$B \to K\eta^{(')}$ decays and NLO contributions in the pQCD approach

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We report our calculations of the partial NLO QCD corrections to the $B \to K\eta^{(')}$ decays in the perturbative QCD (pQCD) factorization approach. The NLO contributions can provide a 70% enhancement (a 30% reduction) to the leading order pQCD predictions for the branching ratios of $B \to K\eta^{(')}$ ($B \to K\eta$) decays. Such NLO contributions play the key role in understanding the observed pattern of branching ratios. The pQCD predictions for the CP asymmetries of $B \to K\eta^{(')}$ decays are also consistent with currently available data.

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1. Introduction

The unexpectedly large branching ratios for $B \rightarrow K\eta'$ decays were firstly reported in 1997 by CLEO Collaboration [1]. 12 years later, three of the four $B \rightarrow K\eta^{(i)}$ decays have been measured with high precision [2]. Besides the branching ratios, the CP violating asymmetries for $B^\pm \rightarrow K^\pm\eta^{(i)}$ and $B^0 \rightarrow K^0\eta^{(i)}$ decays have been measured very recently [2,3].

In the SM the decay $B \rightarrow K\eta^{(i)}$ is believed to proceed dominantly through gluonic penguin processes[4] and has been evaluated by employing various methods [5] [6] [7] [8] [9] [10] [11] [12] [13] [14] [15]. Although great progress have been made during the past decade, but the predictions for $Br(B \rightarrow K\eta')$ from both the QCD factorization (QCDF) approach [13] [16] and the perturbative QCD (pQCD) approach [15] [17] are still smaller than the data.

Furthermore, there is a large disparity between the measured branching ratios: $Br(B \rightarrow K\eta') \gg Br(B \rightarrow K\eta)$. Many efforts have been made to interpret this pattern, which include, for example,

(a) Conventional $b \rightarrow s g \bar{q}$ with constructive (destructive) interference between the $u \bar{u}, d \bar{d}$ and $s \bar{s}$ components of $\eta'$ ($\eta$) [4];

(b) Large intrinsic charm content of $\eta'$ through the chain $b \rightarrow s c \bar{c} \rightarrow s \eta'$ [6] or through $b \rightarrow s c \bar{c} \rightarrow s g^* g^* \rightarrow s(\eta, \eta')$ due to the QCD anomaly [7];

(c) The spectator hard-scattering mechanism through the anomalous coupling of $gg \rightarrow \eta'$ [8,9,10];

(d) A significant flavor-singlet contribution [9,13];

(e) A strong penguin $b \rightarrow s g$ enhanced by new physics [11,12].

In Ref. [15], the authors calculated the branching ratios of $B \rightarrow K\eta^{(i)}$ decays by employing the pQCD approach at leading order. They considered the large corrections from $SU(3)$ flavor symmetry breaking as well as the possible gluonic component of $\eta'$ meson, but their prediction for $Br(B^0 \rightarrow K^0\eta')$ ($Br(B^0 \rightarrow K^0\eta)$ ) is much smaller (larger) than the measured value. A sizable gluonic content in $\eta'$ meson may provide a large enhancement to the decay rate of $B \rightarrow K\eta'$. But the calculation in Ref. [18] showed that such contribution is numerically very small and can be neglected safely.

Besides the possible mechanisms mentioned above, we here consider a new and natural solution: the effects of the next-to-leading order (NLO) contributions in the pQCD approach. The NLO contributions considered here include: QCD vertex corrections, the quark-loops and the chromo-magnetic penguins. We expect that they are the major part of the full NLO contributions in pQCD approach [19].

In the pQCD approach, the decay amplitude is separated into soft ($\Phi_{M_i}$), hard ($H(k_i,t)$), and harder ($C(M_W)$) dynamics characterized by different energy scales ($\Lambda_{QCD}, m_b, M_W$) [17]. The decay amplitude $\mathcal{A}(B \rightarrow M_2 M_3)$ can be written conceptually as the convolution,

$$\mathcal{A}(B \rightarrow M_2 M_3) \sim \int d^4k_1 d^4k_2 d^4k_3 \text{Tr}[C(t) \Phi_{B}(k_1) \Phi_{M_2}(k_2) \Phi_{M_3}(k_3) H(k_1,k_2,k_3,t)]$$

(1.1)

where $k_i$’s are momenta of light quarks included in each meson, and $\text{Tr}$ denotes the trace over Dirac and color indices. $C(t)$ is the Wilson coefficient evaluated at scale $t$. The hard kernel $H(k_1,k_2,k_3,t)$
describes the hard dynamics, and therefore can be perturbatively calculated. The function $\Phi_{M_i}$ is the wave function.

Since the $b$ quark inside the $B$ meson is rather heavy, we consider the $B$ meson at rest for simplicity. It is then convenient to use light-cone coordinate $(p^+, p^-, \mathbf{p}_T)$ to describe the meson’s momenta: $p^\pm = (p^0 \pm p^3)/\sqrt{2}$ and $\mathbf{p}_T = (p^1, p^2)$.

For the studied $B \rightarrow K \eta^{(i)}$ decays, the weak effective Hamiltonian $H_{\text{eff}}$ for $b \rightarrow s$ transition can be written as \cite{20}

$$H_{\text{eff}} = \frac{G_F}{\sqrt{2}} \sum_{q=u,c} V_{qb} V_{qs}^* \left\{ [C_1(\mu)O_1^q(\mu) + C_2(\mu)O_2^q(\mu)] + \sum_{i=3}^{10} C_i(\mu) O_i(\mu) \right\}.$$  \hspace{1cm} (1.2)

where $G_F = 1.16639 \times 10^{-5}$ GeV$^{-2}$ is the Fermi constant, and $V_{ij}$ is the CKM matrix element. $C_i(\mu)$ are the Wilson coefficients evaluated at the renormalization scale $\mu$ and $O_i(\mu)$ are the four-fermion operators.

In PQCD approach, the energy scale “$\tau$” is chosen as the largest energy scale in the hard kernel $H(x_i, b_i, t)$ of a given Feynman diagram, in order to suppress the higher order corrections and improve the reliability of the perturbative calculation. Here, the scale “$\tau$” may be larger or smaller than the $m_b$ scale. In the range of $t < m_b$ or $t > m_b$, the number of active quarks is $N_f = 4$ or $N_f = 5$, respectively. The explicit expressions of the LO and NLO $C_i(m_W)$ can be found easily, for example, in Refs. \cite{21,20}. For the expressions of the wave functions of $B$ meson and the relevant distribution functions of the $K$ and $(\eta, \eta')$ mesons, one can see Ref. \cite{21,22}. The Gegenbauer moments are the following \cite{23}:

$$a^K_1 = 0.2, \quad a^K_2 = 0.25, \quad a^K_4 = -0.015.$$  \hspace{1cm} (1.3)

The values of other parameters are $\eta_3 = 0.015$ and $\omega = -3.0$.

For the mixing of the $\eta - \eta'$ system, we use the the quark-flavor mixing scheme, where the physical states $\eta$ and $\eta'$ are related to the flavor states $\eta_{\mu} = (u\bar{u} + d\bar{d})/\sqrt{2}$ and $\eta_s = s\bar{s}$ through a single mixing angle $\phi$, \hspace{1cm} (1.4)

$$\begin{pmatrix} \eta \\ \eta' \end{pmatrix} = \begin{pmatrix} \cos \phi & -\sin \phi \\ \sin \phi & \cos \phi \end{pmatrix} \begin{pmatrix} \eta_{\mu} \\ \eta_s \end{pmatrix} = \begin{pmatrix} F_1(\phi)(u\bar{u} + d\bar{d}) + F_2(\phi) s\bar{s} \\ F_1'(\phi)(u\bar{u} + d\bar{d}) + F_2'(\phi) s\bar{s} \end{pmatrix}$$

with $F_1(\phi) = \cos \phi/\sqrt{2}$, $F_2(\phi) = -\sin \phi$, $F_1'(\phi) = \sin \phi/\sqrt{2}$ and $F_2'(\phi) = \cos \phi$. The distribution amplitudes $\phi_{\eta}^{A,P,T}$ represent the axial vector, pseudoscalar and tensor component of the wave function respectively \cite{23}, and can be found in Ref.\cite{22}.

2. Decay amplitudes at leading order

At the leading order in pQCD approach, the Feynman diagrams as shown in Fig.1 may contribute to $B \rightarrow K \eta^{(i)}$ decays. From the factorizable emission diagrams 1(a) and 1(b), the corresponding form factors can be extracted by perturbative calculation. For Fig.1(a) and 1(b) with the $B \rightarrow K$ transition, the operators $O_{1,2,3,4}$ and $O_{9,10}$ are $(V - A)(V - A)$ currents, the sum of the
\[ F_{eK} = \frac{8}{\sqrt{2}} \pi G_F C_F m_B^3 \int_0^1 dx_1 dx_2 \int_0^1 b_1 d b_1 b_2 d b_2 \phi_B(x_1, b_1) \]
\[ \times \left\{ \left[ (1 + x_2) \phi_K^a(x_2) + (1 - 2 x_2) r_K (\phi_K^p(x_2) - \phi_K^p(x_2)) \right] \cdot E_e(t_a) h_e(x_1, x_2, b_1, b_2) \right. \]
\[ \left. + 2 r_K \phi_K^p(x_2) \cdot E_e(t'_a) h_e(x_2, x_1, b_2, b_1) \right\} , \quad (2.1) \]

where \( r_K = m_K^2 / m_B \) with \( m_K^2 \) is the chiral scale; \( C_F = 4 / 3 \) is a color factor, and \( x_2 = 1 - x_2 \). The evolution function \( E_e(t) \) and hard function \( h_e \) can be found in Ref. [22]. Also from diagrams 1(a) and 1(b), the decay amplitudes corresponding to the \((V - A)(V + A)\) and/or \((S - P)(S + P)\) currents are the following

\[ F_{eK}^{p_1} = - F_{eK} , \quad (2.2) \]
\[ F_{eK}^{p_2} = \frac{16}{\sqrt{2}} \pi G_F C_F m_B^3 \int_0^1 dx_1 dx_2 \int_0^1 b_1 d b_1 b_2 d b_2 \phi_B(x_1) \]
\[ \times \left\{ r_K \left[ \phi_K^a(x_2) + r_K (2 + x_2) \phi_K^p(x_2) + x_2 \phi_K^p(x_2) \right] \cdot E_e(t_a) h_e(x_1, x_2, b_1, b_2) \right. \]
\[ \left. + 2 r_K r_e \phi_K^p(x_2) \cdot E_e(t'_a) h_e(x_2, x_1, b_2, b_1) \right\} . \quad (2.3) \]

From Fig. 1, one can find the corresponding decay amplitudes: \((M_{eK}, M_{eK}^{p_1, p_2})\) (1(c) and 1(d)), \((M_{aK}, M_{aK}^{p_1, p_2})\) (1(e) and 1(f)) and \((F_{aK}, F_{aK}^{p_1, p_2})\) (1(g) and 1(h)) [22]. By exchanging position of the \( K \) and \( \eta^{(l)} \) in Fig. 1, one can find the corresponding decay amplitudes for the new diagrams easily [22]: \((F_{e\eta}, F_{e\eta}^{p_1, p_2})\), \((M_{e\eta}, M_{e\eta}^{p_1, p_2})\), \((M_{a\eta}, M_{a\eta}^{p_1, p_2})\), and \((F_{a\eta}, F_{a\eta}^{p_1, p_2})\).

For the two \( B \to K \eta \) decays, the total decay amplitude with the inclusion of the corresponding Wilson coefficients can be finally written as

\[ \mathcal{M}(K^0 \eta) = \langle K^0 \eta | H_{eff} | B^0 \rangle = F_{eK} \left\{ \left[ \xi a - \xi \left( 2a_3 - 2a_5 - \frac{1}{2} a_7 + \frac{1}{2} a_9 \right) \right] f_{\eta}^q \right. \]
\[ - \xi \left( a_3 + a_4 - a_5 + \frac{1}{2} a_7 - \frac{1}{2} a_9 - \frac{1}{2} a_{10} \right) f_{\eta}^s \right\} - F_{e\eta} \xi \left( a_4 - \frac{1}{2} a_{10} \right) f_K F_1 (\phi) \]
\[ - \left[ F_{eK}^{p_1} f_{\eta}^s + F_{eK}^{p_2} f_{\eta} F_1 (\phi) \right] \xi \left( a_6 - \frac{1}{2} a_8 \right) - \left[ F_{aK}^{p_1} f_{\eta} + F_{aK}^{p_2} f_{\eta} F_1 (\phi) \right] \xi \left( a_4 - \frac{1}{2} a_{10} \right) \]
\[ + \left[ F_{aK}^{p_1} f_{\eta} F_2 (\phi) + F_{a\eta}^{p_2} F_1 (\phi) \right] \xi \left( a_6 - \frac{1}{2} a_8 \right) f_{\eta} \]

\[ \xi \left( a_7 - \frac{1}{2} a_9 \right) f_{\eta} \]
The NLO hard kernel

The NLO Wilson coefficients

\[ M_{nK} \left\{ \left[ \tilde{\xi}_u C_2 - \tilde{\xi}_t \left( 2C_4 + \frac{1}{2}C_{10} \right) \right] F_1(\phi) - \tilde{\xi}_t \left( C_3 + C_4 - \frac{1}{2}C_9 - \frac{1}{2}C_{10} \right) F_2(\phi) \right\} \]

\[ -M_{\eta K} \tilde{\xi}_t \left( C_3 - \frac{1}{2}C_9 \right) F_1(\phi) - [M_{\eta K}^{P1}] F_2(\phi) + M_{\eta K}^{P1} F_1(\phi) \] \[ -M_{\eta K}^{P1} \xi_t \left( 2C_6 + \frac{1}{2}C_8 \right) F_1(\phi) + (C_6 - \frac{1}{2}C_8) F_2(\phi) \] \[ (2.4) \]

\[ \mathcal{M}(K^+ \eta) = \langle K^+ \eta | H_{\text{eff}} | B^0 \rangle = F_{\text{eff}} \left\{ \left[ \tilde{\xi}_u a_2 - \tilde{\xi}_t \left( 2a_3 - 2a_5 - \frac{1}{2}a_7 - \frac{1}{2}a_9 - \frac{1}{2}a_{10} \right) \right] F_{\eta}^d \right\} \]

\[ -\tilde{\xi}_t \left( a_3 + a_4 - a_5 + \frac{1}{2}a_7 - \frac{1}{2}a_9 - \frac{1}{2}a_{10} \right) F_{\eta}^d + \{ F_{\eta} F_1(\phi) F_K + [F_{\eta} F_1(\phi) + F_{\eta} F_2(\phi)] F_{\phi} \} \tilde{\xi}_t (a_4 + a_{10}) \]

\[ - \left[ F_{\eta}^{P1} F_1(\phi) F_K + (F_{\eta}^{P2} F_1(\phi) + F_{\eta}^{P2} F_2(\phi)) F_{\phi} \right] \tilde{\xi}_t (a_6 + a_8) - F_{\eta}^{P2} F_{\phi} F_{\phi} \tilde{\xi}_t (a_6 - \frac{1}{2}a_8) \]

\[ -M_{\eta K}^{P1} \xi_t \left( C_5 - \frac{1}{2}C_7 \right) + M_{\text{eff}} \left\{ \left[ \tilde{\xi}_u C_2 - \tilde{\xi}_t \left( 2C_4 + \frac{1}{2}C_{10} \right) \right] F_1(\phi) \right\} \]

\[ -\tilde{\xi}_t \left( C_3 + C_4 - \frac{1}{2}C_9 - \frac{1}{2}C_{10} \right) F_2(\phi) \right\} - [M_{\eta K}^{P1} F_2(\phi) + (M_{\eta K}^{P1} + M_{\eta K}^{P1}) F_1(\phi) \]

\[ \times \tilde{\xi}_t (C_5 + C_7) + [M_{\eta K} F_2(\phi) + (M_{\eta K} + M_{\eta K}) F_1(\phi)] \left[ \tilde{\xi}_u C_1 - \tilde{\xi}_t (C_3 + C_9) \right] \]

\[ -M_{\eta K}^{P1} \xi_t \left( 2C_6 + \frac{1}{2}C_8 \right) F_1(\phi) + \left( C_6 - \frac{1}{2}C_8 \right) F_2(\phi) \] \[ (2.5) \]

where \( \xi_u = V_{ub}^* V_{us} \), \( \xi_t = V_{tb}^* V_{ts} \), the coefficients \( a_i \) are the combinations of the Wilson coefficients \( C_i \), and have been defined as usual

\[ a_{1,2} = C_{2,1} + \frac{C_{1,2}}{3}; \quad a_i = C_i + \frac{C_{i+1}}{3}, i = 3, 5, 7, 9; \quad a_i = C_i + \frac{C_{i-1}}{3}, i = 4, 6, 8, 10. \] \[ (2.6) \]

The total decay amplitudes for the two \( B \to K \eta' \) decays can be obtained easily from Eqs. (2.4) and (2.5) by the following replacements

\[ f_{\eta}^d \to f_{\eta'}^d, \quad f_{\eta}^s \to f_{\eta'}^s, \quad F_1(\phi) \to F_1'(\phi), \quad F_2(\phi) \to F_2'(\phi) \] \[ (2.7) \]

3. NLO contributions in pQCD approach

At the NLO level, the following changes or contributions should be considered:

- The NLO Wilson coefficients \( C_i(m_W) \), the NLO RG evolution matrix and the \( \alpha_s(t) \) at two-loop level should be used.
- The NLO hard kernel \( H^{(1)}(\alpha_s^2) \) should be included. All the Feynman diagrams, which may contribute to the hard kernel \( H \) at the order of \( \alpha_s^2 \), as illustrated by Figs. 2-5, should be considered.

At present, the calculations for the vertex corrections, the quark-loops and chromo-magnetic penguins, in relevant with \( B \to K \eta'(\phi) \) decays as shown in Fig. 2, have been done in Ref. [22]. For
The Feynman diagrams as shown in Figs. 3-5, however, the analytical calculations have not been completed yet.

The vertex corrections to the factorizable emission diagrams, as illustrated by Figs. 2a-2d, have been calculated years ago in the QCD factorization approach[16, 13, 14]. The difference of the calculations induced by considering or not considering the parton transverse momentum is rather small [19], say less than 10%, and therefore can be neglected. The vertex corrections can then be absorbed into the re-definition of the Wilson coefficients $a_i(\mu)$ by adding a vertex-function $V_i(M)$ to them[14, 19].

The contribution from the so-called “quark-loops” is a kind of penguin correction with the four quark operators insertion, as illustrated by Fig. 2e-2f. For the $b \to s$ transition, the contributions from the various quark loops are given by:

$$H^{(qf)}_{\text{eff}} = - \sum_{q = u, c, t} \sum_{q' f} \frac{G_F}{\sqrt{2}} V_{qb} V_{qs}^* \frac{\alpha_s(\mu)}{2\pi} C^q(\mu, \Gamma) \left( \bar{s} \gamma_\rho (1 - \gamma_5) T^a b \right) \left( \bar{q'} \gamma_\rho T^a q' \right),$$

(3.1)
It is straightforward to calculate the decay amplitude for Fig. 2e and 2f. We found two kinds of topological decay amplitudes \([22]\): \(M_{K\eta}^{(q)}\) for \(B \to K\) transition and \(M_{\eta K}^{(q)}\) for \(B \to \eta\) transition. For \(B \to K\eta\) decays, we found the similar decay amplitudes. Finally, the total “quark-loop” contribution to the considered \(B \to K\eta^{(l)}\) (\(K = K^0, K^+\)) decays can be written as \([22]\)

\[
M_{K\eta}^{(q)} = \langle K\eta | \mathcal{H}_{eff}^{(q)} | B \rangle = \frac{G_F}{\sqrt{2}} \sum_{q=u,c,d} \lambda_q \left[ M_{K\eta}^{(q)} F_2(\phi) + M_{\eta K}^{(q)} F_1(\phi) \right],
\]

(3.2)

\[
M_{\eta K}^{(q)} = \langle K\eta | \mathcal{H}_{eff}^{(q)} | B \rangle = \frac{G_F}{\sqrt{2}} \sum_{q=u,c,d} \lambda_q \left[ M_{K\eta}^{(q)} F_2(\phi) + M_{\eta K}^{(q)} F_1(\phi) \right].
\]

(3.3)

For the magnetic penguins, the corresponding weak effective Hamiltonian contains the \(b \to s g\) transition,

\[
H_{eff}^{cmp} = -\frac{G_F}{\sqrt{2}} V_{tb} V_{ts}^* C_{8g}^{eff} O_{8g},
\]

(3.4)

with the chromo-magnetic penguin operator,

\[
O_{8g} = \frac{g_s}{8\pi} m_b \bar{d}_i \sigma^{\mu\nu} (1 + \gamma_5) T_{ij}^a G^a_{\mu\nu} b_j,
\]

(3.5)

where \(i, j\) being the color indices of quarks. The corresponding effective Wilson coefficient \(C_{8g}^{eff} = C_{8g} + C_5\).

The decay amplitudes, \(M_{K\eta}^{(q)}\) and \(M_{\eta K}^{(q)}\), have been obtained by evaluating the Feynman diagrams Figs. 2g and 2h \([22]\). The total chromo-magnetic penguin contribution to the considered \(B \to K\eta^{(l)}\) (\(K = K^0, K^+\)) decays can be written as

\[
M_{K\eta}^{(cmp)} = \langle K\eta | \mathcal{H}_{eff}^{cmp} | B \rangle = -\frac{G_F}{\sqrt{2}} \lambda_q \left[ M_{K\eta}^{(q)} F_2(\phi) + M_{\eta K}^{(q)} F_1(\phi) \right],
\]

(3.6)

\[
M_{\eta K}^{(cmp)} = \langle K\eta | \mathcal{H}_{eff}^{cmp} | B \rangle = -\frac{G_F}{\sqrt{2}} \lambda_q \left[ M_{K\eta}^{(q)} F_2(\phi) + M_{\eta K}^{(q)} F_1(\phi) \right].
\]

(3.7)

4. Numerical Results and Discussions

We use the following input parameters \([2, 24]\) in the numerical calculations

\[
f_B = 0.21\text{GeV}, \quad f_K = 0.16\text{GeV}, \quad m_\eta = 547.5\text{MeV}, \quad m_{\eta^0} = 957.8\text{MeV},
\]

\[
m_K = 0.49\text{GeV}, \quad m_{K^0} = 1.7\text{GeV}, \quad M_B = 5.279\text{GeV}, m_b = 4.8\text{GeV},
\]

\[
M_W = 80.41\text{GeV}, \quad \tau_{K^0} = 1.527\text{ps}, \quad \tau_{K^+} = 1.643\text{ps}.
\]

(4.1)

For the CKM quark-mixing matrix elements, we use the values as given in Ref.\([2, 24]\):

\[
V_{ud} = 0.9745, \quad V_{us} = \lambda = 0.2200, \quad |V_{ub}| = 4.31 \times 10^{-3}, \quad V_{cd} = -0.224,
\]

\[
V_{cd} = 0.996, \quad V_{cb} = 0.0413, \quad |V_{td}| = 7.4 \times 10^{-3}, \quad V_{ts} = -0.042, \quad V_{tb} = 0.9991,
\]

(4.2)

with the CKM angles \(\beta = 21.6^\circ, \gamma = 60^\circ \pm 20^\circ\) and \(\alpha = 100^\circ \pm 20^\circ\).

Using the wave functions and the input parameters as specified in previous sections, it is straightforward to calculate the CP-averaged branching ratios for the considered four \(B \to K\eta^{(l)}\)


Table 1: The pQCD predictions for the branching ratios (in unit of $10^{-6}$). The label LO$_{\text{NLOWC}}$ means the LO results with the NLO Wilson coefficients, and $+\text{VC}, +\text{QL}, +\text{MP}$, NLO means the inclusion of the vertex corrections, the quark loops, the magnetic penguin, and all the considered NLO corrections, respectively.

<table>
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<tr>
<th>Mode</th>
<th>LO</th>
<th>LO$_{\text{NLOWC}}$</th>
<th>$+\text{VC}$</th>
<th>$+\text{QL}$</th>
<th>$+\text{MP}$</th>
<th>NLO</th>
<th>Data</th>
<th>QCDF</th>
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<td>$B^+ \to K^+ \eta$</td>
<td>4.7</td>
<td>4.7</td>
<td>4.3</td>
<td>4.9</td>
<td>3.1</td>
<td>3.2$^{+1.2}_{-1.8}$</td>
<td>2.6$^{\pm0.6}$</td>
<td>1.9$^{+3.0}_{-1.9}$</td>
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<tr>
<td>$B^+ \to K^+ \eta'$</td>
<td>30.2</td>
<td>46.8</td>
<td>74.6</td>
<td>48.1</td>
<td>30.2</td>
<td>51.0$^{+18.0}_{-10.9}$</td>
<td>70.5$^{\pm3.5}$</td>
<td>49.1$^{+45.2}_{-23.6}$</td>
</tr>
<tr>
<td>$B^0 \to K^0 \eta$</td>
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<td>3.4</td>
<td>3.1</td>
<td>3.8</td>
<td>2.3</td>
<td>2.1$^{+2.6}_{-1.5}$</td>
<td>&lt; 2.0</td>
<td>1.1$^{+2.4}_{-1.5}$</td>
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<td>46.5</td>
<td>69.7</td>
<td>48.5</td>
<td>20.7</td>
<td>50.3$^{+16.8}_{-10.6}$</td>
<td>68$^{\pm4}$</td>
<td>46.5$^{+41.9}_{-22.0}$</td>
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decays, which are listed in Table 1. For comparison, we also list the corresponding updated experimental results [2] and numerical results evaluated in the framework of the QCDF approach [14].

It is worth stressing that the theoretical predictions in the pQCD approach have relatively large theoretical errors induced by the still large uncertainties of many input parameters, such as quark masses ($m_u, m_d, m_s$), chiral scales ($m_0, m_{b}$), Gegenbauer coefficients ($a_{K, \eta}^{(K, \eta')}$, : : : ), $\theta_V$ and the CKM angles ($\alpha, \gamma$), etc. From the numerical results about the branching ratios, one can see that

- The LO pQCD predictions for branching ratios are much smaller (larger) than the measured values for $B \to K \eta'$ ($B \to K \eta$) decays, show the same tendency as found in Ref. [15].
- The NLO contributions can interfere constructively (destructively) with the corresponding LO part for $B \to K \eta'$ ($B \to K \eta$) decays. For $B^0 \to K^0 \eta'$ and $B^+ \to K^+ \eta'$ decays, the NLO contributions provide a 70% enhancement to their branching ratios. For $B^0 \to K^0 \eta$ and $B^+ \to K^+ \eta$ decays, on the other hand, the NLO contributions give rise to a 30% reduction to their branching ratios and result in the good agreement between the pQCD predictions and the data.
- The NLO pQCD predictions for branching ratios $Br(B \to K \eta^{(K)})$ agree very well with the measured values within one standard deviation. The NLO contributions play an important role in understanding the observed pattern of branching ratios of the four $B \to K \eta^{(K)}$ decays.

It is easy to calculate the direct CP-violating asymmetries for the considered decays, which are listed in Table 2. As a comparison, we also list currently available data [2] and the corresponding QCDF predictions [14]. As to the CP-violating asymmetries for the neutral decays $B^0 \to K^0 \eta^{(K)}$, one can see the numerical results and discussions as given in Ref. [22].

In short, we calculated the branching ratios and CP-violating asymmetries of $B^+ \to K^+ \eta^{(K)}$ and $B^0 \to K^0 \eta^{(K)}$ decays in the pQCD approach. The partial NLO contributions considered here include: QCD vertex corrections, the quark-loops and the chromo-magnetic penguins.

From our calculations and phenomenological analysis, we found the following results:

(a) The NLO contributions in the pQCD approach can provide a 70% enhancement to $Br(B \to K \eta')$, but a 30% reduction to $Br(B \to K \eta)$. The large branching ratio of $B \to K \eta'$ decays, as well as the large disparity $Br(B \to K \eta') \gg Br(B \to K \eta)$ can therefore be understood naturally.


Table 2: The pQCD predictions for the direct CP asymmetries in the NDR scheme (in units of $10^{-2}$), the QCDF predictions [14] and the world average as given by HFAG [2].

<table>
<thead>
<tr>
<th>Mode</th>
<th>LO</th>
<th>+VC</th>
<th>+QL</th>
<th>+MP</th>
<th>NLO</th>
<th>Data</th>
<th>QCDF</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha_{CP}^{dir}(B^\pm \rightarrow K^\pm \eta')$</td>
<td>9.3</td>
<td>31.1</td>
<td>7.8</td>
<td>7.6</td>
<td>11.7</td>
<td>$-27 \pm 9$</td>
<td>$-18.9^{+29.0}_{-30.0}$</td>
</tr>
<tr>
<td>$\alpha_{CP}^{dir}(B^\pm \rightarrow K^\pm \eta)$</td>
<td>-10.1</td>
<td>-10.6</td>
<td>-5.9</td>
<td>-10.4</td>
<td>6.2</td>
<td>$1.6 \pm 1.9$</td>
<td>$-9.0^{+10.6}_{-16.2}$</td>
</tr>
</tbody>
</table>

(b) The pQCD predictions for the CP asymmetries of $B \rightarrow K\eta^{(i)}$ decays are consistent with currently available data.

(c) One should note that Only the partial NLO contributions in the pQCD approach have been taken into account here. To achieve a complete NLO calculations in the pQCD approach, of course, the still missing pieces should be evaluated as soon as possible.

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References

$B \rightarrow K\eta^{(')}$ decays in the pQCD Approach

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