9] the D meson is reconstructed to CP eigenstate final states f_{CP^\pm} , such as $K^+ K^-$ ($CP = +1$) or $K_S^0 \pi^0$ ($CP = -1$). Therefore $A_f / A_{\bar{f}} = 1$, $\delta_D = 0, \pi$ for $CP = 1, -1$, and Eq. 2.2 becomes

$$\Gamma(B^- \rightarrow [f_{CP^\pm}]_D K^-) = A_c^2 A_{f_{CP^\pm}}^2 (1 + r_B^2 \pm 2r_B \cos(\delta_B - \gamma)). \quad (2.3)$$

²The decay $B \rightarrow DK^*$ can also be measured as part of the Dalitz plot analysis of $B \rightarrow DK\pi$. See [8] and references therein.

Equation 2.3 and its CP -conjugate are used to define four observables which depend on the physical parameters γ , δ_B and r_B :

$$A_{CP\pm} = \frac{\Gamma(B^- \rightarrow D_{CP\pm}^0 K^-) - \Gamma(B^+ \rightarrow D_{CP\pm}^0 K^+)}{\Gamma(B^- \rightarrow D_{CP\pm}^0 K^-) + \Gamma(B^+ \rightarrow D_{CP\pm}^0 K^+)} = \frac{\pm 2r_B \sin \delta_B \sin \gamma}{1 + r_B^2 \pm 2r_B \cos \delta_B \cos \gamma}, \quad (2.4)$$

$$R_{CP\pm} = \frac{\Gamma(B^- \rightarrow D_{CP\pm}^0 K^-) + \Gamma(B^+ \rightarrow D_{CP\pm}^0 K^+)}{2\Gamma(B^- \rightarrow D^0 K^-)} = 1 + r_B^2 \pm 2r_B \cos \delta_B \cos \gamma, \quad (2.5)$$

where $\Gamma(B^- \rightarrow D_{CP\pm}^0 K^-) \equiv \Gamma(B^- \rightarrow [f_{CP\pm}]_D K^-) / BF(D^0 \rightarrow f_{CP\pm})$ and $\Gamma(B^- \rightarrow D^0 K^-) \equiv A_c^2 \simeq \Gamma(B^- \rightarrow [K^- \pi^+]_D K^-) / BF(D^0 \rightarrow K^- \pi^+)$. $A_{CP\pm}$ and $R_{CP\pm}$ are bound by the relation $R_{CP+} A_{CP+} + R_{CP-} A_{CP-} = 0$.

Both BaBar and Belle have reconstructed $B^- \rightarrow D^0 K^-$ and $B^- \rightarrow D^{*0} K^-$ decays with $D^{*0} \rightarrow D^0 \pi^0$. BaBar has also selected the mode $D^{*0} \rightarrow D^0 \gamma$ and the decay $B^- \rightarrow D^0 K^{*-}$ with $K^{*-} \rightarrow K_S^0 \pi^-$. D^0 mesons have been reconstructed in CP -even ($K^+ K^-$ and $\pi^+ \pi^-$) and CP -odd ($K_S^0 \pi^0$, $K_S^0 \phi$ and $K_S^0 \omega$) eigenstates. Recently also CDF has measured $B^- \rightarrow D^0 K^-$ with $D^0 \rightarrow K^+ K^-$ and $\pi^+ \pi^-$. The data samples used by BaBar and Belle consist of 382 and 275 million $B\bar{B}$ pairs, respectively, while CDF has used 1.0 fb^{-1} of data. The results are summarized in Table 1.

Due to the discrete 8-fold ambiguity in the extraction of γ from $R_{CP\pm}$ and $A_{CP\pm}$, and the still large uncertainty on r_B , the GLW measurements have a poor constraining power on γ when they are considered alone. However, they generally improve the knowledge of r_B , γ and δ_B when combined with the results of the Dalitz method discussed in Sec. 2.3. This aspect is further discussed in Sec. 3.

Table 1: Summary of $R_{CP\pm}$ and $A_{CP\pm}$ measurements.

Mode	Experiment	A_{CP+}	A_{CP-}	R_{CP+}	R_{CP-}
$B \rightarrow D^0 K$	BaBar [10]	$0.27 \pm 0.09 \pm 0.04$	$0.09 \pm 0.09 \pm 0.02$	$1.06 \pm 0.10 \pm 0.05$	$1.03 \pm 0.10 \pm 0.05$
	Belle [11]	$0.06 \pm 0.14 \pm 0.05$	$0.12 \pm 0.14 \pm 0.05$	$1.13 \pm 0.16 \pm 0.08$	$1.17 \pm 0.14 \pm 0.14$
	CDF [12]	$0.39 \pm 0.17 \pm 0.04$	—	$1.30 \pm 0.24 \pm 0.12$	—
$B \rightarrow D^{*0} K$	BaBar [13]	$0.11 \pm 0.09 \pm 0.01$	$0.06 \pm 0.10 \pm 0.02$	$1.31 \pm 0.13 \pm 0.03$	$1.09 \pm 0.12 \pm 0.04$
	Belle [11]	$0.20 \pm 0.22 \pm 0.04$	$0.13 \pm 0.30 \pm 0.08$	$1.41 \pm 0.25 \pm 0.06$	$1.15 \pm 0.31 \pm 0.12$
$B \rightarrow D^0 K^*$	BaBar [14]	$0.09 \pm 0.13 \pm 0.06$	$-0.23 \pm 0.21 \pm 0.07$	$2.17 \pm 0.35 \pm 0.09$	$1.03 \pm 0.27 \pm 0.13$

2.2 The ADS method

In the method proposed by Atwood, Dunietz and Soni (ADS) [15] the D meson is reconstructed in doubly Cabibbo-suppressed (DCS) states. We consider $D^0 \rightarrow K^+ \pi^-$ as an example in the following discussion. The decay rate of the process $B^- \rightarrow [K^+ \pi^-]_D K^-$ is the result of the interference between $B^- \rightarrow D^0 K^-$ followed by the DCS $D^0 \rightarrow K^+ \pi^-$, and the suppressed $B^- \rightarrow \bar{D}^0 K^-$ followed by the Cabibbo-allowed $\bar{D}^0 \rightarrow K^+ \pi^-$. From Eq. 2.2 we find

$$\frac{\Gamma(B^\mp \rightarrow [K^\pm \pi^\mp]_D K^\mp)}{\Gamma(B^\mp \rightarrow [K^\mp \pi^\pm]_D K^\mp)} = r_B^2 + r_D^2 + 2r_B r_D \cos(\delta_B + \delta_D \mp \gamma), \quad (2.6)$$

where both $r_D = A_f/A_{\bar{f}} = |A(D^0 \rightarrow K^+\pi^-)/A(D^0 \rightarrow K^-\pi^+)|$ and the strong phase difference δ_D are measured independently [2, 16]. Defining R_{ADS} and A_{ADS} as

$$R_{ADS} = \frac{\Gamma(B^- \rightarrow [K^+\pi^-]_D K^-) + \Gamma(B^+ \rightarrow [K^-\pi^+]_D K^+)}{\Gamma(B^- \rightarrow [K^-\pi^+]_D K^-) + \Gamma(B^+ \rightarrow [K^+\pi^-]_D K^+)}, \quad (2.7)$$

$$A_{ADS} = \frac{\Gamma(B^- \rightarrow [K^+\pi^-]_D K^-) - \Gamma(B^+ \rightarrow [K^-\pi^+]_D K^+)}{\Gamma(B^- \rightarrow [K^+\pi^-]_D K^-) + \Gamma(B^+ \rightarrow [K^-\pi^+]_D K^+)} \quad (2.8)$$

it follows

$$R_{ADS} = r_B^2 + r_D^2 + 2r_B r_D \cos \gamma \cos(\delta_B + \delta_D), \quad (2.9)$$

$$A_{ADS} = 2r_B r_D \sin \gamma \sin(\delta_B + \delta_D) / R_{ADS}. \quad (2.10)$$

Since $r_D(K\pi) = (5.78 \pm 0.08)\%$ [17] and r_B is expected to be around 10%, the interference effect can be quite large. Similar relations, with small modifications, are derived for $B^- \rightarrow D^{*0}K^-$ and $B^- \rightarrow D^0K^{*-}$ decays (see [6] and refs. in Table 2) and for multi-body D^0 final states [18, 19]. Both BaBar and Belle have reconstructed the decay $B^- \rightarrow D^0K^-$ followed by $D^0 \rightarrow K^+\pi^-$ on datasets of 467 and 657 million $B\bar{B}$ pairs, respectively. BaBar has also selected $B^- \rightarrow D^{*0}K^-$ with $D^{*0} \rightarrow D^0\pi^0$ and $D^{*0} \rightarrow D^0\gamma$ ($467 \times 10^6 B\bar{B}$), and $B^- \rightarrow D^0K^{*-}$ with $K^{*-} \rightarrow K_S^0\pi^-$ ($379 \times 10^6 B\bar{B}$). On a dataset of 465 million $B\bar{B}$ pairs BaBar has performed the first measurement of flavor-tagged decays $B^0 \rightarrow D^0K^{*0}$ with $K^{*0} \rightarrow K^+\pi^-$, selected in the D^0 final states $K^+\pi^-$, $K^+\pi^-\pi^0$ and $K^+\pi^-\pi^-\pi^+$. The results are summarized in Table 2. Due to the smallness of the involved branching fractions no evidence of signal has been observed so far and the null measurements have been used to set upper limits on r_B . The strongest hint of signal has been reported by BaBar with a statistical significance of 2.6σ for $B^\mp \rightarrow D^0[K^\pm\pi^\mp]K^\mp$. Even when a signal is observed the constraining power on γ of the ADS method is weak when it is used alone, but it becomes significant when the ADS information is combined with the other methods. This aspect is discussed in Sec. 3.

2.3 The Dalitz or GGSZ method

If D^0 decays to a 3-body final state such as $D \rightarrow K_S^0\pi^+\pi^-$, the decay amplitudes of D^0 and \bar{D}^0 can be written as $A_f e^{i\delta_f} = f(m_-^2, m_+^2)$ and $A_{\bar{f}} e^{i\delta_{\bar{f}}} = f(m_+^2, m_-^2)$, where m_-^2 and m_+^2 are the squared masses of the $K_S^0\pi^-$ and $K_S^0\pi^+$ combinations. The rate in Eq. 2.2 becomes

$$\Gamma(B^\mp \rightarrow [K_S^0\pi^-\pi^+]_D K^\mp) \propto |f(m_\mp^2, m_\pm^2)|^2 + r_B^2 |f(m_\pm^2, m_\mp^2)|^2 + 2r_B |f(m_\mp^2, m_\pm^2)| |f(m_\pm^2, m_\mp^2)| \cos(\delta_B + \delta_D(m_\mp^2, m_\pm^2) \mp \gamma), \quad (2.11)$$

where $\delta_D(m_\mp^2, m_\pm^2)$ is the strong phase difference between $f(m_\mp^2, m_\pm^2)$ and $f(m_\pm^2, m_\mp^2)$. The amplitudes $f(m_\pm^2, m_\mp^2)$ are measured through a Dalitz plot analysis on a large sample of flavor-tagged D^0 decays. Therefore, the B^\pm yields in Eq. 2.11 only depend on the unknowns γ , δ_B and r_B . The great advantage of this method [23] is that the relation between the signal yields and the physics parameters varies over the Dalitz plot, making possible the extraction of γ with only a 2-fold ambiguity ($\gamma \rightarrow \gamma + 180^\circ$). Furthermore, the $D^0 \rightarrow K_S^0\pi^+\pi^-$ branching fractions is relatively large ($\sim 3\%$). Since the direct extraction of r_B , δ_B and γ through a maximum likelihood fit (MLF) using Eq. 2.11 overestimates r_B and underestimates the statistical error of γ and δ_B , it is convenient to express Eq. 2.11 in terms of the cartesian coordinates $x_\pm = \text{Re}[r_B e^{i(\delta_B \pm \gamma)}]$, $y_\pm = \text{Im}[r_B e^{i(\delta_B \pm \gamma)}]$,

$$\Gamma(B^\mp \rightarrow D^0[\rightarrow K_S^0\pi^+\pi^-]K^\mp) \propto |f_\mp|^2 + (x_\mp^2 + y_\mp^2)|f_\pm|^2 + 2[x_\mp \text{Re}[f_\mp f_\pm^*] + y_\mp \text{Im}[f_\mp f_\pm^*]] \quad (2.12)$$

Table 2: Summary of R_{ADS} and A_{ADS} measurements, and limits on r_B .

Mode	Experiment	$R_{ADS}[10^{-2}]$	A_{ADS}	r_B
$B^- \rightarrow D^0 K^-$ $D^0 \rightarrow K^+ \pi^-$	BaBar [20]	$1.36 \pm 0.55 \pm 0.27$	$-0.70 \pm 0.35^{+0.09}_{-0.14}$	[0.09, 0.193] @ 95% CL
	Belle [21]	$0.8 \pm 0.6^{+0.2}_{-0.3}$	$-0.13^{+0.97}_{-0.88} \pm 0.26$	< 0.19 @ 90% CL
$D^0 \rightarrow K^+ \pi^- \pi^0$	BaBar [19]	$1.2 \pm 1.2 \pm 0.9$	—	< 0.19 @ 95% CL
$B^- \rightarrow D^{*0}[D^0 \pi^0]K^-$ $D^0 \rightarrow K^+ \pi^-$	BaBar [20]	$1.76 \pm 0.93 \pm 0.42$	$0.77 \pm 0.35 \pm 0.12$	[0.007, 0.176] @ 95% CL
$B^- \rightarrow D^{*0}[D^0 \gamma]K^-$ $D^0 \rightarrow K^+ \pi^-$	BaBar [20]	$1.3 \pm 1.4 \pm 0.5$	$0.36 \pm 0.94^{+0.25}_{-0.41}$	
$B^- \rightarrow D^0 K^{*-}$ $D^0 \rightarrow K^+ \pi^-$	BaBar [14]	$6.6 \pm 3.1 \pm 1.0$	$-0.34 \pm 0.43 \pm 0.16$	[0.17, 0.43] @ 95% CL (combined with GLW)
$B^0 \rightarrow D^0 K^{*0}$ $D^0 \rightarrow K^+ \pi^-$	BaBar [22]	$6.7^{+7.0}_{-5.4} \pm 1.8$	—	[0.07, 0.41] @ 95% CL
$D^0 \rightarrow K^+ \pi^- \pi^0$	BaBar [22]	$6.0^{+5.5}_{-3.7} \pm 0.9$	—	
$D^0 \rightarrow K^+ \pi^- \pi^- \pi^+$	BaBar [22]	$13.7^{+11.3}_{-9.5} \pm 2.2$	—	

where the notation was simplified using $f_{\pm} = f(m_{\pm}^2, m_{\mp}^2)$. The extraction of x_{\pm} and y_{\pm} with a MLF using Eq. 2.12 is unbiased. The physics parameters γ , r_B and δ_B are extracted from x_{\pm} and y_{\pm} with a frequentist statistical procedure.

BaBar has used this approach to measure the angle γ with $B^- \rightarrow D^0 K^-$, $D^{*0} K^-$ ($D^{*0} \rightarrow D^0 \pi^0$ and $D^0 \gamma$) and $D^0 K^{*-}$ ($K^{*-} \rightarrow K_S^0 \pi^-$), with $D^0 \rightarrow K_S^0 \pi^+ \pi^-$ and $K_S^0 K^+ K^-$ using a sample of 386 million $B\bar{B}$ pairs [24]. Belle has selected $B^- \rightarrow D^{(*)0} K^-$ decays ($D^{*0} \rightarrow D^0 \pi^0$ and $D^0 \gamma$) with $D^0 \rightarrow K_S^0 \pi^+ \pi^-$ on a sample of 657 million $B\bar{B}$ pairs [25]. The $D^0 \rightarrow K_S^0 \pi^+ \pi^-$ and $D^0 \rightarrow K_S^0 K^+ K^-$ decay amplitudes are determined with Dalitz plot analyses of large and very pure samples of D^0 mesons from $D^{*+} \rightarrow D^0 \pi^+$ decays produced in $e^+ e^- \rightarrow c\bar{c}$ events. The amplitudes are described using an isobar model, consisting of a coherent sum of two-body amplitudes (parameterized using relativistic Breit-Wigner) plus a “nonresonant” term. BaBar has described the $\pi\pi$ and $K\pi$ S-wave amplitudes in $D^0 \rightarrow K_S^0 \pi^+ \pi^-$ using a K -matrix formalism (see [24] for a detailed discussion).

The results for x_{\pm} and y_{\pm} are reported in Table 3. From the (x_{\pm}, y_{\pm}) confidence regions both BaBar and Belle determine the 1σ confidence intervals of γ , δ_B and r_B using a frequentist procedure. BaBar finds $\gamma = (76 \pm 22 \pm 5 \pm 5)^\circ$ and Belle $\gamma = (76^{+12}_{-13} \pm 4 \pm 9)^\circ$ ($B^{\pm} \rightarrow D^{(*)} K^{\pm}$), where the solution closest to the SM average has been quoted. All results are reported in Table 4. Figure 3 shows the (x_{\pm}, y_{\pm}) contours for $B \rightarrow D^0 K$ as measured by Belle (top-left) and BaBar (bottom-left), the projections of confidence regions onto the (γ, r_B) and (γ, δ_B) plane obtained by Belle, and the confidence-level as a function of r_B and γ found by BaBar. The combined $B \rightarrow D^{(*)0} K^{(*)}$ measurements of BaBar and Belle correspond to 3.0σ and 3.5σ evidence of CP violation, respectively.

It is interesting to ask why the statistical error measured by Belle is about twice smaller than what BaBar finds, despite the fact that the experimental observables x_{\pm}, y_{\pm} have similar uncertainties. The error on γ scales roughly as $1/r_B$. Since Belle measurements have central values of r_B between 1.5 and 3.5 times larger than BaBar values, though consistent within the errors, the result-

Table 3: Summary of x_{\pm} and y_{\pm} measurements. The third error is the systematic uncertainty associated to the Dalitz model of the D final state.

Mode	Experiment	x_+ [10^{-2}]	y_+ [10^{-2}]	x_- [10^{-2}]	y_- [10^{-2}]
$B \rightarrow D^0 K$	Dalitz				
	BaBar [24]	$-6.7 \pm 4.3 \pm 1.4 \pm 1.1$	$-1.5 \pm 5.5 \pm 0.6 \pm 0.8$	$9.0 \pm 4.3 \pm 1.5 \pm 1.1$	$5.3 \pm 5.6 \pm 0.7 \pm 1.5$
	Belle [25, 26]	$-10.7 \pm 4.3 \pm 1.1 \pm 5.5$	$-6.7 \pm 5.9 \pm 1.8 \pm 6.3$	$10.5 \pm 4.7 \pm 1.1 \pm 6.4$	$17.7 \pm 6.0 \pm 1.8 \pm 5.4$
GLW	BaBar [10]	$-9 \pm 5 \pm 2$	—	$10 \pm 5 \pm 3$	—
$B \rightarrow D^{*0} K$	Dalitz				
	BaBar [24]	$13.7 \pm 6.8 \pm 1.4 \pm 0.5$	$8.0 \pm 10.2 \pm 1.0 \pm 1.2$	$-11.1 \pm 6.9 \pm 1.4 \pm 0.4$	$-5.1 \pm 8.0 \pm 0.9 \pm 1.0$
	Belle [25, 26, 27]	$13.3 \pm 8.3 \pm 1.8 \pm 8.1$	$13.0 \pm 12.0 \pm 2.2 \pm 6.3$	$2.4 \pm 14.0 \pm 1.8 \pm 9.0$	$-24.3 \pm 13.7 \pm 2.2 \pm 4.9$
GLW	BaBar [13]	$11 \pm 6 \pm 2$	—	$0 \pm 6 \pm 2$	—
$B \rightarrow D^0 K^*$	Dalitz				
	BaBar [24]	$11.3 \pm 10.7 \pm 2.8 \pm 1.8$	$12.5 \pm 13.9 \pm 5.1 \pm 1.0$	$11.5 \pm 13.8 \pm 3.9 \pm 1.4$	$22.6 \pm 14.2 \pm 5.8 \pm 1.1$
	Belle [28]	$-10.5_{-16.7}^{+17.7} \pm 0.6 \pm 8.8$	$-0.4_{-15.6}^{+16.4} \pm 1.3 \pm 9.5$	$-78.4_{-29.5}^{+24.9} \pm 2.9 \pm 9.7$	$-28.1_{-33.5}^{+44.0} \pm 4.6 \pm 8.6$
GLW	BaBar [14]	$18 \pm 14 \pm 5$	—	$38 \pm 14 \pm 5$	—

ing γ uncertainty of Belle is smaller. The concept is illustrated in Fig. 4a. Fluctuations of the γ error due to the “ $1/r_B$ ” effect are expected to decrease as the relative uncertainty on r_B will become smaller.

At present the Dalitz method has the best sensitivity to γ . However, the uncertainty associated to the Dalitz model of the D final state is already the dominant contribution to the systematic error and it may be difficult to reduce it greatly in the future. To bypass this limit a model-independent analysis is required [23]. At the price of a small loss of statistical power [29], the method is free of model-dependent assumptions on the D decay and therefore it is a promising approach to follow at LHCb and at the next generation B -factories. It requires the use of entangled $\Psi(3770) \rightarrow D\bar{D}$ decays at tau-charm factories such as CLEO-c, BES-III or next generation Super Flavor factories [4]. It has been recently shown that using 818 pb^{-1} of CLEO-c data an error on γ of $\sim 2^\circ$ associated to the knowledge of the relative strong phase difference of $D^0 \rightarrow K_S^0 \pi^+ \pi^-$ and $\bar{D}^0 \rightarrow K_S^0 \pi^+ \pi^-$ can be obtained [30].

BaBar has applied the Dalitz method also to flavor-tagged $B^0 \rightarrow D^0 K^{*0}$ decays, with $K^{*0} \rightarrow K^+ \pi^-$ and $D^0 \rightarrow K_S^0 \pi^+ \pi^-$. On a dataset of 371 million $B\bar{B}$ pairs 39 ± 9 signal candidates have been selected. Using the D^0 Dalitz plot model obtained in the charged B analysis and imposing an external measurement of r_B , a loose constraint on γ was set [31]. Self tagging $B^0 \rightarrow D^0 K^{*0}$

Table 4: Measurement of γ , δ_B and r_B in $B^- \rightarrow D^{(*)0} K^{(*)-}$ decays reconstructed by BaBar and Belle.

Parameter	BaBar	Belle
γ	$76 \pm 22 \pm 5 \pm 5$	$76_{-13}^{+12} \pm 4 \pm 9$
$r_B(D^0 K)$	$0.086 \pm 0.035 \pm 0.010 \pm 0.011$	$0.161_{-0.038}^{+0.040} \pm 0.011 \pm 0.049$
$\delta_B(D^0 K)$	$(109_{-31}^{+28} \pm 4 \pm 7)^\circ$	$(137.4_{-15.7}^{+13.0} \pm 4.0 \pm 22.9)^\circ$
$r_B(D^{*0} K)$	$0.135 \pm 0.051 \pm 0.011 \pm 0.005$	$0.196_{-0.069}^{+0.072} \pm 0.012_{-0.012}^{+0.062}$
$\delta_B(D^{*0} K)$	$(297_{-30}^{+28} \pm 5 \pm 4)^\circ$	$(341.9_{-19.6}^{+18.0} \pm 3.0 \pm 22.9)^\circ$
$r_B(D^0 K^*)$	$0.163_{-0.105}^{+0.088} \pm 0.037 \pm 0.021$	$0.564_{-0.155}^{+0.216} \pm 0.041 \pm 0.084$
$\delta_B(D^0 K^*)$	$(104_{-41}^{+43} \pm 17 \pm 5)^\circ$	$(242.6_{-23.2}^{+20.2} \pm 2.5 \pm 49.3)^\circ$

decays and time-dependent measurements of $B^0 \rightarrow DK_S^0$ decays are expected to be powerful tools to measure γ at LHCb and at the next generation B -factories, where their production will be abundant and the large interference ($r_B \sim 0.4$) can be fully exploited.

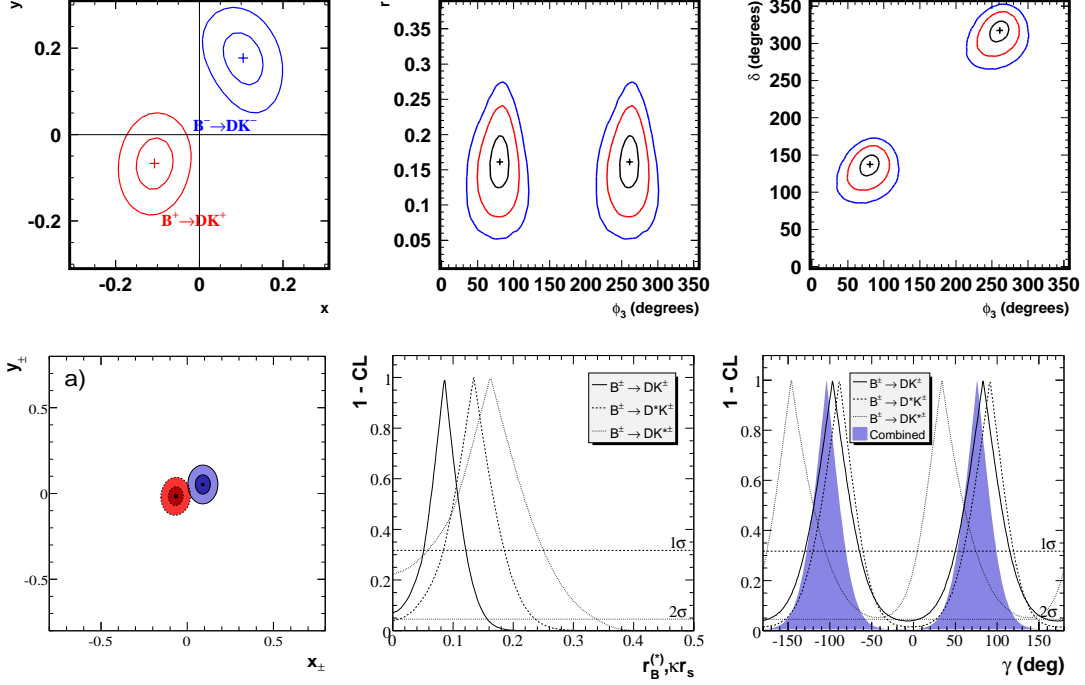


Figure 3: Top row: Belle; bottom row: BaBar. 1σ and 2σ contours in the (x_\pm, y_\pm) plane of $B^\pm \rightarrow D^0 K^\pm$ (top-left); projections of confidence regions for the $B \rightarrow DK$ mode onto the (r_B, γ) plane (top-center) and onto the (δ_B, γ) plane (top-right); 1- and 2-standard-deviation regions in the (x_\pm, y_\pm) plane of $B^\pm \rightarrow D^0 K^\pm$ (bottom-left); 1-CL as a function of r_B (bottom-center) and γ (bottom-right) for $B \rightarrow D^0 K, D^{*0} K$ and $D^0 K^*$.

3. Note on the cartesian coordinates

The output of the GLW and ADS methods can be expressed in terms of the same cartesian coordinates $x_\pm = r_B \cos(\delta_B \pm \gamma)$ and $y_\pm = \sin(\delta_B \pm \gamma)$ measured with the Dalitz method. This alternative way to quote the results can be useful when different methods are compared or combined. In this section we will derive the alternative parameterization and will discuss some of the advantages with respect to the classic observables.

From the definitions in Sec. 2.1 we can write $\Gamma(B^- \rightarrow D_{CP\pm}^0 K^-) / \Gamma(B^- \rightarrow D^0 K^-) = 1 + x_-^2 + y_-^2 \pm 2x_-$, from which it follows

$$\frac{1}{4} \left(\frac{\Gamma(B^- \rightarrow D_{CP+}^0 K^-)}{\Gamma(B^- \rightarrow D^0 K^-)} - \frac{\Gamma(B^- \rightarrow D_{CP-}^0 K^-)}{\Gamma(B^- \rightarrow D^0 K^-)} \right) = x_-, \quad (3.1)$$

$$\frac{1}{2} \left(\frac{\Gamma(B^- \rightarrow D_{CP+}^0 K^-)}{\Gamma(B^- \rightarrow D^0 K^-)} + \frac{\Gamma(B^- \rightarrow D_{CP-}^0 K^-)}{\Gamma(B^- \rightarrow D^0 K^-)} \right) - 1 = x_-^2 + y_-^2. \quad (3.2)$$

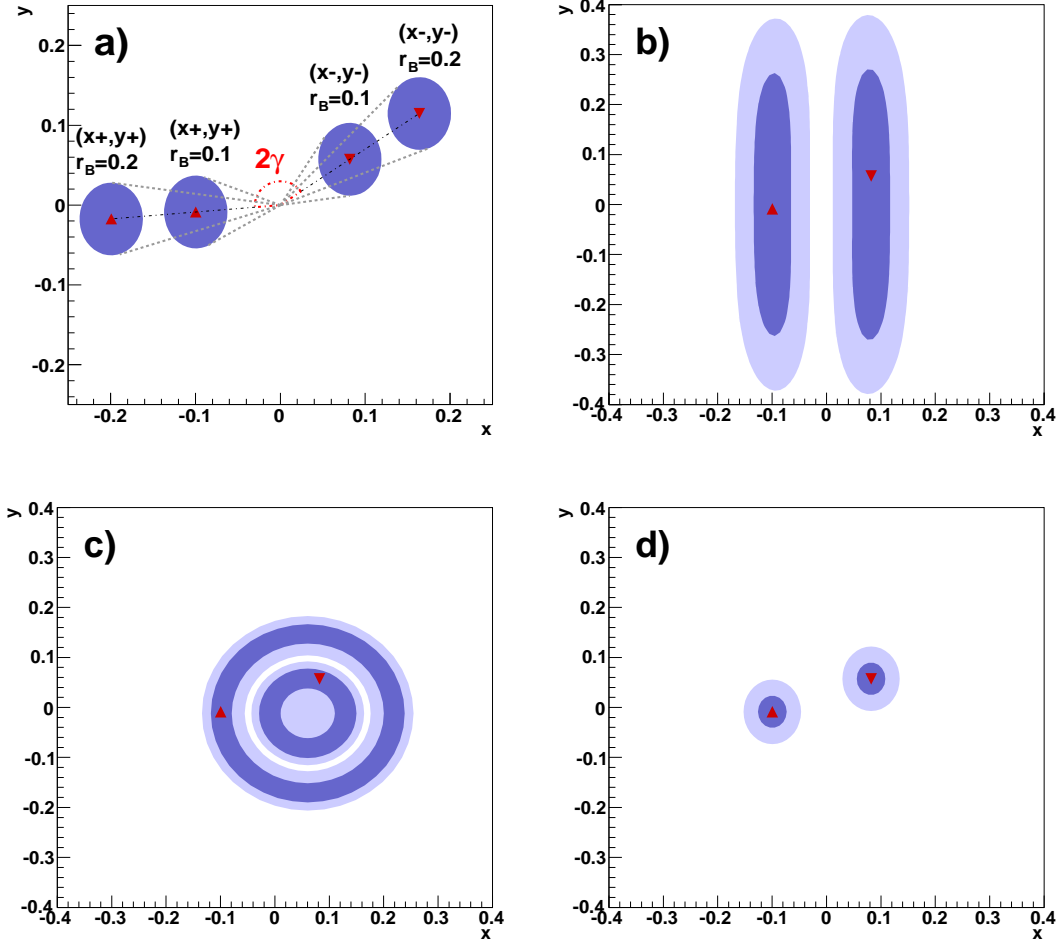


Figure 4: a): Geometric definition of γ in the (x_{\pm}, y_{\pm}) plane and pictorial description of how its uncertainty changes as a function of r_B . Circles represent x_{\pm}, y_{\pm} error regions for B^+ (\blacktriangle) and B^- (\blacktriangledown) and the dashed lines delimit the range of variation of 2γ . b,c,d): 1- and 2-standard deviation regions in the (x_{\pm}, y_{\pm}) plane using the GLW (b), ADS (c) and Dalitz (d) methods, on a dataset of 1 ab^{-1} . The assumptions used are discussed in the text.

The same relations hold for x_+ and $x_+^2 + y_+^2$ after replacing B^- with B^+ . Equation 3.1 has been used by BaBar to measure x_{\pm} with the GLW analysis³, whose results are reported in Table 3. It is interesting that the measurements of x_{\pm} performed with the GLW and Dalitz methods have about the same uncertainty on datasets with similar size. The constraint given by Eq. 3.2 is much looser because of the quadratic dependence on x, y and the fact that $r_B \ll 1$. Therefore, the GLW method measures x_{\pm} quite precisely but not y_{\pm} , and this is the reason why it can hardly constrain γ when it is considered alone. When combined with the Dalitz method, however, the overall error of γ can improve significantly.

³The relation was expressed in terms of $R_{CP\pm}$ and $A_{CP\pm}$.

Proceeding in a similar way, Eq. 2.6 of the ADS method can be written as

$$\frac{\Gamma(B^\mp \rightarrow [K^\pm \pi^\mp]_D K^\mp)}{\Gamma(B^\mp \rightarrow [K^\mp \pi^\pm]_D K^\mp)} = (x_\mp + r_D \cos \delta_D)^2 + (y_\mp - r_D \sin \delta_D)^2, \quad (3.3)$$

that represents two circles in the (x_\pm, y_\pm) plane centered at $(-r_D \cos \delta_D, r_D \sin \delta_D)$ and with radii $R_\mp = \sqrt{\frac{\Gamma(B^\mp \rightarrow [K^\pm \pi^\mp]_D K^\mp)}{\Gamma(B^\mp \rightarrow [K^\mp \pi^\pm]_D K^\mp)}}$. It is not possible to determine γ with only the ADS analysis of $B \rightarrow D^0 K$ because the true x_\pm, y_\pm points are delocalized over two circles⁴. However, the measurement can be combined with the Dalitz and GLW analyses to reduce the overall error of x_\pm, y_\pm , and therefore the uncertainty of γ . Figure 4 shows the constraints provided by the GLW, ADS and Dalitz measurements with $B \rightarrow D^0 K$ decays on a dataset of 1 ab^{-1} , assuming $\{\gamma, r_B, \delta_B, r_D, \delta_D\} = \{75^\circ, 0.1, 110^\circ, 0.06, 191^\circ\}$ in a scenario where the measured observables are centered to their true value. The uncertainty on the values of r_D and δ_D was neglected when drawing the ADS constraint.

4. Conclusions

Despite the fact that BaBar and Belle have collected almost 1.5 ab^{-1} of data, a precise measurement of the CKM phase γ is not yet available. This is not surprising if one thinks that the typical branching fractions of the involved decays are of the order of 10^{-6} or smaller and the interference term is found to be around 10% in the main decay modes. In fact, the sensitivity reached by the B -factories is much better than it was initially foreseen. The decays $B^\pm \rightarrow D^{(*)0} K^{(*)\pm}$ are currently the most sensitive tool. The method where the D meson decays to $K_S^0 h^+ h^-$ ($h = \pi, K$) has the smallest uncertainty, ranging between 15° and 24° including systematics. The dominant source of systematic uncertainty currently comes from the Dalitz model of the D final state.

Using a Bayesian statistical procedure and combining BaBar and Belle results of GLW, ADS and Dalitz methods, the UTfit collaboration finds $\gamma = (78 \pm 12)^\circ$ [32]⁵, while CKMfitter using a frequentist approach quotes $\gamma = (73_{-25}^{+22})^\circ$ [33]. The results are reported in Fig. 5. To reduce the error to a few degrees we have to wait for LHCb ($\sigma_\gamma \approx 2 - 3^\circ$ with 10 fb^{-1} [34]) or a Super Flavor factory ($\sigma_\gamma \approx 1^\circ$ with 75 ab^{-1} [4]).

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⁴In the case of $B \rightarrow D^{*0} K$ the circles associated to $D^0 \pi^0$ and $D^0 \gamma$ are centered at opposite points $(\pm r_D \cos \delta_D, \pm r_D \sin \delta_D)$ because of the effect pointed out in [6]; therefore the resulting constraint in the (x_\pm, y_\pm) plane is the intersection of two circles and in principle it is possible to determine γ up to a discrete ambiguity.

⁵The average is not updated with the most recent BaBar's $B \rightarrow D^{(*)0} K^{(*)}$ ADS and Belle's Dalitz measurements.

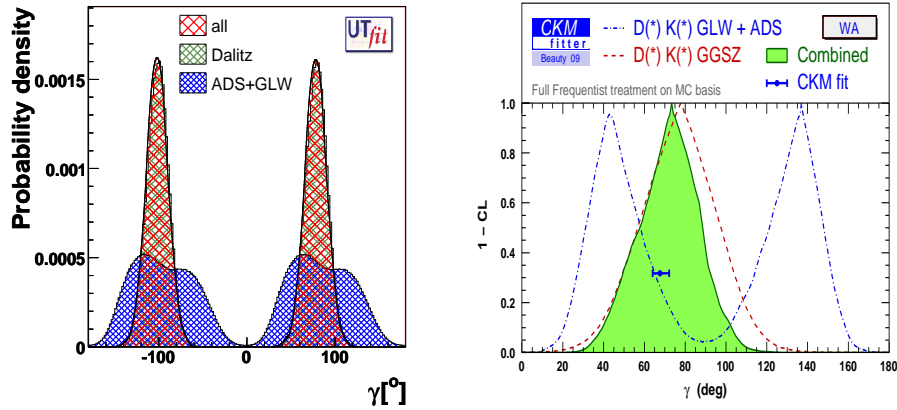


Figure 5: Left: probability density function of γ from the combination of BaBar and Belle measurements of $B^\pm \rightarrow D^{(*)0}K^{(*)\pm}$ decays as obtained by UTfit [32]. Right: 1-confidence-level as derived by CKMfitter [33].

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